PREFACE.

THE only excuse that can be offered for adding another book to the already full catalogue of Electrical literature, especially under the heading of Electrical Measurements, is that, in spite of the numerous excellent works on this subject, there seemed room for one dealing more particularly with the requirements of electrical engineers as distinguished from electrical physicists. Several valuable handbooks exist dealing with the subject of Electrical Testing from the point of view of the telegraphist or physicist, but these generally contain either too much or too little for the purposes of those who have to deal with the class of electrical measurements which it is necessary to make in electrical stations or factories.

For this latter purpose, what is required is not a multiplication of methods gathered together without regard to their applicability or accuracy, but a selection of approved and well-tried methods.

In many text-books on Electrical Measurements there is an absence of critical discussion on the intrinsic utility of the various methods of measurement given. A process which looks well on paper does not always work out well in practice, and the practical engineer, therefore, requires to have placed before him a series of selected methods of measurement rather than a collection made as inclusive as possible. The present treatise has therefore been divided into a series of chapters, each of which deals with one particular class of measurements. No attempt, however, has been made to include a description of all the methods under that particular heading, far less of all the implements or instruments, but certain processes which experience has

shown to give good results are described as fully as possible, and a detailed description given of certain typical forms of widely used instruments.

In the case of electrical instruments there is a process of evolution and a survival of the fittest. Many ingenious or otherwise interesting instruments for some particular reason drop out of existence, whilst other forms survive, and it is to these surviving forms that attention has been most directed.

In the First Volume, in addition to a chapter on the Equipment of electrical laboratories, the subject of the measurement of Electrical Resistance, Electric Current, Electromotive Force and Electric Power is dealt with in the remaining chapters.

The Second Volume is devoted to the measurement of Capacity and Inductance, Electric Quantity and Energy, including Battery and Meter testing, the Magnetic testing of iron, Photometric and Electric lamp testing, and the testing of Dynamos, Motors and Transformers.

Each chapter is as far as possible complete in itself, and where tables or numerical data are given they are placed at the end of the chapter to which they belong, and not, as usual, at the end of the book.

As the object has been to place in the hands of the reader a practical handbook rather than a theoretical treatise, such brief mathematical discussions as are introduced have generally been placed in smaller type so that the non-mathematical reader may leave them out of consideration ; and as the desire has been to produce a handbook useful in the test rooms of electrical factories and stations, descriptions of instruments more usually found in a physical laboratory than a testing room have been omitted.

J. A. F.

University College, London,
 August, 1901.

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HANDBOOK

FOR THE

ELECTRICAL TESTING-ROOM AND LABORATORY.

CHAPTER I.

THE EQUIPMENT OF AN ELECTRICAL TESTING ROOM.

§ 1. The Equipment of an Electrical Testing Room or Laboratory.—In the following pages the class of electrical measurements we shall chiefly consider are those required in the engineering applications of electricity and magnetism as far as regards that range of work included in the so-called heavy electrical engineering. We shall omit all references to measurements and tests particularly limited to Telegraphic and Telephonic work, as on this part of the subject several excellent text-books already exist.

In the establishment of an electrical testing laboratory the electrician will not often be called upon to design the structure or enjoy the advantages of a building erected especially to meet his own views. Usually he will have to adapt or utilise for the purpose some existing rooms in a factory, electric supply station, college, or technical institution. If, however, the opportunity presents itself of being able to begin by designing the laboratory buildings to be occupied, a great advantage is gained at the outset. If,

moreover, the space available can be arranged in the most convenient and suitable way, and if initial outlay is not an obstacle to the possession of the most desirable arrangements, the following conditions may be realised. We shall assume that, in the testing room or electrical laboratory being arranged, at least one or more large and convenient rooms are available for the purposes of a general testing laboratory, and that, in addition, a dynamo room and also an accumulator room are provided. If the laboratory is in a technical teaching institution, an apparatus room, at least one, and preferably several, private rooms, as well as a lecture and preparation room, will also be required. It is a great convenience to have, if possible, all these rooms located on one level. Time and labour are economised in the day's work, and it is easier to take heavy apparatus backwards and forwards.

At one end of each electrical laboratory a partition should be run up separating off a space at least 5ft. wide and 30ft. long to form a photometric gallery. Then with regard to the special arrangement and equipment of each apartment, these should be as follows:—

Dynamo Room.—Even although current is obtainable from public electric supply circuits or from factory lighting dynamos, it will generally be necessary to provide special means for generating the testing currents. Many tests are impossible unless the electric pressure is exceedingly steady, and ordinary public electric supply or that from a gas engine and dynamo is generally quite useless for the purposes of incandescent lamp tests, transformer tests, and numerous other purposes. Hence provision has to be made in the first place for continuous current supply. This should be taken from secondary batteries which are regularly charged by a dynamo set apart for the purpose. It is better, if possible, to charge by a steam engine than a gas engine, as the charging current is then more uniform and the cell plates are less rapidly deteriorated. If continuous current is supplied from

public electric supply circuits at 100 or 200 volts it may be used to run motor-generators or " boosters " by which sufficient charging voltage may be obtained for charging storage cells. A booster consists of an electromotor and a dynamo coupled together and bolted on to the same bedplate. The motor takes current at 100 or 200 volts and drives the dynamo. The dynamo armature can pass a current equal to the maximum required for charging the cells, this being drawn from the supply circuits, and it adds to this circuit voltage the additional E.M.F. required for charging a local battery of 53 or 106 cells. Automatic Boosters are now made which keep the charging current constant irrespective of the gradually accumulating back E.M.F. of the cells being charged. In addition to means for providing continuous current at 100 volts, it is necessary to be able to obtain a supply at pressures of 10 or 15 volts or less, and at about 200 volts or more. This is best obtained by having two similar dynamo machines bolted on one bedplate, so that the machines can be coupled with their shafts in one line. One of these machines is a motor, and takes current at 100 or 200 volts from the secondary battery resistances being provided in the armature circuit and fields for regulating the speed; the other coupled machine is a dynamo, and should be provided with three separate armatures, which can be inserted at pleasure, one giving, say, 10 volts at the standard or proper motor speed, one giving, say, 100 volts, and the other 200 or 250 volts. The fields of each machine are best separately excited at 100 volts. A combined motor-generator, consisting of these two coupled machines, each of one or two kilowatts output and running at 1,800 or 2,000 revolutions per minute, is a very convenient appliance.

In the next place, provision must be made for generating alternating currents. This is best done by means of an alternator coupled directly to a continuous current motor. A very convenient arrangement, designed by the Author for use

in the electrical laboratory at University College, London, consists of a pair of alternators of 5-kilowatt capacity, each coupled to a continuous current motor, so as to be driven by it, and provision made by a coupling to join the shafts of both sets in one line. The four machines may be bolted to cne bedplate, and a flanged pulley on the inner end of each shaft has the flange pierced with holes so that the pulleys can be coupled in such positions that the alternating currents from the two alternators are in any desired relative phase. Resistances are provided in the field and armature circuit of all machines, so that the speeds and electromotive forces are under complete control.

This compound machine provides alternating current of single or two-phase kind, and of any required electromotive force up to 200 volts. Machines so coupled should be well bedded down. It is worth while to expend the necessary sum on good foundations to secure perfect steadiness of running and freedom from vibration, as the commutators are thus more easily kept in order. One or both ends of the double alternator shaft can be provided with a curve tracer, to be described later on, by means of which the curves of alternating currents can be taken.

Well insulated cables from the dynamo terminals should be brought along covered chases in the floor, from the dynamo room to the electrical laboratories to double-pole switches and terminals suitably placed. Whilst laying these leads it is a good plan to run a number of pairs of spare cables and wires of different sizes for voltmeter wires, telephone or bell wires, extra circuits, and other purposes, so that the use of temporary cables lying about on the floor may be avoided. Positive and negative cables should be distinguished by being coloured red and black as usual.

In the selection of dynamo plant for purely experimental purposes much must depend on the resources and purpose of the laboratory. As far, however, as regards the generation of current for most purposes in the electrical laboratory, nothing

is so convenient as the coupled motors and continuous or alternating current generators; the motors being worked off a secondary battery. This arrangement leaves nothing to be desired in steadiness of pressure and ease of working. The speed is more easily regulated than in the case of a dynamo driven off counter-shafting with coned pulley speed regulator.

For the dynamo room at University College a combined motor-alternator plant, as above described, was built in 1893, by Messrs. Johnson and Phillips, to the Author's specification and to Mr. Kapp's designs, of which a general description is as follows:—

"The machine consists of four separate machines bolted on to the same bedplate, viz., two Kapp alternators and two continuous-current motors (*see* Figs. 1 and 2). Each alternator is coupled permanently to its own motor, the commutators of the continuous-current machines being on the outside end of the shaft. The shafts of each pair of machines are truly lined, and the inside ends each carry a flange pulley. These pulleys can be coupled together through the flanges by bolts, so as to drive the whole as one machine, or they can be separated for use as two machines. By coupling them together with the armatures in the proper relative position, two-phase currents can be got out of the united machine. Each of the continuous-current motors is a 5-H.P. motor, designed to work at 100 volts. The armature is ring-wound, and there are 216 turns of wire on it, connected to a 72-part commutator. The magnets of the motors are of cast steel, $6\frac{3}{4}$in. in diameter. Each armature can carry 35 or 40 amperes comfortably. The alternator armatures contain an iron core, and are wound over with eight coils, each having 16 turns of wire. The field magnets have eight poles, and corresponding poles are opposite to one another. The alternators, when driven at a speed of 1,250 revolutions, give an electromotive force of 100 volts.

"The four machines are fixed on a cast-iron bedplate 9ft. long and 2ft. wide, which is carried on slide rails in the usual manner. The ends of all the armature and field magnet circuits are brought to terminals fixed in a box on the front of the machine. In the field-magnet circuit of each machine is an appropriate resistance, and in the

armature circuits of the motors there are also resistances for starting the motors. To the shafts of both motors is fixed a hydraulic speed indicator. A small centrifugal pump is
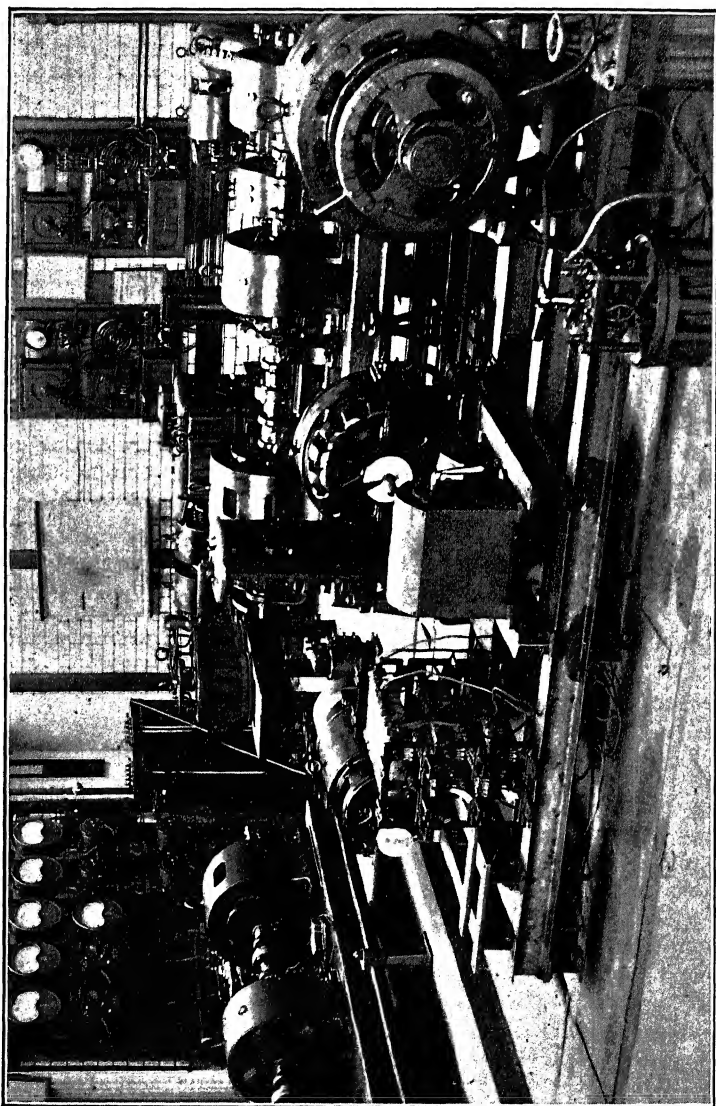
FIG. 1.—View of part of the Dynamo Room in University College, London, showing the Motor-Alternator Plant.
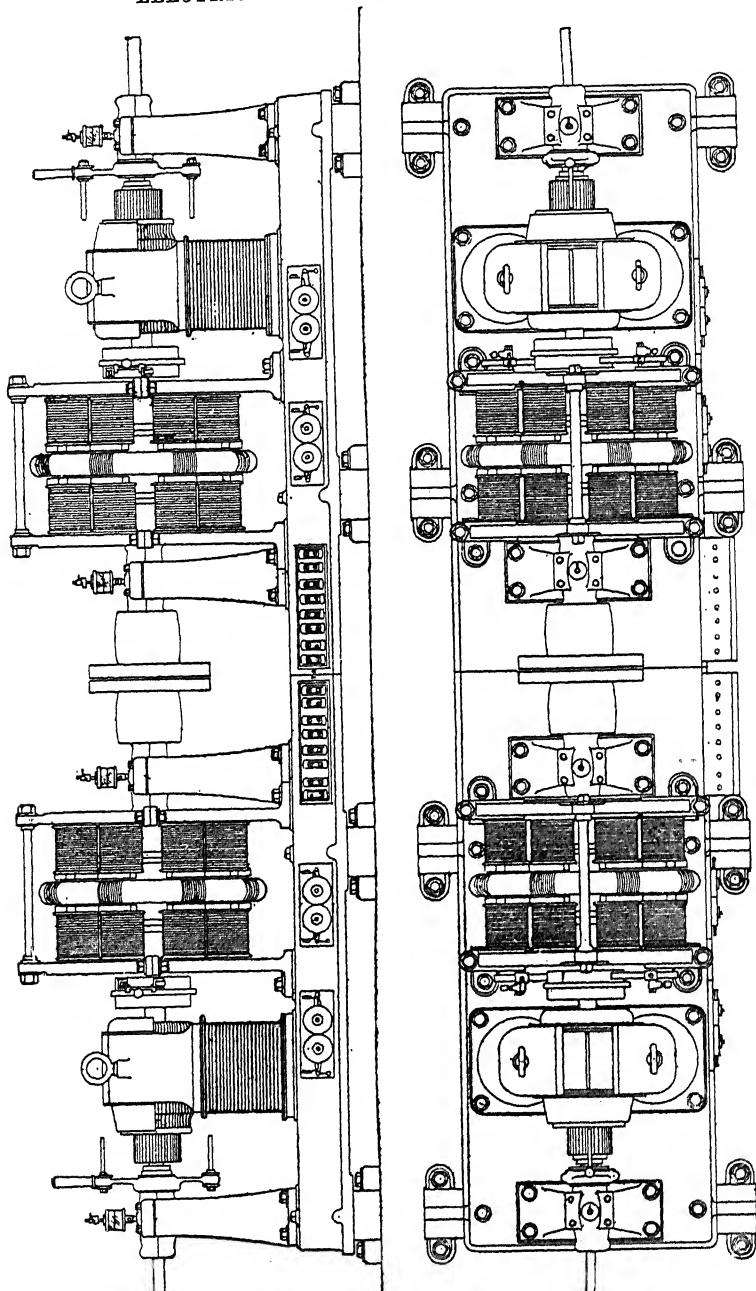
FIG. 2.—Combined Motor-Alternator Plant.

driven by the shaft, and this pump forces coloured water from a small reservoir placed over the pump through a pipe into which are connected two vertical glass tube pressure gauges, one of which is placed on the wall of the dynamo room, and the other is placed on the wall of the electrical laboratory 30ft. away. When the motor alternator is running, the centrifugal pump forces the water up these tubes, until the hydrostatic pressure of the column of liquid supported in the tube balances the pressure due to the pump. The height of the column of liquid, therefore, can be made to measure the speed of the machine after these gauges have been carefully calibrated. The special advantage of this hydraulic speed indicator is that it is so exceedingly responsive to changes in speed. A change in speed of less than one per cent. can be certainly detected and measured, whilst the accuracy of the indication is independent of the density of the liquid used. The speed of the motor can be regulated from the laboratory by the use of a carbon rheostat inserted in the field or armature circuit and can thus be kept exceedingly constant."

The above described motor-alternator plant has many uses The alternators can be run in parallel either coupled or free. They can be coupled and joined in series, so as to give a current at 200 volts pressure. They can be set to give a two-phase current. Efficiency tests can be made with the direct current motors, driven as coupled machines, or with the alternators. After some years' use in the above form it was found more convenient to separate the two motor-alternators above described into two entirely separate plants. The use of the coupled alternators to produce two-phase currents was rendered unnecessary by the possession of other two-phase machines.

Electrical Laboratory.—In the design of the electrical laboratory the important matter is to provide sufficiently steady tables or supports for galvanometers and instruments. It is not a good plan to build up, as is sometimes done, brick tables in the middle of the laboratory, because these cannot afterwards be moved if space is required for special work. It

is best to provide round the room, at a standard height of say 3ft. 6in. from the floor, strong stone or slate slabs, let into the main walls of the building. If the main walls are strongly built, and have good foundations, very steady supports can in this way be obtained. Circumstances, however, must decide what is best to be done. In any case, several firm slabs must be provided, either by building up brick pillars, covered with a slate surface, on independent foundations, thus forming solid tables, or by building up stone supports to the level of, but not touching, the floor, on which can rest the legs of special steady wooden tables for carrying the galvanometers, ampere balances, and other instruments. All that is necessary is to secure steadiness for certain particular pieces of apparatus. In some instances special precautions may have to be taken to eliminate the effects of vibration due to adjacent machinery or traffic. In these cases the galvanometers may be placed on slabs of slate which are suspended at the four corners by stout india-rubber bands from brackets let into the wall, and heavier instruments, such as ampere balances, may be placed on slabs of Yorkshire stone carried on three or four blocks of india-rubber.

One of the greatest difficulties which generally presents itself in arranging an electro-technical laboratory is the contrivance of suitable means to prevent the mirror instruments being disturbed by vibrations of the building due to machinery or traffic in the neighbourhood. The usual method of securing steadiness is to build up brick pillars on very solid foundations formed in the ground, as supports for the legs of ordinary stout deal tables, and to keep the laboratory floor from contact with these pillars. This plan, however, is not always entirely a success, and then sometimes a remedy may be found by placing the galvanometer or mirror instruments, as above suggested, on a slab of slate or flagstone of considerable weight, resting the slab on four india-rubber blocks. Or it may be possible to suspend from the ceiling, by india-rubber door springs, a heavy wooden slab or board on which the instrument is placed, and so take up the vibrations. A combination of the independent brick pillar with capstone resting on india-rubber blocks is the best method in difficult cases. One device, said to be very effective, is to support the galvanometer on a wooden base placed on a thick pad of hair felt laid in a tray, the corners of the base board and tray being connected by stretched india-rubber bands. See *Electrical Review*, 1898, Vol. 42, p. 592.

In the next place, it is convenient to arrange a standard height and size of working table. A convenient size is 5ft. long, 2ft. 6in. wide, and 3ft. 3in. in height. These tables can be arranged together as required. Around the room, however, should be fixed benches or tables with strong top surfaces, having cupboards and drawers underneath, and on which may be arranged certain sets of apparatus never to be moved. It is essential that such standard sets of apparatus as the Bridge for resistances measurements, the Potentiometer for electromotive force measurements, and the Ballistic Galvanometer, should never be moved, but should be kept always connected up and be available at a moment's notice for use. Apparatus stored in glass cases, and therefore not ready when required, is a fruitful source of waste of time and energy, and the plan of so keeping it should be avoided. Dust and light may be kept off the arranged apparatus by simply throwing over the articles a black velveteen cloth, or, better still, keeping every important piece of apparatus in a wooden or cardboard box, which is shut up when the apparatus is not in use. The laboratory should be provided, if possible, with double windows, and these windows be darkened when required by blinds made of black American oilcloth or some material impervious to light. The laboratory can then be kept in the proper state of illumination necessary to see well the spots of light on galvanometer scales. As far as possible an equable temperature should be maintained in the rooms all the year round. A sink with hot and cold water, and a fume cupboard must be provided, and a table, having a top covered with sheet lead, with a narrow fillet or edge round it, and drainage tube, is useful for experiments with primary batteries or secondary cells, and other things likely to be messy. It is a good plan in preparing the room to form covered floor chases around and across the room in which wires and cables can be laid. The permanent leads and cables are placed in casing fixed to the floor of these chases. The cover boards of the chase can be made to take up in sections

so as to lay temporary cables, and thus avoid the danger and nuisance of loose electric cables lying about all over the floor. Round the room should run several separate insulated circuits, having terminals, fuses, and switches at each place where current is likely to be required. One of these should be a circuit from the battery for supplying current to incandescent lamps to be used for galvanometer scale lamps, and for a special table lamp if required.

The incandescent lamps best adapted for galvanometer purposes are small Tungsten lamps with single horse-shoe shaped filament. The lamp bulb should be covered with a cylindrical asbestos hood having a slit in it which permits the light from one leg of the filament loop to pass out. The galvanometer is then arranged so that the mirror, with or without the assistance of a lens, throws an image of this straight incandescent carbon on to a divided ground glass or semi-transparent celluloid scale suitably placed. In this manner an exceedingly sharp, bright line, being the image of a part of the filament is thrown on the divided scale. This image can be seen in daylight or in a slightly darkened room. If the scale is placed at the right focal distance from the mirror, and if the scale is divided into millimetres, a reading to one quarter of a millimetre can be taken, provided the galvanometer mirror is a good one.

This circuit should be provided with numerous wall plugs (preferably of the concentric pattern) by which a lamp with a socket and flexible cord can be plugged in where required. Another circuit should run from the main secondary battery and provide, where required, at several places larger currents at 100 volts pressure. A third circuit should give continuous current at various pressures from the continuous motor generator in the dynamo room, and a fourth circuit should bring alternating currents from the alternators. In laying out these circuits, blank terminals with double-pole porcelain cut-outs and double-pole switches should be provided in as many places as possible. Comfort and convenience in subsequent work will greatly depend on the care with which all these little details are thought out by the electrician in arranging his laboratory. In order to avoid disturbing magnetic needle galvanometers by strong stray magnetic fields, it is desirable to run all these laboratory supply circuits with concentric cable.

Accumulator Room.—This room may be at any convenient and necessary distance from the laboratory working rooms. It should, if possible, have walls built with glazed brick, and a ceiling of glazed tiles. In any case, all exposed wood and metal work and all cables should be thickly painted with anti-sulphuric paint or enamel. The cells should be placed on low brick table supports covered with slate slabs, or on the usual painted wooden supports. In any case, they should be well insulated. Cables joined to various points on the battery should be brought out through the walls of the battery room, to a switchboard outside, by leading them through porcelain tubes plugged with slag-wool, so that they are spray-tight. Changes in the electromotive force of the working circuit can then be made as required without entering the battery room. If possible, a separate battery should be provided for incandescent lamp tests or for any special purposes when great uniformity of pressure is required. The choice and size of cell must be left to the user of the laboratory to determine. One or more large accumulator cells are useful. These can be charged from the low voltage side of the motor-generator set, and are very useful in yielding the large currents required for testing ammeters.

An additional essential is a series of small secondary cells, for giving high electromotive forces. Of these the most convenient are the cells known as Lithanode cells.

Lithanode is not compressed peroxide of lead, as is sometimes stated, for, however strongly lead peroxide may be compressed, the resulting mass will disintegrate when immersed in a liquid electrolyte. It is produced from litharge, made into a pasty mass with a solution of sulphate of ammonia, which causes the material to "set," so that it will no longer disintegrate when placed in a fluid. The "forming," according to the original idea, was performed in a bath of sulphate of magnesia. In ordinary practice the elements are made up of a number of small slabs of lithanode, whose outer edges are V-shaped. These slabs or pellets are arranged in a casting mould of any suitable dimensions, and are placed at such a distance apart and from the edges of the casting frame as to allow of sufficient space for the requisite quantity of metal to run in and impart adequate mechanical strength to the completed element. After the pellets have been arranged

in this manner, an alloy of lead antimony is more suitable for this use, and than a complete plate of lead. Before using as support accumulators are inserted into perforated ends of cells, which are filled in ... plate are simply ... for ... being reduced to a minimum of ... method.

These cells are contained in small glass tubes or cylinders about 1in. in diameter and 4in. high. A series of ... of these cells, contained in a portable box or case, can be charged through an incandescent lamp off a 200 volt circuit. Two or more sets of 50 cells each of these small testing cells are most useful for insulation tests and voltmeter testing. Finally, some sets of single or double cells, having a capacity of about 40 or 50 ampere-hours, are requisite for the potentiometer testing. In a well-organised laboratory there should be regular days when all these cells are recharged by being suitably arranged in series with resistances or lamps, so that the cells are charged with their proper charging current taken from the dynamo circuit. The particular type and size of cells are matters which must be determined by the work to be done. If many tests are likely to be made on complete batteries, it is an advantage to have the accumulator room as near as possible to the electrical laboratory. In any case, it must have well fitting double doors, to prevent the egress of acid vapours. Assuming a certain area available on the ground floor to be arranged as an electrical laboratory, the space might be conveniently divided up as shown in Fig. 3.

The battery room should be separated completely from the engine and dynamo room, and from the private room, but cables should be carried spray tight through the walls, as above suggested, to the regulating switchboard and to the various laboratories.

In each electrical laboratory should be a photometric gallery, forming a completely closed and darkened space of about 30ft. long, 6ft. wide, and 8ft. high. The cables bringing current from the dynamo room to each laboratory should be laid along a channel in the floor of the passage, and

distributed to each room as required.   Every room except the dynamo and battery room should have windows, provided with black opaque cloth blinds, which can be drawn down so as to darken the room more or less as required.   Dimensions are not added in the sketch plan below, because various circumstances must determine the space required, but an arrangement of rooms as here suggested forms a convenient one for a small testing or teaching electrical laboratory.
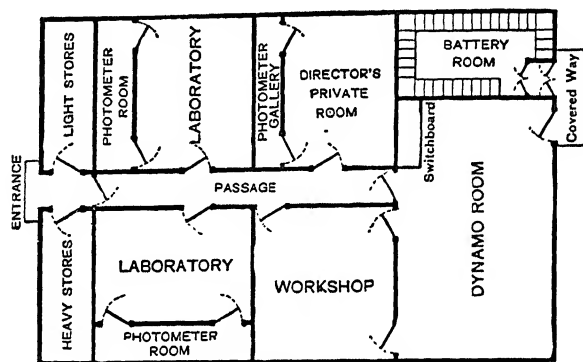
Fig. 3.

As an example of the arrangements of an electrical laboratory, which, though not large, is well equipped, a brief description may here be given of the Pender Electrical Engineering Laboratories in University College, London, the internal arrangements for which were designed by the Author, and embody some thought and experience in this work* :—

"The portion of the buildings allotted for the purpose of an electrical engineering laboratory consists of ten rooms, in all

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* The architect of these buildings was Prof. Roger Smith, F.R.I.B.A., Professor of Architecture in University College, London. A full description of these laboratories, given by the architect and the professors who collaborated with him in their internal design and fittings, is given in the *Journal* of the Royal Institute of British Architects, Vol. I., 3rd series, p. 294, from which, by kind permission of the Council of the Institute, this description of the electrical laboratories, written by the Author, is taken.

of which the interior arrangements have been very carefully designed. The principal rooms open into one another. These are the two dynamo rooms and the general students' working electrical laboratory. The arrangement of the building permitted one dynamo room floor to be placed on solid ground, and thus secured the possibility of making both floor and machine foundations of great steadiness. This dynamo room is 31 ft. long and 22 ft. wide, built in white glazed brick. The plant placed in this room comprises the following machines : There are two combined plants by Messrs. Crompton, each consisting of a bedplate, in the centre of which is a 5 B.H.P. direct-current electric motor running at a speed of 1,500 revs. per min., and taking current at 220 volts as required either from the storage batteries or from the electric mains of the St. Pancras Borough Council which supply the College with electric current. This motor is in each case coupled direct to a generating machine on either side of it by couplings which can be made or broken as required.

" In both cases the motor can be coupled on one side to a polyphase generator of 3 k.v.a. output and the fixed armature coils can be so coupled up as to give either one, two or three-phase currents at a frequency of 50 *p.p.s.*

" By running both sets the two polyphase generators can be run in parallel and other experiments made with them. In the case of both plants the opposite end of the shaft of the driving motor can be coupled at pleasure to a direct current generator of 3 kw. output. In the case of one plant this direct-current generator is a compound wound dynamo giving a current of 30 amperes at 100 volts as a maximum. In the case of the other plant it is a shunt-wound dynamo giving a maximum current of 6 amperes at 500 volts. The two plants, therefore, provide a large number of combinations for teaching work. The fields of all machines are excited by currents controlled by rheostats, and the currents and voltages of the driving motors and of the coupled dynamos can be read on ammeters and voltmeters on the switchboards.

" These switchboards, placed on the walls of the dynamo room, are skeleton structures of angle iron across which are bolted teak beams to which are attached the necessary instruments, field rheostats, starting switches and controllers.

" The boards are so arranged that the motors can be run

either with current taken from St. Pancras supply or from the local storage batteries.

"These batteries, placed in a battery room beneath one of the dynamo rooms consist of four sets of E.P.S. cells having about 200 ampere-hours' capacity. The necessary charging voltage and current are obtained from the St. Pancras Supply by the aid of two boosters, made by the Lancashire Dynamo & Motor Co. These boosters consist of a motor taking current at 210 to 240 volts and running at 1,200 revs. per min. The generator portion or dynamo which is coupled to this motor is of 2·5 kw. capacity, and can pass 24 amperes through its armature and boost the voltage from 0 to 100 volts as regulated. The current from the St. Pancras supply at 220 volts passes through the generator armature into the storage cells, and the booster fields are so wound that the charging current into the cells is kept constant, or nearly so, at about 24 amperes. The booster, therefore, adds driving E.M.F. as the counter E.M.F. of the cells builds up. There are two of these boosters which are worked alternately so as to charge up in about 8 hours the two batteries, each composed of 108 cells. These cells provide the steady voltage for photometric and transformer tests.

"The same dynamo room contains a high frequency alternator of 3 k.v.a. output, which supplies current at 1,000 frequency. This machine is built on the same bedplate with a direct-current driving motor of 2·7 B.H.P., which takes current at 220 volts and runs up to 2,000 revs. per min. The alternator has a desk armature with zigzag winding laid in slots. The field magnet has 60 poles placed radially on the inside of a ring-shaped frame. These magnets are excited at 220 volts. The alternator yields a maximum current of 30 amperes at 100 volts. The E.M.F. curve is not quite sine form, but has a somewhat pronounced third and fifth harmonic. This, however, has proved to be an advantage. By means of suitable resonator circuits it is possible to filter out and separate these harmonics and to obtain currents of nearly perfect sine form having frequencies of 1,000, 3,000 and 5,000 respectively. This motor alternator has proved itself to be of great value in investigations on telephony and wireless telephony.

"There are also in the same dynamo room two other machines of considerable interest. One is a single-phase commutator

alternating-current motor of 4·5 B.H.P., which can be varied to be either a single series motor capable of running on direct current or alternating current, or else a simple repulsion motor. Also it can be changed to be a compensated series or repulsion motor.

" It is provided with a flywheel or pulley with hollow water-cooled rim, and a large variety of experimental exercises can be carried out with it by the students. It takes current at 25 frequency and 150 volts. Another interesting and useful machine in the same room is a C.M.B. converter, built by Messrs. Crompton.

" This machine is a motor-generator having a doubly-wound armature with two commutators. On one side it takes in current at constant voltage of 220 volts from the supply circuits of the laboratory. On the other side it gives out a constant current up to 50 amperes, which can be used for instance, to work large arc lamps or small electric furnaces, without wasting energy in external resistance.

" The floor of this dynamo room consists of concrete 24 in. in thickness. The concrete is finished 2 in. from the wall all round the room, and slag wool packed into the interspace. On the concrete are laid, 18 in. apart, teak beams 6 in. deep by 4 in. wide. These beams are held down by 24 in. holding-down bolts, which pass right through the concrete, and are terminated in anchor plates at the bottom. The space between the beams is filled in with granolithic cement. The cement is cupped out between the beams, and given a slight cant towards a main drain running down the room formed in the cement. By this means oil or water spilt on the floor is easily got rid of and the floor kept dry. The floor has proved itself to be so satisfactory that no sensible vibration is propagated up the building when the motors and dynamos are at work ; and delicate electrical instruments can be used in the room when the machines are in operation.

" Adjacent to the above-described dynamo room lies a room, formerly used as a lecture room, but since converted to a dynamo room by putting down a new floor on steel joists.

" In this latter room are now placed the pair of motor-driven alternators built by Messrs. Johnson & Phillips, described on p. 5. The bedplates have, however, now been divided, and

each coupled pair is used separately. In addition to this there is on one bedplate a pair of coupled variable speed dynamos, by Crompton, which can be run at speeds varying from 600 to 1,200 revs. per min., when either machine is made the motor. The machine which functions as a motor takes current at 220 volts, and is direct coupled to the machine which acts as a dynamo. This latter machine has a pair of slip-rings on its shaft, connected to two opposite points on the armature winding, so that from this alternating current at a frequency of 25 *p.p.s.* can be taken. There is also an exploring coil on the armature, the ends of which are joined to another pair of slip-rings on the shaft. This set has a capacity of about 5 kw. It is mostly used for differential efficiency tests and for supplying the low-frequency alternating current required to drive the single-phase alternating-current motor in the adjacent room. In the same dynamo room there is also a similar but smaller coupled motor-generator set, which takes current at 220 volts, and gives a current up to 100 amperes at 20 volts or less. This set is chiefly used for charging a special battery of eight storage cells, which in turn are used to supply current at 16 volts to induction coils.

" In the same room is placed another rotary converter, which provides one, two, or three phase alternating currents.

" It consists of a dynamo with a gramme ring armature. From six points on the armature winding connections are made to six slip-rings on the shaft, and from these rings by means of brush contacts alternating current can be drawn off, which is, as required, a single, two or three-phase current.

" In addition to these various generators, the laboratory is well provided with motors, both direct and alternating, for test work. There are in the first place several polyphase induction motors, with squirrel-cage rotor, for two and three-phase currents. A three-phase Westinghouse induction motor is provided with a wound rotor, having on it also a search coil for various experiments. Also a variable speed induction motor, the number of stator poles being variable from four to eight, together with a controller for pole-changing. Both the last machines are three-phase, working at 110 volts and 50 *p.p.s.*

" There are several direct-current motors which can be used

for various experiments. Although none of these machines are very large, yet they are of sufficient size for useful investigations to be made on them.

" In those cases in which work has to be put upon a motor a rope brake is used, the pulley generally being hollow and water circulated through it. There is also one magnetic brake for experimental purposes.

" The electrical laboratory opens out of the dynamo room. This is a room 50ft. long and 32ft. wide. In order to secure quietness and to keep out the dust, the room has been built with double windows, the outer ones being ordinary sashes, and the inner ones French windows. Black blinds are provided, so as to darken each window when required. All round the room a series of stout stone slabs are let into the wall at a height of 4ft. above the floor, between the windows. These stone slabs are intended as steady tables to carry various measuring instruments. It is found by experience that it is better to arrange the steady tables in this way than to build them up as brick pedestals through the floor of the room, because advantage is then taken of the greater steadiness of the footings of the main walls, and the central portion of the room is kept clear. Across and around the floor of the laboratory chases are left, covered in by floor boards, in which electrical mains are carried, and these terminate in lock-up switch and fuse boxes in various parts of the room. At one end of the laboratory is a long photometric gallery, 30ft. long, 10ft. high, and 6ft. wide. In this gallery is placed the photometer and various apparatus required in testing arc and incandescent lamps. One end of this gallery is formed into a small dark room for photographic purposes. Around the room are placed a series of strong tables having drawers and cupboards, and each of these tables is provided with electric currents from the mains, and with a special circuit for working the incandescent lamps required by the galvanometers. In the centre of this room numerous other tables are arranged for special work. On the stone steady slabs are placed all the standard electrical instruments, and the general principle has been adopted of having each particular piece of apparatus required for each special electrical measurement set up and arranged so that it is never disturbed, and is always ready at a moment's notice for use and experiments. One side of

the room is devoted to the current-weighing instruments and standard voltmeters. The laboratory is provided with a very fine set of Lord Kelvin's standard electrical balances and electrostatic voltmeters. These balances are checked by weighing the copper deposit produced in a voltmeter in circuit with them, and for this purpose an Oertling chemical balance has been specially built, and which is a remarkably fine instrument. On other tables are set up the apparatus for the measurement of resistances, insulation tests, magnetic induction, electrical capacity and potential, and as these permanent pieces of apparatus are never disturbed, a great economy of time is effected in setting up and taking down apparatus. In addition to the above, the experimental apparatus is provided for complete tests of alternating current transformers and other alternating current appliances. Beneath the lecture-room is a large accumulator room built in white glazed brick. On stone shelves round the room are placed fifty-four cells of a D.P. battery. This battery has a storage capacity of 250 ampere-hours, and will discharge at the rate of 50 amperes. The current from the battery is laid on to all the working benches of the laboratory.

"The general plan of the laboratory, of having all the rooms opening one into another, is an immense convenience, and saves much time. The arrangement of apparatus in permanent groups for special measurement effects also a great economy in time, as apparatus once set up is not unnecessarily disturbed. The care with which the heating and ventilation have been considered, as well as the universal adoption of the electric light, has made these laboratories exceedingly comfortable to work in. The rooms are all excellently lighted. Although the laboratory stands in a main street, no difficulty has been found to arise from vibration. The stone steady shelves let into the main walls provide all that is necessary in the way of support for the instruments which must be kept steady.

"In addition to the above described dynamo rooms and working laboratory, there is a well-equipped and special photometer room for advanced work. This room has in it a photometer bench 15 ft. long, furnished with Lummer-Brodhun, Bunsen and Flicker photometers, standard lamps, Pentane and Fleming large bulb incandescent and a Crompton potentiometer for reading the electrical quantities.

" Adjacent to it is the lecture room seating 50 to 60 students, and beyond that a research laboratory, which is especially equipped for research in connection with wireless telegraphy and high-frequency currents.

" Besides the above rooms, a diagram and model room, apparatus room, and professor's private room are included. The apparatus room opens into the lecture room close to the lecture table. The apparatus room is well provided with dust-tight apparatus cases and cupboards for the laboratory apparatus. Access is obtained to the accumulator room when required by a cellar-flap door opening out of the lecture room, and through this opening any cells can be hoisted up or let down which are required for examination."

§ 2. **The Fundamental Standards of Length, Mass, and Time.**—The outfit of an electrical laboratory or testing room must include, in the first place, means for making comparisons with the fundamental standards of Mass, Length, and Time. These comparisons involve the possession of standard weights, a good balance, standard lengths, scales and callipers, and an adjusted chronometer. The degree of refinement necessary in these fundamental measurements will depend upon the nature of the work to be done in the electrical laboratory. In a standardising laboratory the actual working instruments—viz., the scales, weights, and watches used for observations—should be checked by comparison with certain more carefully preserved and seldom used principal standards. These last should be brought into careful comparison with the legal standards kept at the Standards Office of the Board of Trade in Great Britain or with the principal legal standards in other countries.

The Unit or Standard of Length to which all other measurements of length in scientific work are now referred in Europe is the *Mètre International*, or Standard Metre. It is a bar of platinum-iridium deposited with the International Committee of Weights and Measures, and is carefully preserved at

Sèvres, near Paris. The length between two marks on it, taken at 0°C., is defined to be the standard length, and is called *the Metre.*

The metre is equal to 39·37011 British standard inches or to 1·09361426 British standard yards.

Comparisons of actual metre scales or yard measures are, or can be, made at the Standards Office of the Board of Trade with a certified copy of the standard yard or metre. Every electrical laboratory in which accurate work is being carried out should possess certain metre scales for working use, and a standard comparison scale which has been thus compared and certified at the Standards Office of the Board of Trade.

Similarly, the Standard or Unit of Mass in scientific work is the *Kilogramme International,* preserved at the same place as the standard metre. The kilogramme is equal to 2·20462 British standard pounds.

In the metric system, multiples and fractions of the standard unit are all decimal, and expressed by the following prefixes to the proper substantive :—

The prefix *mega* means a millionfold.
,, ,, *kilo* ,, thousandfold.
,, ,, *hecto* ,, hundredfold.
,, ,, *deka* ,, tenfold.
,, ,, *deci* ,, one-tenth.
,, ,, *centi* ,, one-hundredth.
,, ,, *milli* ,, one-thousandth.
,, ,, *micro* ,, one-millionth.

Thus a dekametre is 10 metres and a micrometre is one-millionth of a metre.

The following are equivalents in the metric and Imperial systems of measurement :—

1 metre .... = 1·093614 British Standard yards.
1 ,, ... = 3·280842 ,, ,, feet.
1 ,, ... = 39·37011 ,, ,, inches.
1 yard .... = 0·914399 metre.
1 foot .... = 0·304797 ,,
1 inch .... = 0·025400 ,,

1 centimetre........................	=	0·393701	inch.
1 „	=	0·032808	foot.
1 foot	=	30·4797	centimetres.
1 inch	=	2·5400	„
1 kilometre	=	3,280·842	feet.
1 „	=	0·6214	mile.
1 mile	=	160,933	centimetres.
1 square metre	=	10·7639	square feet.
1 square foot........................	=	929·03	square centimetres.
1 litre	=	1	decimetre cube.
1 „	=	61·024	cubic inches.
1 cubic foot	=	28,316	cubic centimetres.

The Metric Standard of Capacity is the litre, and is the volume of 1 kilogram of pure water at 4°C.   The mass of 1 cubic decimetre of water at 4°C. is 0·9999707 kilogram.

The Imperial Standard of Capacity is the gallon, which is the volume of 10 pounds avoirdupois of pure water at 62°F., and is equal to 277,274 cubic inches.

The Unit of Time in scientific operations is the *mean solar second* as defined by the mean solar clock at Greenwich Observatory.

The time of one complete revolution of the earth on its polar axis is called a sidereal day, and is the ultimate unit or standard of reference in measuring duration.   It is equal to 86164·09 mean solar seconds.   The mean solar day is the average time interval between two successive transits of the sun across a meridian, and it is divided into 86,400 mean solar seconds.

A good chronometer should be provided, and should be adjusted to show Greenwich mean time or mean solar time, and, from time to time, should be rated by comparison with the mean solar clock at Greenwich.

It seems hardly necessary to say that the standard chronometer, unless a non-magnetic one, should be kept carefully at a distance from all dynamos and strong magnets or currents.

The electrical laboratory should also be provided with a good standard metre scale, divided into centimetres and

millimetres; a box of standard gramme weights, multiples, and submultiples; and the above-mentioned standard chronometer, showing Greenwich mean time.

Comparisons with the principal standards of length, mass, and time are made by means of working scales, various working balances, and stop-watches or pocket chronometers. The laboratory must, in the first place, be provided with a number of suitable measuring instruments for determining lengths. A number of metre scales, divided into centimetres and millimetres, engraved on steel, brass, or boxwood, are essential. Outside and inside callipers, measuring by a vernier to one-ten-thousandth of an inch or one-four-hundredth of a millimetre, are required. A spherometer for measuring the thickness of thin metal plates is useful.

For measuring the diameters of exceedingly fine wires a microscope micrometer may occasionally be required. In its best form this consists of an ordinary achromatic microscope with a calliper eye-piece. It is provided with a slide consisting of a glass slip on which has been engraved with a diamond a scale in hundredths and thousandths of an inch. The eye-piece of the microscope has two needle points projecting inwards and moved by external screws, so that these points can be made to approach or recede from each other in one plane. These needles are fixed in the focus of the eye lens of the eye-piece. To measure the diameter of a fine wire or, say, the filament of an incandescent lamp, the wire is placed in the focus of the microscope and the needle points screwed in or out until they appear to touch the two edges of the image of the object to be callipered. The eye-piece must, of course, be turned round until the needles are exactly at right angles to the image of the wire. This being done, the wire is removed from the microscope stage and the divided glass scale placed on it. The image of the scale will then be formed in the plane of the needle points. The observer can read off on the scale the length of the interval between the needle points, which is therefore

the diameter of the wire or filament. The arrangement forms an optical calliper, and is a very convenient one to employ in determining the diameter of a very fine wire.

For measuring short lengths or large diameters, a calliper of the following kind is found to be useful.* A steel tube has two flat surfaces planed on it at right angles to each other. The tube has a scale of centimetres and millimetres engraved on it. Two brass blocks slide on this tube, and can be clamped. One of them has a slow motion screw, and carries a vernier. These blocks carry straight steel transverse bars, which act as calliper jaws and remain parallel when placed any distance apart. The object to be callipered is placed between the jaws, and contact is judged as usual by the "feel." One or two of the ordinary inside and outside steel vernier callipers should also be provided, and the best forms have a loose head screw which prevents undue pressure being put upon the object measured.

The laboratory must be provided with a standard balance and with other rougher balances for ordinary weighing. The chief standard balance should be one of the type called a short-beam chemical balance, which is adjusted to carry at least 200 grammes and turn to a tenth of a milligramme. Another balance, to carry 500 grammes and turn to a milligramme, may be used for less accurate weighings.

These balances should stand on stone shelves let into the wall of the laboratory, and be enclosed in glass cases. In each case should stand a beaker containing some lumps of well-baked pumice stone saturated with strong sulphuric acid. The use of this is to dry the air in the balance case and keep all steel parts bright. The standard balance case should be kept locked when not in use. Young students or assistants should not be allowed to use a very good balance without proper instruction as to its employment. On the side of the case should be fastened a notice with four rules

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* Made by the Cambridge Scientific Instrument Company.

distinctly printed thereon for the guidance of the inexperienced :—

1. Never put any object directly on the balance pans, but always weigh it in a tared porcelain crucible, watch glass, or on a disc of glazed writing paper.

2. Never add weights to or remove from the pans unless the balance is clutched or off its knife edges.

3. Never leave the balance case open when making the final adjustment of the rider.

4. Never omit to return the weights to their proper places in the weight box as they are taken off the pan, or to remove the balance from its knife edges when the weighing is finished.

From time to time a careful adjustment of the balance should be made by an experienced person.

The balance should be provided with a special short suspension scale pan called a specific gravity pan, and a glass stoppered bottle should be kept by the case containing well-boiled distilled water for use in taking specific gravities.

The object of which the specific gravity is required should be suspended from the hook at the bottom of the short pan and so hung by a fine hair taken from a horse's tail that it hangs freely in the distilled water. The temperature of the balance case should be taken at the same time. In using the balance to determine the mean diameter of a wire —which is one of the operations the electrician may occasionally have to perform—the known length of wire may be folded up into a suitable compact coil so that it can be suspended in the water in a small beaker. Air is then entangled in the meshes. This should be removed by boiling the wire in the water and letting the wire cool in the water and not removing it until the weight of the mass hanging freely in the water has been taken.

In many operations in the electrical laboratory, such as in meter testing, time has to be determined very accurately. For this purpose a good stop-watch or working chronometer

(preferably non-magnetic) is required. It is often possible to purchase a second-hand ship's chronometer by a first-class maker at a reduced price. If the chronometer is in good order it may be sent to Kew Observatory or to Greenwich to be rated and its true time and temperature rate errors determined. The laboratory will then be provided with a good timekeeper by which other watches may be checked.

It is useless to make very careful observations of meter errors or electrolytical experiments with a common watch having a possible large rate error. Under the head of time standards, reference may be made to the speed indicators and speed counters which are in frequent use in the dynamo room. A most useful form of speed counter is one supplied by Messrs. O. Berend & Co., consisting of a revolution counter of the usual kind combined with a stop-watch counting seconds. The stop-watch is thrown into action by the act of pressing the contact point against the end of the shaft, the speed of which is required. The time and number of revolutions in that time are thus recorded in the same period. There are many forms of mechanical speed indicators or tachometers which record directly on a dial the revolutions per minute. None of these, however, are accurate instruments or can be depended upon to give the actual speed within 5 per cent. In careful dynamo and motor trials the actual revolutions per minute must always be taken by one observer.

A very accurate speed indicator which can be practically applied to dynamo machines is one depending on the centrifugal pressure of a revolving mass of liquid. A small, flat cylindrical box has in it a small four-vane paddle-wheel, the shaft coming out through a stuffing box or gland. The paddle shaft is attached to the axis or shaft, the speed of which is to be measured. The box is carried on an independent support. Two pipes lead to this box, one entering near the centre and one leaving near the circumference. The

pipe to the centre communicates with a small reservoir of coloured water. The pipe from the circumference leads to a vertical glass stand-pipe which may be fixed at any distance from the dynamo. When the paddle revolves it acts as a centrifugal pump, and pumps water from the reservoir up the stand-pipe. Equilibrium is established when the hydrostatic pressure of the column of liquid in the stand-pipe just balances the centrifugal pressure at the circumference of the revolving mass of liquid in the paddle-box. The apparatus can be calibrated once for all, and the true speeds, corresponding to various heights of liquid in the stand-pipe, marked on a wooden scale-board behind the pipe. An arrangement of this kind attached to an alternator is very useful in enabling the operator to set the machine to give a certain frequency and to keep it at this frequency during the test.

In cases where the above hydrostatic tachometer cannot be fixed on account of the power required to drive it, recourse must be had to a simple worm-wheel gearing by which the speed of the shaft is made to turn a wheel at one-tenth, one-hundredth, or less of its own speed. This slow-revolving wheel can be made to close the circuit of a single-stroke electric bell once every revolution, and thus strike a blow at every ten or hundred revolutions of the shaft.

By counting the bell strokes per minute or the time in seconds extending over ten bell strokes the speed of the shaft is ascertained.

§ 3. **The Principal Electrical Units and Standards.**— Next in importance in the equipment of an electrical testing room to the standards and instruments for making the fundamental measurements of length, mass, and time come the principal standards of electrical resistance, electromotive force, and the means for recovering the standard or unit electric current.

These principal electrical units have been given legal definition in Great Britain, and identical units have been adopted in other countries. They are accordingly now called the *International units* of resistance, electromotive force and current, and are defined as follows :—

> *The electrical resistance offered to an unvarying electric current by a column of pure mercury at the melting point of ice, 106·3 centimetres long and weighing 14·4521 grammes, and of constant cross-section, is defined as the unit of resistance, and is called one ohm.*

The ohm so defined is intended to represent 10^9 absolute electromagnetic units (C.G.S.) of resistance.

The unit electric current is defined by an operation of electrolysis performed with a solution of a silver salt, as follows :—

> *The electric current of unvarying strength which, when passed through a neutral solution of nitrate of silver containing 15 parts by weight of the salt to 85 of water, using a silver anode and a platinum cathode, deposits silver at the rate of 0·001118 of a gramme per second, is defined as the unit of electric current and is called one ampere.*[*]

The ampere so defined is intended to represent or reproduce 10^{-1} of an absolute electromagnetic (C.G.S.) unit of current.

From the unit of resistance and current is defined the unit of electromotive force :—

> *The unvarying electromotive force which creates in a circuit having a resistance equal to one ohm an unvarying current of one ampere is defined as the unit of electromotive force, and is called one volt.*

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[*] See remarks on page 58 with reference to more recent determinations of the Electro-chemical Equivalent of Silver.

The electromotive force so defined is intended to represent or reproduce 10^8 absolute electromagnetic units (C.G.S.) of electromotive force.

The above units are defined and recovered in the laboratory by the use of certain practical standards of resistance, electromotive force and appliances or processes, for electric current measurement.

A principal necessity in an electrical laboratory is, then, the possession of satisfactory means for recovering or reproducing the International ohm, the volt, and ampere.

Carefully constructed apparatus made for this purpose is in use at the British Board of Trade Electrical Laboratory, in accordance with the following regulations :—

I.—*Standard of Electrical Resistance.*

The standard of electrical resistance denominated one ohm shall be and is the resistance between the copper terminals of the instrument marked "Board of Trade Ohm Standard, verified 1894," to the passage of an unvarying electrical current when the coil of insulated wire forming part of the aforesaid instrument and connected to the aforesaid terminals is in all parts at a temperature of 15·4°C.

The ohm so reproduced is generally called the *Board of Trade Standard ohm*, to distinguish it from the *theoretical* or *true ohm*, which is defined as 10^9 centimetres per second in C.G.S. measure. The density of pure mercury at 0°C. is 13·5956. Hence a volume of mercury having the form of a right circular cylinder 106·3cms. in length and a mass of 14·4521 grammes, has a cross-sectional area of one square millimetre or 0·01 sq. cm. The actual principal standard ohm of the Board of Trade has been constructed to represent as nearly as possible the International ohm. This last may be otherwise defined as the true electrical resistance at 0°C. of a column of mercury 106·3cms. long and one square millimetre in section. Present knowledge seems to show that the actual Board of Trade ohm standard is 0·02 per cent. larger than the true ohm.

II.—*Standard of Electrical Current.*

The standard of electrical current denominated one ampere shall be and is the current which is passing in and through the coils of wire forming part of the instrument marked "Board of Trade Ampere Standard, verified 1894," when on reversing the current in the fixed coils the change in the

forces acting upon the suspended coil in its sighted position is exactly balanced by the force exerted by gravity in West-minster upon the iridio-platinum weight marked A and forming part of the said instrument.

III.—*Standard of Electrical Pressure.*

The standard of electrical pressure denominated one volt shall be and is one-hundredth part of the pressure which when applied between the terminals forming part of the instrument marked "Board of Trade Volt Standard, verified 1894," causes that rotation of the suspended portion of the instru-ment which is exactly measured by the coincidence of the sighting wire with the image of the fiducial mark A before and after application of the pressure and with that of the fiducial mark B during the application of the pressure, these images being produced by the suspended mirror and observed by means of the eyepiece.

In the use of the above standards the limits of accuracy attainable are as follows:—

For the ohm, within one-hundredth part of 1 per cent.

For the ampere, within one-tenth part of 1 per cent.

For the volt, within one-tenth part of 1 per cent.

§4. The Practical Standard of Electrical Resistance.— The practical standard or unit of electrical resistance is officially defined as above-stated in terms of a certain length and mass of pure mercury. Great labour has been expended, both in England and in other countries, on the determination of the absolute specific resistance or the resistivity of pure mercury.* The following are some of the values which have

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* *See* "The Specific Resistance of Mercury," Lord Rayleigh and Mrs. Sidgwick, *Phil. Trans.* Roy. Soc., 1883, Part I., p. 173. Also R. T. Glaze-brook, *Phil. Mag.*, October, 1885. Also R. T. Glazebrook and T. C. Fitz-patrick, "On the Specific Resistance of Mercury," *Phil. Trans.* Roy. Soc., June, 1888; or *Proc.* Roy. Soc., Vol XLIV., No. 270; or *The Electrician*, Vol. XXI., p. 538, 1888 ; Kohlrausch, *Abhandt der K. Bayer Akad der Wiss*, Vol. XVI., Abth III., 1887 ; Mascart, Nerville and Benoit, *Journal de Phy-sique*, 1884 ; Strecker, *Wiedemann Annalen*, Vol. XXV., 1885 ; L. Lorenz, *Wiedemann Annalen*, Vol. XXV., 1885; Rowland, *Proc.* British Association, 1887. Also Hutchinson and Wilkes, "John Hopkin's University Circular," May, 1889. Also "Recent Determinations of the Absolute Resistance of Mercury," R. T. Glazebrook, *The Electrician*, Vol. XXV., pp. 543, 588, 1890. Also *see* Prof. J. V. Jones, "On the Determination of the Specific Resistance of Mercury in Absolute Measure," *Phil. Trans.* Roy. Soc., 1891, A, p. 2.

been obtained for the resistivity of pure mercury at 0°C. in C.G.S. units per centimetre cube :—

Observer.	Date.	Resistivity.
Lord Rayleigh and Mrs. Sidgwick	1883	 94,133
Mascart, Nerville and Benoît	1884	 94,096
Strecker	1885	 94,057
L. Lorenz	1885	 94,108
Rowland	1887	 94,072
Kohlrausch.....................	1887	 94,054
Glazebrook and Fitzpatrick	1888	 94,074
Hutchinson and Wilkes.......................	1890	 94,064
J. V. Jones	1890	 94,067

The present accepted value of the specific electrical resistance or resistivity of pure mercury at 0°C. is 94,070 C.G.S. units. Hence, a column of mercury one metre in length and everywhere of 1 sq. millimetre in cross section has at the temperature of melting ice a resistance of 0·94070 ohms, as far as the mean of the best determinations will allow us to pronounce.

The following are the values obtained by the above observers for the length in centimetres of a column of mercury one square millimetre in section and having a resistance of one ohm at 0°C. :—

Observer.	Length of column in centimetres.
Lord Rayleigh and Mrs. Sidgwick	106·23
Mascart, Nerville and Benoit	106·33
Strecker	106·32
L. Lorenz	106·26
Rowland ...._ ..	106·32
Kohlrausch	106·32
Glazebrook and Fitzpatrick	106·29
Hutchinson and Wilkes	106·34
J. V. Jones...................	106·31

The value now accepted for the length of the column of mercury one square millimetre in section, which has at 0°C a resistance of one International ohm, is 106·3 cms.

Continental physicists have devoted considerable time and knowledge to the construction of primary and secondary mercury resistances, consisting of mercury specially purified and preserved in glass tubes of known dimensions, and intended to practically realise the Board of Trade or International definition of the practical unit of resistance.

For the most part, English investigators have given their attention to the re-determination in absolute measure of

the electrical resistance of certain wire standards, notably that called the mean British Association unit, and hence from the known resistivity of mercury proceeded to construct the most closely approximate realisation of the ohm in wire resistances.*

In the majority of instances the standards of resistance used in an electrical laboratory are preserved in the form of a uniform metallic wire drawn from an alloy of permanent composition, and having a small change of resistance with temperature.
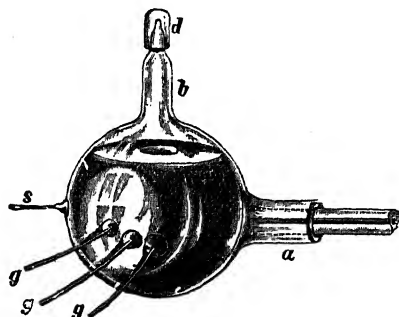
a End of mercury tube. s Platinum wire electrode. g Platinum wire, galvanometer wires.

FIG. 4.—Glass Spherical Terminal Vessel of the Berlin Reichsanstalt Standard Mercury Ohm.

Owing to its fragility and non-portability, a standard of resistance consisting of mercury in a glass tube is not likely to be constructed or used in any but a national standard-ising laboratory. The construction of an original mercury-in-glass resistance by which to reproduce the standard or international ohm from its definition is a matter requiring the highest skill and knowledge. Broadly speaking, it consists in preparing and filling a glass tube of known dimensions with pure mercury, the weight of which at 0°C. is

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* For additional information on the determination of resistance in absolute measure *see* Chap. II., RESISTANCE MEASUREMENT ; also Tables at the end of the same chapter.

accurately determined.* This tube has its ends included in large spherical vessels of mercury to which electrical connection is made by means of platinum wires (*see* Fig. 4). In calculating the resistance of the column of mercury from the dimensions of the tube and the resistivity of the mercury, a correction has to be made in accordance with known principles by which the effective length of the tube is taken as greater than the actual length by 0·82 of the diameter of the tube. For the full details of the construction of an original mercury-in-glass standard of electrical resistance the reader must be referred to the detailed accounts which have been given of the production of the mercury standard set up in the Physikalisch-Technische Reichsanstalt, in Berlin, and the similar work carried out in Paris† by Benoit (*see* Fig. 5).

Mercury-in-glass standards are, however, used only as original reference standards. The labour involved in their construction and the necessity for great precautions in their employment to reduce the whole mass of glass and mercury to a known temperature before making a measurement with them renders them unfit for ordinary laboratory use. For all ordinary standards, multiples and submultiples of the ohm, a wire resistance standard is much more convenient and is usually employed. The two principal alloys which have been found sufficiently permanent and of sufficiently small temperature coefficient to use in the construction of such a wire standard are a platinum-silver alloy and a ternary alloy of copper-manganese and nickel called *Manganin*. There are therefore, preserved in the various national electrical laboratories numerous copies of the ohm in the form of wire standards, which are intended to have, as nearly as possible, a resistance equal to the true ohm.

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* For instructions for the purification of the mercury, see the Reichsanstalt directions, as set forth by Dr. Jaeger, *The Electrician*, Vol. XXX., p. 395.

† See *The Electrician*, Vol. XXXVII., p. 569, " On the Standard Mercury Ohm of the Physikalisch-Technische Reichsanstalt," by Dr. Jaeger, translated from the *Zeitschrift für Instrumentenkunde*, Vol. XVI., pp. 134-146, May, 1896. Also see Benoit, *Comptes Rendus*, Vol. XCIX, p.. 864, 1884.
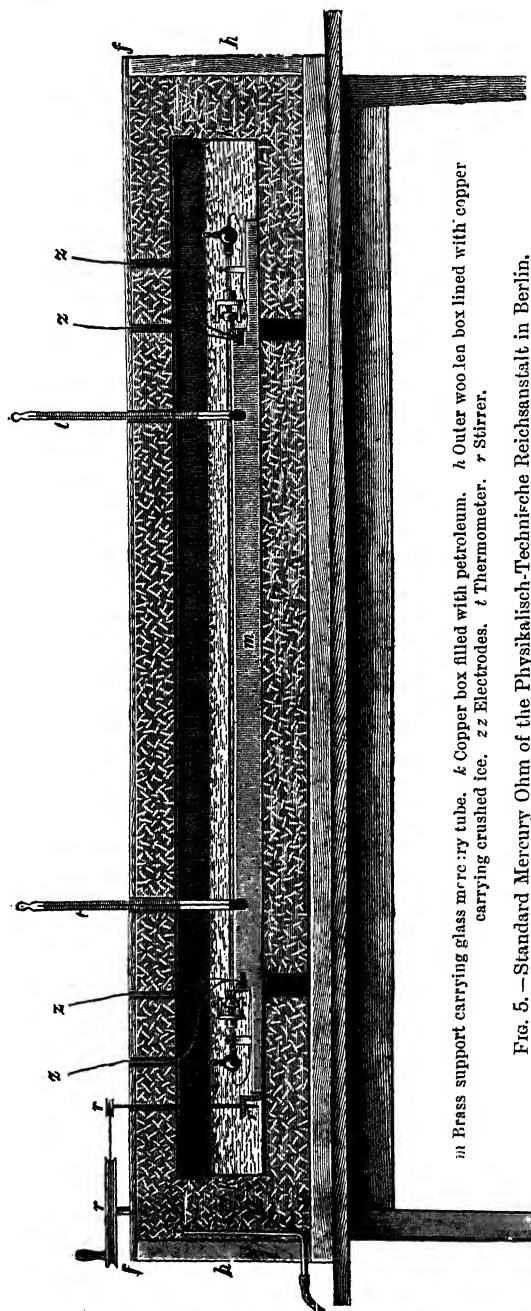
m Brass support carrying glass mercury tube. *k* Copper box filled with petroleum. *h* Outer wooden box lined with copper carrying crushed ice. *zz* Electrodes. *t* Thermometer. *r* Stirrer.

FIG. 5.—Standard Mercury Ohm of the Physikalisch-Technische Reichsanstalt in Berlin.

The original British Association wire resistance standards are in the custody of the Secretary of the British Association Committee on Electrical Standards; the Board of Trade standards are kept at the Government Electrical Laboratory, Whitehall, London; and carefully constructed standards of electrical resistance of the same character are preserved at the Physikalisch-Technische Reichsanstalt, in Berlin, and at the Bureau International des Poids et Mesures, near Paris.

FIG. 6.—Secondary Standard Mercury Ohm.

Secondary standards or copies of the ohm, consisting of mercury contained in tubes of Jena glass with platinum electrodes sealed through the glass, have also been constructed at the Physikalisch-Technische Reichsanstalt, in Berlin (*see* Fig. 6) and by Benoit, in Paris. These secondary mercury standards consist of U or W shaped tubes of glass

most carefully filled in vacuo with mercury and then fixed in a suitable case.

A comparison in 1892 and 1894 of all the mercury and manganin copies of the ohm, made at the Reichsanstalt in Berlin, showed that these standards had remained constant in resistance for two years to within one or two parts in one hundred thousand.

A series of very elaborate experiments were made in the year 1892 on the variation of resistance with temperature of mercury-in-glass tubes between 0° and 28°C., and it was found by Drs. Kreichgauer and Jaeger[*] that the resistance at t°C. (R_t) of mercury in Jena glass tubes can be expressed between 0°C. and 28°C. as follows :—

$$R_t = R_0 (1 + 0.000875\, t + 0.00000125\, t^2),$$

where R_0 is the apparent resistance at 0°C.

The true resistance of mercury (r_t) at t°C. is related to its resistance (r_0) at 0°C. by the relation

$$r_t = r_0 (1 + 0.0008827\, t + 0.00000126\, t^2)$$

for a range of temperature between 0°C. and 28°C. These values are not very different from those found by Benoit, Guillaume, and others in 1890.[†]

In spite, therefore, of the advantages gained by the use of a pure, unchangeable and fluid metal, such as mercury, as against the supposed possible non-permanency of structure of a solid metallic alloy, we have to take into account the difficulties raised by the larger temperature coefficient of the pure metal.

Experience is as yet wanting to show how far the various alloys, now used for the resistance wires of standard

[*] See *Wied. Ann.*, No. 1, 1895 ; also *The Electrician*, Vol. XXX, p. 567.

[†] Researches carried out in 1889-1890 by M. C. E. Guillaume, at the Bureau International des Poids et Mesures (see *The Electrician*, Vol. XXIX., p. 565), gave by two determinations the true temperature variation of mercury between 0°C. and 61°C. as follows :—

(a) $r_t = r_0 (1 + 0.00088745\, t + 0.0000010181\, t^2)$
(b) $r_t = r_0 (1 + 0.00088879\, t + 0.0000010022\, t^2)$

coils, will remain absolutely unaltered over long periods of time. It is certainly to a considerable degree dependent on the manner in which the standards are made, and how far they are allowed to undergo changes of temperature in use.

In the employment of a wire of a metallic alloy as a standard of electrical resistance there is always an element of uncertainty as to the extent to which time may affect the electrical qualities of the alloy. Experiments made by the author in 1878 and 1879 on the original British Association standard coils (fourteen in number) made of various alloys showed that this fear was not altogether groundless, some coils in the course of 14 or 15 years having certainly changed in electrical resistance. Attention has, therefore, been devoted to this question especially in connection with the use of manganin and platinum-silver. In 1894 a number of resistance coils of manganin (42 coils) and of constantan (three coils) were tested at Berlin which had been in use about three years. Of these—

25 coils showed a variation of from 0·00 to 0·01 per cent.

13	,,	,,	,,	,,	0·01 to 0·02	,,
5	,,	,,	,,	,,	0·02 to 0·05	,,
2	,,	,,	,,	,,	0·05 to 0·25	,,

when re-compared with primary standards.

Confidence in the absolute permanency of electrical resistance in the case of wire standards can hardly be said to have been established beyond doubt. The balance of evidence seems to show that for coils carefully made of well aged manganin or platinum-silver it will not be serious.

One argument which has been used to support the contention that an ultimate standard should take the form of mercury in glass is the assumed unalterable character of the materials thus used. There is a possibility, however, that the platinum electrodes may in time slightly dissolve and thus contaminate the mercury.

The alloys which have been found practically satisfactory

as materials to employ in the construction of wire-resistance standards are

1. *Platinum Silver.*—An alloy consisting of two parts of silver and one of platinum. This alloy was originally recommended by the Electrical Standards Committee of the British Association. It has a resistivity of 25 to 30 microhms per centimetre-cube and a temperature coefficient of nearly 0·0003, or nearly 0·03 per cent. per degree centigrade.

2. *Manganin.*—This alloy is composed of 84 parts of copper, 12 of manganese and 4 of nickel. Its electrical properties and suitability for employment as an electrical resistance material were made known by experiments conducted in the Physikalisch-Technische Reichsanstalt in Berlin. Its specific resistance is about 42 microhms per centimetre-cube. Between 0°C. and 10°C. its temperature coefficient is 0·000025. At some temperature generally lying between 15°C. and 30°C. it has a nearly zero temperature coefficient, and beyond about 30°C. a negative temperature coefficient. It has not nearly so great a thermo-electric power with copper as German silver or platinoid. It must, however, be *aged* for use as a resistance material by first heating it to a temperature of 140°C. for five or ten hours in an air bath to prevent subsequent changes in resistance. It is somewhat easily oxidised, and when used exposed to air must be either gilt or varnished. It must not be soldered with solder containing zinc.*

In the manufacture of commercial resistance coils not intended as standards the following alloys are useful:—

3. *German Silver.*—This is an alloy of copper (50 to 66 parts), nickel (13 to 18 parts) and zinc (19 to 31 parts), and is of somewhat variable composition in different specimens. It is not suitable for the construction of other than commercial

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* *See* Dr. St. Lindeck, *The Electrician*, Vol. XXX., p. 119, "On Alloys for Resistance Coils." "On the Permanency of Manganin Resistances," *see* Prof. W. E. Ayrton, *The Electrician*, Vol. XL., p. 39, also p. 227 ; also W. Watson, *Proc.* Phys. Soc., Lond., Vol. XVI., p. 25 ; also Drs. Jaeger and St. Lindeck, *Science Abstracts*, Vol. I., p. 336.

resistances, on account of its rather large temperature coefficient (0·0004). It has a resistivity varying from 20 to 30 microhms per centimetre-cube.

4. *Platinoid.*—An alloy resembling German silver in composition, but containing in addition about 2 per cent. of tungsten. It has a smaller temperature coefficient (0·0003) than German silver, and preserves a better surface in air. In some specimens, however, time and temperature produce a certain brittleness, and it seems to be attacked when exposed in moist air containing carbonic acid.* It has in some specimens as high a resistivity as manganin, viz., 40 to 45 microhms per centimetre-cube, but in other samples the resistivity is as low as 30 microhms per centimetre-cube.

In addition to the above well-known alloys of definite composition, a number of patented and commercial alloys, the composition of which is not made public, are in use for the construction of resistances in which a small temperature coefficient is not of the greatest importance. These alloys are employed in the manufacture of regulating resistances, but are not used for resistance standards. Such alloys are

5. *Rheostene.*—A nickel-steel alloy, having a resistivity of about 77 microhms per centimetre-cube and a temperature coefficient of 0·00119.†

6. *Kruppin.*—An alloy having a resistivity of 83 microhms per centimetre-cube and a temperature coefficient of 0·0013.‡

7. *Hadfield's Resista.*—An alloy having a resistivity of 76 microhms per centimetre-cube and a temperature coefficient of 0·0011.

8. *Eureka.*—An alloy having much the same electrical qualities as *platinoid*, but not so liable to brittleness. Its

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* *See* Mr. Rollo Appleyard, "On the Failure of German Silver and Platinoid Wires," *The Electrician,* Vol. XL., p. 227, 1897 ; also *ibid. Electrical Review,* Vol. XLII., p. 536 ; *Science Abstracts,* Vol. I., p. 412 ; *Proc.* Phys. Soc., Lond., Vol. XVI., p. 17.

† *See* Van Aubel, *The Electrician,* Vol. XL., p. 315 ; or *Science Abstracts,* Vol. I., p. 19.

‡ See *The Electrician,* Vol. XXXII., p. 351 ; also *Electrotechnische Zeitschrift,* December 15, 1893 ; also see Van Aubel, *The Electrician,* Vol. XXXIII., p. 122.

chief mechanical fault is its considerable extensibility and non-elastic yielding under tension.

9. *Brunton's Brocon Alloy.* An alloy having a resistivity of 60 to 80 microhms per centimetre cube and a temperature coefficient of 0·0007.

A large number of alloys have been found which have high resistivity and small temperature coefficient, but in many cases they have also poor mechanical qualities and are brittle, or ill suited for being drawn into fine wire.*

As made at present, a standard of electrical resistance consists of a carefully-drawn and well-annealed wire of manganin or platinum-silver. If made of manganin it has to be *aged*, as above described. This wire is then spun over with two or three layers of white silk, and is wound on a bobbin and preserved in a case in such manner that its temperature can be controlled.

The actual resistance wire has to be attached at both ends to terminal blocks or rods of copper, and has to be enclosed for security in a metal case of some form or immersed in an insulating oil.

Many English electricians are still somewhat sceptical as to the permanency of manganin.† It has been stated that all alloys containing zinc are liable to erratic changes of resistance. Zinc solder or soft solder should never be employed in the construction of standard coils. Silver solder, containing 75 per cent. of silver, should be used in soldering manganin, and shellac varnish, and not paraffin wax, should be used as an insulator.

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* For further information see "Alloys for Resistance Standards," J. T. Willyoung, *Journal of Franklin Institute*, or *The Electrician*, Vol. XXIX., p. 277; "Experiments on Nickel Steel Alloys," C. E. Guillaume, *The Electrician*, Vol. XL., p. 543, "The Composition of Various Alloys for Resistance Coils," Drs. Feussner and St. Lindeck, *The Electrician*, Vol. XXXI., p. 495. Also *The Electrician*, Vol. XXX., p. 119.

† In all these more or less complicated alloys there may be variations of electrical and mechanical qualities caused by irregularities in manufacture. It is not unusual to find samples of German silver and platinoid, and perhaps manganin, which seem quite inferior to the general run of specimens of these particular materials.

The authorities of the Berlin Reichsanstalt have devoted great attention to the consideration of the best form to give to a wire standard of electrical resistance, and the following is a description of the form adopted at present (1900)   A section of the standard is shown in Fig. 7.
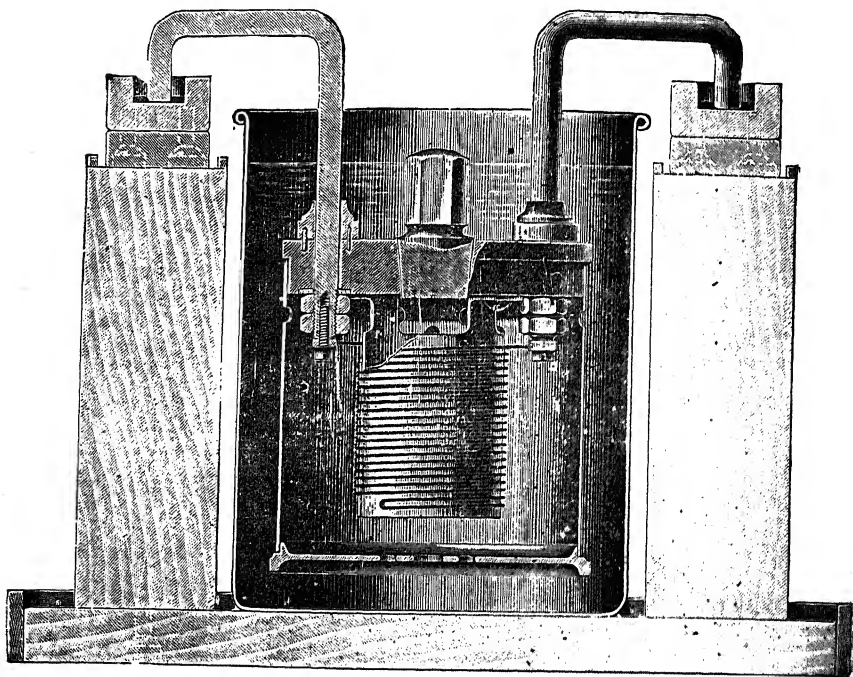
FIG. 7.—Berlin Reichsanstalt Wire Standard of Electrical Resistance or Standard Ohm.

The resistance wire is a carefully insulated wire wound on a brass cylinder, the wire being doubled on itself to annul inductance as much as possible. The wire is laid on in a single layer. The ends of the wire are silver soldered to square nuts, which are in turn soft soldered to heavy copper terminal rods.   The final adjustment of resistance is made by shortening a much finer parallel wire. Thus the 0·1 ohm standard consists of two wires in parallel: one the principal

wire, and another wire one-tenth of the section and ten times as long as the first. Hence a change in length of one metre of the fine wire is equivalent to only one millimetre change in length of the principal wire. The coil so wound and adjusted is hung in a bath of paraffin oil and contact is made with the copper leads of the coil by means of the mercury cups. When in use the oil is kept well stirred by a stirrer driven by a small electromotor. In preparing the coil the silk-covered resistance wires are wound on the brass core and made to adhere to it by melted shellac. The coil is then heated for ten hours to 140°C. This baking not only gives the coil the necessary insulation from layer to layer (which is of the order of a million megohms), but *ages* the manganin and prevents subsequent secular change in resistance.

In the use of a coil of the above kind as a standard of resistance it is desirable, if extreme accuracy of comparison is required, that the power wasted in the resistance coil shall not exceed one watt. The oil in which the coil is immersed must be kept in thorough circulation, preferably by means of a stirrer driven by a small electromotor, when a measurement of resistance is being made. For further and fuller information on the details of the construction and mode of use of the Reichsanstalt standard wire coils, the reader is referred to a paper by Drs. K. Feussner and St. Lindeck in the *Zeitschrift für Instrumentenkunde,* November and December, 1895.[*]

In addition to the principal standards of resistance in a laboratory, which must always be carefully used, it is necessary to possess certain working standards for general use. A convenient form for these secondary reference standards has been designed by the author.[†]

This laboratory standard resistance coil is made as follows:— The brass case or shell which contains the coil is in the form

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[*] See *The Electrician,* Vol. XXXVI., p. 509, 1896 ; also Drs. Jaeger and K. Kahle, *Wied. Ann.,* No. 3, 1898 ; *The Electrician,* Vol. XL., p. 847 ; *Science Abstracts,* Vol. I., p. 412.

[†] See *Phil. Mag.,* January, 1889.

of a ring (*see* Fig. 8).   This ring consists of a pair of square-sectioned circular troughs provided with flanges which can be screwed together so as to form a square-sectioned, hollow, circular ring.
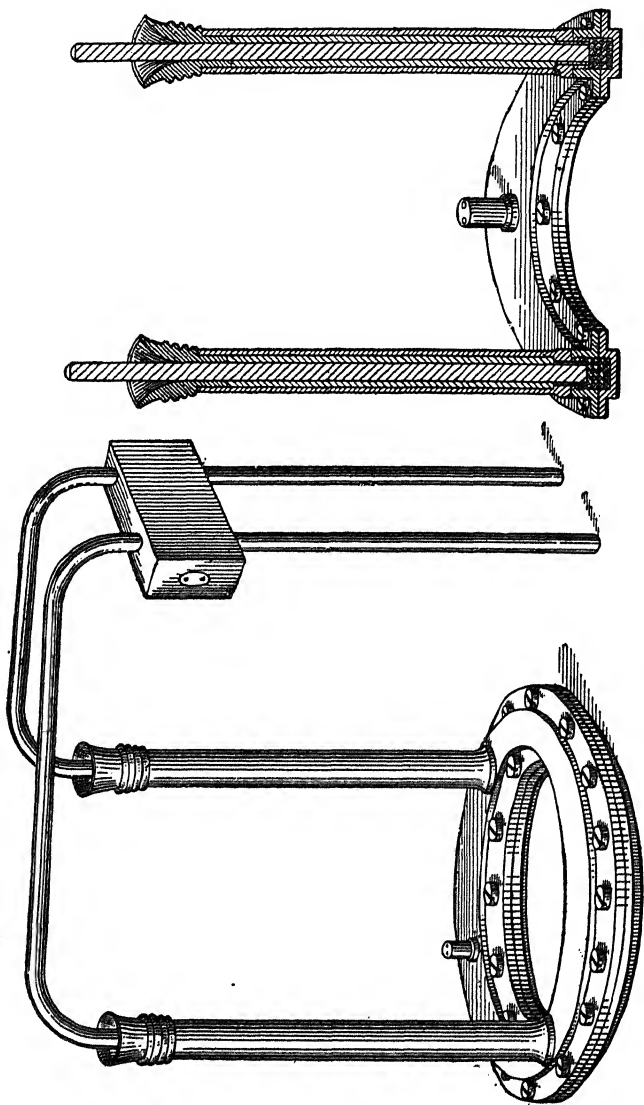

Fig. 8.—Fleming Standard Laboratory Resistance Coil.

From this ring proceed upwards two brass tubes about five or six inches in length. Down these brass tubes pass the copper electrodes or rods, and these rods are insulated from the tubes at the top and bottom by ebonite insulators. The insulator at the bottom of the tube, where it enters the ring, is a simple collar, that at the top has the form of a funnel corrugated on its outer surface. The use of this funnel will be referred to presently. The actual resistance-coil is a length of platinum-silver wire three-fold silk-covered. The silk-covered wire is first baked above 100°C. to dry it completely, and then immersed in melted ozokerit, or better, in shellac varnish made up with absolute alcohol.

The so insulated wire is cut about the proper length and laid double or folded once upon itself, and then rolled up on a wooden mandril so as to form a circular coil of diameter suitable to drop into the hollow of the brass ring. The wire being wound double, its coefficient of self-induction is rendered very small. This coil of wire is then wrapped over with white silk and again dipped in melted ozokerit. The ends of the wire are next soldered into nicks in the ends of the copper rods, they having been previously pushed a little way through the brass tubes for the purpose, and afterwards drawn back into proper positions. The coil is then packed into the circular groove, and, after adjusting the resistance to the proper value, the bottom half of the ring is placed over it. A thin washer of indiarubber is inserted between the flanges, and the whole screwed tightly together. The resistance-coil is thus enclosed in a thin ring of metal, and can be placed wholly below the surface of water or ice. In order to test the tightness of the joints, a little test-pipe is provided on the upper surface of the ring. By placing the ring coil below water and blowing into the test-pipe, the good fitting of the joints can be assured. The aperture of this test-pipe is afterwards closed by solder or a screw (*see* Fig. 8).

Apart from the insulation of the coil itself, it will be apparent that the insulation is limited by the amount of insulation resistance secured at the ebonite insulators at the top end of the brass tubes. Any leakage from the copper rod over these insulators to the brass tube destroys to that extent the insulation of the coil. The object of making these external insulators funnel-shaped is to prevent surface-creeping due to condensation of moisture on them, by placing paraffin oil or insulating liquid in the funnel-shaped cavity. When

this is done, even if dew should collect on the outer surface of the funnels, the inner surface is kept dry by the paraffin oil placed in them, the action being the same as that in the well-known Johnson and Phillips fluid insulator.

The ring coils when in use are placed in rather shallow zinc troughs, which can be filled with water, and are closed with a wooden lid. When so placed the whole of the actual coil or resistance part is down beneath the liquid at one level, where the temperature can be accurately ascertained. The insulators and point of emergence of the electrodes are away up above the level of the water, and well protected from any action which might permit of leakage over them. The large metallic mass of the ring assists in bringing the resistance-coil quickly back to the temperature of the surrounding water, and the coil therefore "tests quickly." In all other respects these standards of resistance are as compact and portable, and not more expensive to construct, than the old form of B.A. standard, whilst obviating the difficulties which present themselves in the use of the old form in very accurate comparisons of resistance.

It is quite possible to have two or more coils of wire inside the same ring, each coil having its separate pair of electrodes. A useful coil of this form can be made up containing 1, 10, and 100 ohms, so that comparisons can be quickly made at the same temperature with these three multiples of the same unit of resistance.

In an ordinary electrical laboratory the electrician will not often be called upon to make and adjust the principal standards of resistance himself. This is an operation requiring great care and patience and some skill. Suffice it to say that one of the best means of making the final adjustment of resistance coils to an exact value is to wind on the coil two insulated wires of the same material in parallel. One of these is the principal resistance wire, and the other is a much longer wire of smaller section, and is the adjusting wire. The principal wire is, in the first instance, cut to a length which shall be greater than the resistance required. The following table may serve as a rough guide in making

resistance coils of platinum silver, but a margin should always be allowed in cutting off wire :—

To make a resistance coil of		Take platinum-silver wire of		
		Length.	Gauge in S.W.G.	
1 ohm	....	9ft.	No. 22	0·036in. dia.
10 ohms	....	42·5ft	No. 24	0·025in. dia.
100 ohms	....	133ft.	No. 30	0·014in. dia.
1,000 ohms	....	675ft.	No. 34	0·010in. dia.

The principal wire being roughly adjusted to a resistance a little greater than the resistance desired, it is joined in parallel with a wire of one-tenth its diameter and ten times the length. The final adjustment is then made by lengthening or shortening this finer wire until the two wires in parallel have exactly the required resistance. The resistance wires are wound doubled or non-inductively on themselves on a metallic core. The greatest care must be taken in insulating the different turns of wire from each other and from the core. The wire should be double-covered with white silk and never be touched with the moist hand. When the coil is prepared as above described it must be placed for use in a vessel of water or paraffin which is kept well stirred and the temperature of which can be taken with a correct thermometer. In all very accurate measurements it is better to work with the coils in melting ice. The true temperature of the wire is then known with a considerable degree of exactness if the coil is allowed to remain for a sufficient time in the melting ice to arrive exactly at 0°C. The source of error to be avoided is a difference in temperature between different parts of the resistance wire and between the wire and the external bath. In making a resistance measurement sufficient time must always be allowed to elapse between two measurements to permit the coil to reach its original and desired temperature. The only method of making such a measurement with certainty as to the temperature of the wire is to make the resistance measurement after keeping the coils for 12 hours immersed in melting ice.

In consequence of the slight uncertainty which still exists as to the true value of the various practical standards of resistance, it has been proposed by Prof. J. Viriamu Jones that the ultimate standard of resistance shall not be embodied in the form of a wire, but shall be recovered when required by an absolute determination (see *The Electrician*, Vol. XXV., p. 552; also *Proc.* British Assoc., Leeds, 1890, " Suggestions Towards the Determination of the Ohm "). With this object he has perfected the Lorenz apparatus for the absolute determination of resistance. His proposal is that such an apparatus shall be set up in national standardising laboratories, and that when it is necessary to evaluate a secondary standard of resistance this shall be done by an absolute determination of the same. In the Lorenz apparatus the essential portion consists of a metallic disc revolving in the interior of a carefully arranged spiral coil of wire.   If an electric current is sent through the coil, and if the disc is set in revolution, the disc becomes the seat of an electromotive force.   This electromotive force may be balanced against the fall of potential down a certain resistance in circuit with the coil.   When once the dimensions of the apparatus have been determined, the actual observations necessary to obtain the absolute value of the resistance of the inserted conductor are reduced to the determination of the speed of revolution of the disc, and the absolute value of the resistance is given as the product of a calculated coefficient of mutual induction and an observed number of revolutions per second.   Since the things actually observed are merely a speed and the absence of a current in a circuit, extreme accuracy of measurement can be obtained.

These necessary measurements can be carried out with a very high degree of precision, and with a well-made Lorenz apparatus it is possible to determine the value of a resistance of the order of an ohm with an accuracy of one part in 10,000.*

* For a full description of the Lorenz apparatus the reader is referred to a lecture given by Prof. J. V. Jones at the Royal Institution, May 24, 1895, see *The Electrician*, Vol. XXXV., pp. 231 and 253 ; also the *Proceedings* of the Royal Institution, Vol. XIV., p. 601. Also to an account of " A Deter-

§ 5. Current-Carrying Standard Resistances.—In addition to resistance coils, which are intended to be used merely with very small currents passing through them, the electrical laboratory must be provided with a graduated series of resistances capable of carrying or passing large currents and for use in measurements in which the fall in potential down a conductor of known resistance is made to yield a knowledge of the current passing through a conductor in series with the resistance.

These resistances are called *Current Resistances,* or sometimes *Mho Standards.* Resistances of the above kind are now made by instrument makers in the form of thick
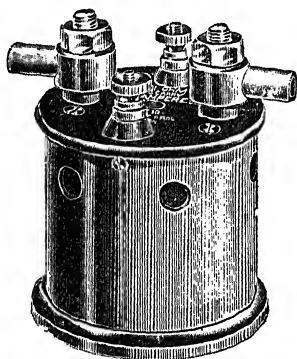
FIG. 9.—Standard 0·1 Ohm Resistance to carry 10 amperes.

manganin wires in well-ventilated brass cases (*see* Fig. 9), and in the form of manganin strip soldered to end blocks (*see* Figs. 10 and 11), or in the shape of tubes of manganin with end terminals (*see* Fig. 12). The great aim in the construction of these resistances for large currents is to afford

mination of the Ohm made in Testing the Lorenz Apparatus of the McGill University, Montreal," Profs. Ayrton and Jones, see *The Electrician*, Vol. XI., p. 150, 1897. These experiments seemed to show that the Board of Trade standard ohm is between two and three parts in 10,000 larger than the true ohm. In other words, it is 0·02 per cent. greater than 10^9 centimetres per second. (*See* Prof. Ayrton on "Our Knowledge of the Value of a Resistance," *The Electrician*, Vol. XL., pp. 133 and 149.) *See* also Chapter II., RESISTANCE MEASUREMENT.

sufficient surface to carry off the heat generated by the
current, either by air convection or by radiation.

. With the object of obtaining great current-carrying
capacity, some makers, such as Mr. Crompton, make these
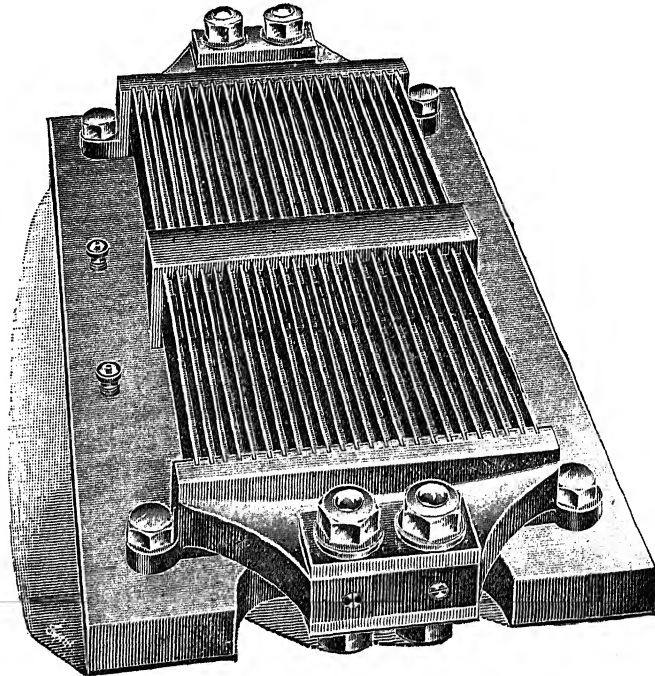
FIG. 10.—Bare Metal Strip Resistance for Currents up to 1,500 amperes.

FIG. 11.—Metal Strip 0·1 Ohm Resistance for Currents up to 15 amperes.

standards in the form of tubes of manganin through which
water can be passed, and by this means they construct
standards of resistances capable of carrying without sensible

heating very large currents. In Fig. 12 is shown a tubular resistance made to carry 1,500 amperes. The resistance is 0·001 of an ohm. Hence the fall in potential down it at full
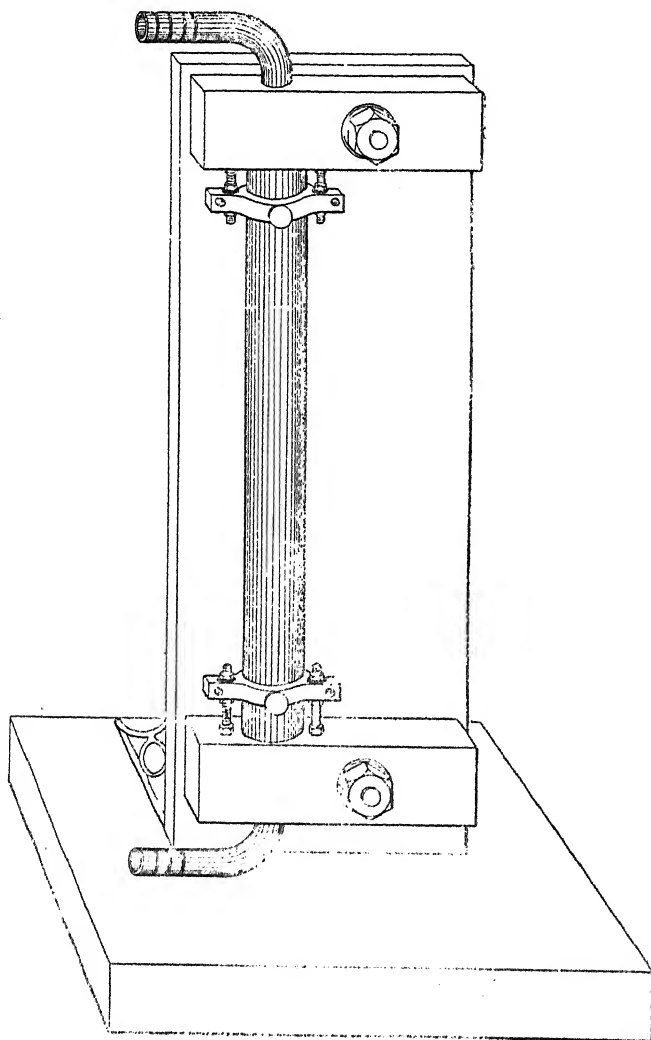
FIG. 12.—Crompton Water-Tube Low-Resistance Standard

load is 1·5 volts. The water used for cooling is circulated through the pipe by a rubber hose, and the current density in the metal rises to 13,000 to 20,000 amperes per square inch without risk. The potential measuring connections are attached to terminals on clamps which are fixed at such distances as to define exactly the resistance required.

These current-carrying resistances are always provided with two pairs of terminals, one pair called the *current terminals*, by which the current is led in and out, and another pair called the *potential terminals*, between which the resistance has its stated value.

A most essential possession in an electrical laboratory or testing room is a set of low resistances of the following capacity :—

1. One-ohm resistance, carrying one ampere.
2. One-tenth of an ohm resistance, carrying 10 amperes.
3. One-hundredth of an ohm resistance, carrying 100 amperes.
4. One-thousandth of an ohm resistance, carrying 1,000 amperes.

These resistances should be of manganin strip or wire, the surface being gilt to prevent oxidation and the surface of such sufficient area that the resistances do not become heated beyond 60°C. when kept in circuit for some time.

In using them the electrician may either take their value for granted from the reputation of the maker, or, if possible, and far better, get them checked by some authority. In default he may proceed *ab initio*, and construct for himself low-resistance standards by means of the resistance standards prepared for use in Wheatstone bridge measurements.

The method which may be adopted of creating a standard of low resistance, which shall have approximately a defined value, is as follows:—Let 1, 2, 3, &c. (Fig. 13), be mercury cups, connected, as shown, by resistance coils r_1, r_2, &c. Let there be an even number of mercury cups on each side, and therefore an odd number of resistance coils. Arranged in

this way, the coils r_1, r_2, &c., are in series. If r be the resistance of each coil such as r_1, and there be n coils, the total resistance in series will be $n\,r$. Let two copper combs, A and B, made of very thick high-conductivity copper rod, be provided with claws, which enable them to connect together the mercury cups on each side when dipped into them. Suppose the cups 1, 3, 5, &c., connected on one side, and the cups 2, 4, 6, &c., on the other, then the coils r_1, r_2, &c., will now be in parallel between A and B, and the joint resistance between A and B will be $\dfrac{r}{n}$. The mercury cups must be made of copper amalgamated in the interior, and the ends of the comb
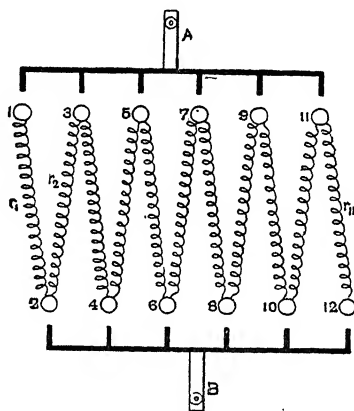
FIG. 13.

terminals be pressed down very strongly on the bottom of these cups when the resistances are thrown into parallel. If R be the joint resistance of the n coils in series, then $\dfrac{R}{n^2}$ will be the joint resistance in parallel. If, however, the resistance of each coil is different, then, if k_1, k_2, &c., be the conductivities of each coil in mhos—that is to say, the reciprocal of its resistance measured in ohms—the total conductivity K, when arranged in parallel, will be $k_1 + k_2 + \&c. = K$, or the total parallel mho conductivity is

the sum of the separate mho conductivities of each coil. Suppose that a current is passed through such resistances in parallel, it distributes itself between the coils, and, if we know the conductivity of each coil, and can measure the difference of potentials between A and B, we have at once the means of calculating the current.

Moreover, we can discover whether the initial resistance of the combined resistance has appreciably changed by the heat due to the passage of the current, for, by raising the forks out from the mercury cups, the n resistances, which were a moment ago in parallel, are now in series, and the actual change in resistance of the n in series is n^2 times the actual change in resistance of the n in parallel. We get, therefore, a resistance of considerable, or, at least, not very small magnitude to determine, and by performing the operation of throwing the coils into series and measuring their resistance on a bridge we can determine within a small amount any change in resistance due to heat generated in any time in the conductors. Hence, generally speaking, the method consists in providing an arrangement by which resistances can be arranged in parallel and traversed by the current to be measured, and then thrown into series to determine what change, if any, the conductor has experienced in resistance, and calculating from this change, when measured in series, the change when arranged in parallel.

Accordingly, the procedure is as follows :— Suppose it is desired to construct a resistance of one-tenth of an ohm and capable of carrying ten amperes. The first step is to measure on the Wheatstone bridge, as subsequently explained, the resistance of a series of wires, and to make them exactly equal. For this purpose select platinoid wire No. 18 S.W.G. and measure the resistance of any length of this wire on the bridge. Then calculate the length of the wire, which will have a resistance of slightly more than one ohm. Prepare in this way ten similar wires. Each platinoid wire is then silver soldered to a square nut of well tinned copper,

which is drilled with a central hole. The resistance of each wire is then made exactly the same by rubbing down with glass-paper all the wires until they are exactly equal in resistance to each other and to the highest in resistance of the ten. This process needs much care and patience. The ten wires having been made absolutely equal in resistance at the same temperature, the true resistance of each one is determined. The ten wires are then joined in parallel by passing a screw through the holes in the ten terminal nuts, and by heating these tinned blocks to the melting point of tin, they are compressed and soldered together into one mass. We have then a resistance of one-tenth of an ohm constructed of ten one-ohm wires in parallel. A potential wire or terminal is then soldered to the terminal blocks as near as possible to the place of contact of the resistance wires and the terminal blocks.

A more careful comparison must then be made between this one-tenth ohm standard and a one-ohm standard coil by means of a low resistance bridge or potentiometer, as subsequently described, and the nearest value determined for the resistance at a known temperature of the one-tenth ohm standard. A much more tedious process of the same nature would result in the construction of a one-hundredth of an ohm standard. When once original standards of low resistance have been standardised or measured by means of the Lorenz apparatus, it is comparatively an easy matter to copy or reproduce them.*

The minimum outfit of carefully standardised resistances which any electrical laboratory should possess is as follows :—

1. A 100-ohm standard.
2. A ten-ohm standard.
3. A one-ohm standard.

the above to be coils suitable for comparison on a

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* For determinations of the value of low resistances by the Lorenz apparatus *see* Prof. J. Viriamu Jones "On Standards of Low Resistance," *The Electrician,* Vol. XXXI., p. 620.

Wheatstone Bridge, but not necessarily adapted for carrying large currents. In addition should be provided :—

4. A one-ohm standard, capable of carrying one ampere.
5. A one-tenth of an ohm standard, for carrying up to ten amperes.
6. A one-hundredth of an ohm standard, capable of carrying up to 100 amperes.
7. A one-thousandth of an ohm standard, capable of carrying up to 1,000 amperes.

The last four coils or strips must be suitable for measurement of current by the fall of potential method, and have potential terminals attached to them.

ᵀn addition to resistances of the above kind, for much work, especially on insulators, it is necessary that the laboratory should possess certain high resistance standards A standard wire resistance *megohm* is, however, a somewhat expensive article. Many cheap substitutes have been proposed in the form of carbon or other admixtures of semi-conductors. For really careful work nothing, however, is satisfactory except a high resistance wire standard, preferably of manganin in the form of ten coils each having a resistance of 100,000 ohms. In the case of these high resistance standards the insulation of the separate coils and of their terminals becomes a very important matter. The coil terminals must be carried on tall corrugated ebonite pillars, and these must be placed a considerable distance apart on an ebonite base. A standard megohm box of this kind must be carefully protected from light and dust. Exposure to light destroys the surface insulation of ebonite.* The coils themselves must be insulated with well-baked solid shellac, and be suspended or carried on glass rods, also varnished. or covered with well-baked shellac.

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* The action of light and air oxidises the sulphur in the ebonite and produces a conducting layer, probably of sulphuric acid. This can be removed by washing with a dilute solution of soda, then thoroughly drying the surface and rubbing with a clean rag moistened with perfectly dry paraffin oil.

§ 6. The Recovery of the Standard or Unit Electric Current.

—We proceed, in the next place, to consider the means and instruments necessary to recover the standard electric current, and to compare currents with the authorised unit of electric current, called the ampere. The practical and legal definition of the ampere or unit of current is based upon the power of the current to cause electrolytic decomposition in a standard electrolyte, such as a neutral solution of the nitrate of silver when electrolysed with a silver anode and a platinum cathode. The British Board of Trade have issued a specification for the electrolytic measurement of unvarying electric currents, and the arrangement of apparatus and process is as follows:—

BRITISH BOARD OF TRADE SPECIFICATION FOR MEASURING ELECTRIC CURRENT.

In the following specification the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time the time average of the current, or if the current has been kept constant, the current itself can be deduced.

In employing the silver voltameter to measure currents of about one ampere the following arrangements should be adopted. The cathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10cm. in diameter, and from 4cm. to 5cm. in depth.

The anode should be a plate of pure silver some 30 sq. cm. in area and 2mm. or 3mm. in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the cathode, the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

Method of Making a Measurement.

The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support to which a binding screw is attached. This copper support must be insulated.

The anode is then immersed in the solution so as to be well covered by it and supported in that position; the connections to the rest of the circuit are made.

Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half-an-hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

The solution is now removed from the bowl, and the deposit is washed with distilled water and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol and dried in a hot-air bath at a temperature of about 160°C. After cooling in a desiccator it is weighed again. The gain in weight gives the silver deposited.

To find the current in amperes, this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed, and by 0·001118.

There seems reason to believe that the above officially-adopted value of the electrochemical equivalent of silver is about one part in a thousand too small, if the ampere so defined is to represent one-tenth of the absolute C.G.S. unit of current.

A careful discussion of the different values obtained for the mechanical equivalent of heat ($=J=$Joule's equivalent) has shown that the best results obtained by the heating of water electrically are in excess of the best results obtained by mechanical friction methods by about one part in 400. Thus at 19·1° by the Paris nitrogen thermometer the values of J are as follows per degree Fahr. :—

By Mechanical Methods.	*By Electrical Methods.*
Joule .... 774 foot-pounds.	Griffiths .... 779·1 foot-pounds.
Rowland .... 776·1 ,,	Schuster & Gannon 778·5 ,,

More recent determinations of the electro-chemical equivalent of silver gives the value 0·0011192 grammes per ampere-second (see *Electrical Review,* September 30, 1898) and the adoption of this constant instead of 0·001118 removes nearly all difference between the value of J when determined mechanically and electrically.

The result of the measurement carried out as above specified will be the time-average of the current if during the interval the current has varied.

In determining by this method the constant of an instrument the current shall be kept as nearly constant as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time-average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading.

The every-day practical measurement of current is best performed by some instrument which gives not merely the time-average, but the precise value at any instant. Such instruments are called *amperemeters* or *ammeters*. Of the many principles employed in the construction of amperemeters none is so well adapted to the construction of a standard instrument as the utilisation of the fact that forces of attraction and repulsion exist between conductors carrying electric currents. These instruments, which are variously called electro-dynamometers, current balances or ampere balances, may all be said to be modifications of the electro-dynamometer, which was first described by Weber in his " Electro-dynamische Maasbestimmungen " (Leipsic, 1846). The principle on which these instruments act was made known, however, twenty-five years previously. On September 25, 1820, Ampère announced to the Academy of Sciences in France that he had discovered the existence of forces of attraction and repulsion between conductors of non-magnetic material traversed by electric currents. The experiments of Ampère and the subsequent elaborate researches of Weber (W. Weber, "Electro-dynamische Maasbestimmungen," Thl. I., s. 10, 1846, Auszug in *Pogg. Annal.*, Bd. LXXIII., s. 193) established the fact that, if there be a fixed and a movable conductor in the neighbourhood of each other, traversed by the same electric current, there will in general be a mechanical stress brought into existence, which will tend to displace the movable conductor and to bring it into a position relatively to the fixed conductor in which the mechanical stress between them is zero. The mutual force

or stress between these mutually influencing portions of the same circuit is, in any given position, proportional to the square of the strength of the current traversing the two portions of the circuit. In the ampere balances invented by Lord Kelvin this stress is measured by comparing it with or balancing it against the weight of certain standard masses; in other words, the electro-dynamic stress between electric circuits is compared with the gravitation stress on a given mass.

The great difficulty which has hitherto presented itself to all who have attempted to design instruments on the electro-dynamometer principle for any but very small currents has been that of getting the current into and out of the movable conductor. The device on which most experimentalists have fallen back is that adopted by Ampère himself, viz., to use mercury cups as a means of constructing a flexible and conducting joint. The use of mercury is open to many objections. The surface gradually becomes oxidised, the cups must be filled and emptied each time the instrument has to be transported, and the joint with impure mercury is by no means exceedingly flexible The great novelty in the ampere balances of Lord Kelvin was the invention of a joint or electric coupling which is excessively flexible, and at the same time capable of being constructed so as to carry with safety any current desired. This was accomplished by the introduction of a device which may be called a metallic ligament. The general principle of its construction, and the mode of rendering a circuit freely movable, yet accessible to a large current, may be described as follows :— Let A A (Fig. 14) be a pair of semi-cylindrical fixed trunnions, which are carried on some form of supporting frame and held with the flat sides downwards. Let B B be two similar trunnions, which project out from the sides of two strips, connecting together a pair of rings C C. The pair of rings and the connecting strips constitute the circuit which is to be rendered movable. A current entering by the trunnion

+ B flows round the two halves of the circuit, as shown by the arrows, and emerges at the trunnion — B. In Fig. 14 the current is shown, for simplicity, dividing round the two rings. The circuit should in reality be shown so that the current goes round both rings in series in figure of eight fashion. This is the case in all but the kilo- and hecto-ampere balances, in which the current divides round the ring, as shown in Fig. 14. To the upper surface of the upper trunnion are soldered a very large number of exceedingly fine copper wires (No. 60 B.W.G.), which are laid close together. These wires are also soldered to the under surface of the lower trunnion. The movable circuit C C thus hangs from the upper trunnion by two ligaments, which appear like thin strips, but which are really composed of an immense
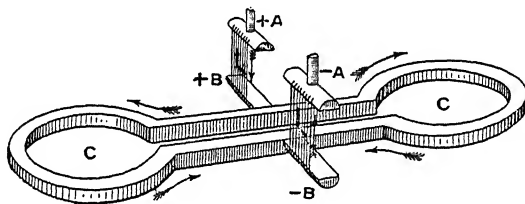
FIG. 14.

number of very fine wires. In Fig. 14 these ligaments are intentionally drawn much longer in proportion to the rest of the figure than they really are in the instrument, the object being to render the mode of suspension clear. This method of suspension enables the conductor C C to vibrate freely like a balance by a motion which is partly a bending of the flexible ligaments, and partly a sort of rolling and unrolling of the lower trunnion on the ligament attached to it. By this ingenious method not only can a heavy copper conducting circuit of the shape shown be suspended as freely as the beam of a good balance, but at the same time a very large current density can be permitted in the flexible ligament, since its great radiating surface, and the freedom with

which heat is taken out of it by conduction into the mass of
the trunnions, allows a proportionately very large current
to be transmitted. If, then, the trunnions A and B are the
electrodes, the method above described affords the means of
passing a very large current into the circuit C C, which yet
at the same time retains within certain narrow limits great
freedom of movement. Let such a suspended conductor be
arranged so as to have a circular conducting circuit of
annular form, briefly called an ampere ring, placed above and
below each of the movable rings of the balanced arm. Let
the connections be made as shown diagrammatically in
Fig. 15, in which it will be seen that the current entering by
the + electrode flows in series through all four fixed
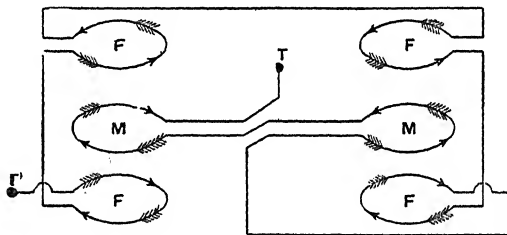
Fig. 15.

ampere rings F, and through the two movable ampere
rings M. An examination of the direction of current flow in
each ring will then show that, in consequence of Ampère's
law (parallel currents in same direction attract, in opposite
direction repel), forces of attraction and repulsion will be
brought into play between each fixed ring and the movable
rings, which tend to lift one ring M and depress the other,
and tilt over the balance arm to which they are attached.
The amperian forces thus exert a couple on the movable
part. To bring back the movable part to its initial position,
which we may suppose to be half-way between each fixed
ring, an equal and opposite mechanical couple must be
applied. We shall in what follows call the movable part

of the conducting circuit the balance coils, and the other portion of the circuit the fixed coils. The operation of weighing an electric current consists, therefore, in bringing first of all the balance coils into a definite sighted position between the fixed coils, then passing the current, and bringing back the balance coils into the sighted position against the displacing electro-dynamic forces by applying to the balanced part a couple produced by a standard weight. The restoring couple is applied as follows :—Attached to the balanced part or movable portion is a stiff metal bar, turned up at the bottom edge so as to form a sort of long shelf. This shelf or tray extends the whole length of the movable beam, and moves or tilts with it when the balanced part is displaced (*see* Fig. 16). At one end of the balanced part is a small V-shaped tray, in which a weight is placed. A standard weight is then placed at the opposite end of the shelf. This weight is of such form as to slide along the shelf easily. When placed at the zero position on the shelf it exactly balances the counterpoise in the V-trough, and the balanced part should be in equilibrium when no current is passing. If it is not so, then a small adjustment can be made by means of a metal flag on the beam similar to that on a gravimetric balance, so as to alter slightly the position of the centre of gravity of the balanced part. If the sliding weight on the shelf is moved along towards the middle the equilibrium is disturbed and a couple brought to bear upon the balanced part, which is proportional to the displacement of the sliding weight from its zero position. If a current is passed through the circuit, and the sliding weight displaced, so as to restore the balance to its sighted position, the current strength is proportional to the square root of the distance by which the sliding weight has to be shifted in order to restore equilibrium. The shifting of this weight is performed by a very ingenious piece of mechanism, which will be understood from the enlarged view (Fig. 17). On the base board of the instrument, and just underneath the shelf on which the
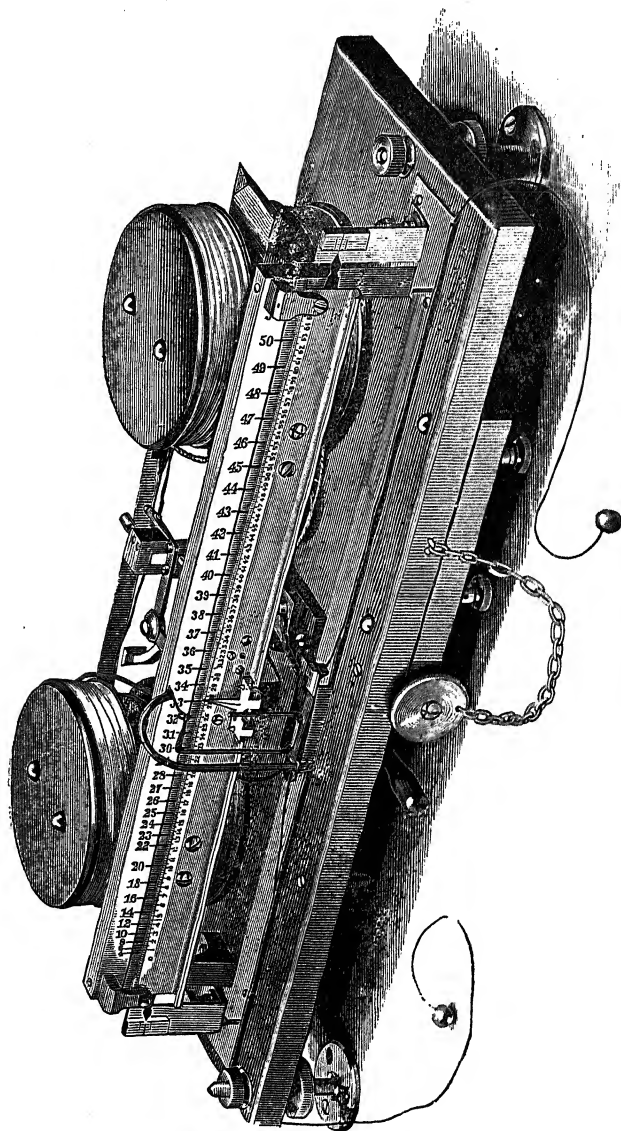
Fig. 16.—Kelvin Ampere Balance.

weight slides, is placed a little metal block, which carries a stout vertical wire extending a little above the shelf carried on the balanced beam. From the top of this wire is carried a pendent wire, which hangs down through a notch in the weight which slides on the shelf. This block, carrying the pendant, is pulled along the railway by a silk cord. When pulled it drags with it the weight on the shelf; but when the string is released the pendant returns to its vertical position and disengages itself automatically from the weight, leaving the balance free to tilt according to the direction of the forces acting upon it. By this means the weight can be shifted along until a balance is obtained, whilst at the same time the whole balance is covered with a glass case to protect it from currents of air.
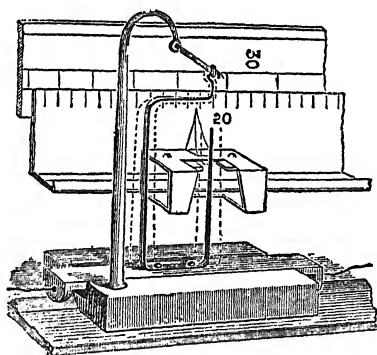
FIG. 17.—Shifting-Weight Apparatus of Kelvin Ampere Balance.

With each current balance four sliding weights and four corresponding counterpoises are provided; the weights being in the ratio of $1:4:16:64$. Thus if with the ampere balance the lightest weight on the sliding tray in a certain position corresponds to a current of half an ampere through the instrument, the next weight at the same place corresponds to one ampere, and with the third weight on it the current is two amperes. The upper edge of the shelf on which the weights slide is graduated into equal divisions, and

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the weight is provided with a sharp tongue of metal, in order that its position on the shelf may be readily and accurately determined. For the purpose of avoiding continual reference to square root tables, another fixed scale is placed behind the shelf, called the inspectional scale. On the upper edge of the shelf, along which the graduations are made, a small notch is cut at those divisions whose numerical denominations are exact squares; thus, corresponding to divisions 1, 4, 9, 16, 25, 36, &c., on the scale engraved on the shelf are cut little notches in its upper edge. The inspectional scale is fixed close behind this, and yet not touching, and at those points on it exactly behind the notches are engraved numbers which are twice the square roots of the corresponding numbers on the shelf; thus, corresponding to 1, 4, 9, 16, 25, &c., on the shelf, the numbers 2, 4, 6, 8, &c., are engraved on the inspectional scale. Since the current passing through the balance when equilibrium is obtained with a given weight is proportional to the square root of the couple due to this weight upon the balance, it follows that the current strength when equilibrium is obtained is proportional to the product of the square root of weight used, and the square root of the distance of this weight from its zero position; but the inspectional scale is so graduated as to show at a glance the square root of the distance of the weight from its zero position, and hence the numbers on the inspectional scale indicate half-amperes, amperes, or double amperes, according to the weight used.

In using these and all other gravity instruments it should be remembered that the acceleration of gravity, and therefore also the weight of a given mass, varies with latitude and with the height above the earth's surface. If M is the mass of a body and g is the acceleration of gravity, then its weight $W = Mg$, and

$$g = 980.6056 - 2.5028 \cos 2\lambda - 0.000003h,$$

where λ = latitude and h = height above sea level in centimetres.
If, then, an ampere-balance is set to read correctly at Glasgow, it will need a correction to be applied if used in other places. The weight of each *weight*, and therefore the corresponding current, will **vary** proportionately with **gravity**

The following table shows for each type of instrument the value per division of the inspectional scale corresponding to each of the four pairs of weights :—

	I.	II.	III.	IV.
	Centi-amperes per division.	Deci-amperes per division.	Amperes per division.	Amperes per division.
1st pair of weights	0·25	0·25	0·25	1·5
2nd ,, ,,	0·50	0·5	0·5	3·0
3rd ,, ,,	1·0	1·0	1·0	6·0
4th ,, ,,	2·0	2·0	2·0	12·0

The fixed inspectional scale shows approximately enough for many purposes the strength of the current; the notches in the top of the aluminium sliding scale or shelf show the precise position of the weight corresponding to each of the numbered divisions on the inspectional scale, and practically annuls error of parallax due to position of the eye. When the pointer on the weight is not exactly below one of the notches corresponding to integral divisions of the inspectional scale, the proportion of the space on each side to the space between two divisions may be estimated inspectionally with accuracy enough for all practical purposes. Thus, we may readily read off 3±·2 or 3±·7 by estimation, with little chance of being wrong by 1·0 in the decimal place. But when the utmost accuracy is required, the reading on the fine scale of equal divisions must be taken, and the strength of the current estimated by aid of a table of square roots supplied with each instrument.

These general principles being understood, we proceed to describe the details of the several types of instruments adapted for various ranges of useful measurement. The range of each instrument is from 1 to 100 or 1 to 25 times, the smallest current for which its sensibility suffices. The ranges of the different types of instrument regularly made are :—

. The	Centi-ampere	balance from	1	to	100	centi-amperes.
,,	Deci-ampere	,, ,,	1	to	100	deci-amperes.
,,	Ampere	,, ,,	1	to	100	amperes.
,,	Deca-ampere	,, ,,	1	to	100	,,
,,	Hecto-ampere	,, ,,	6	to	600	,,
,,	Kilo-ampere	,, ,,	100	to	2,500	,,

Fig. 16 shows the general appearance of the centi-ampere, deci-ampere and ampere balances, and Fig. 18 that of the standard kilo-ampere balance. In these instruments the outer diameters of the fixed coils are slightly greater, and the inner diameters slightly less than those of the movable rings attached to the balance arms. The position of the movable rings when in equilibrium and equi-distant from the fixed ones above and below it, is a position of minimum force, and the sighted position, for the sake of stability, is above it, at one end of the beam and below it at the other, in each case being nearer to the repelling than to the attracting ring by such an amount as to give about $\frac{2}{10}$ per cent. more than the
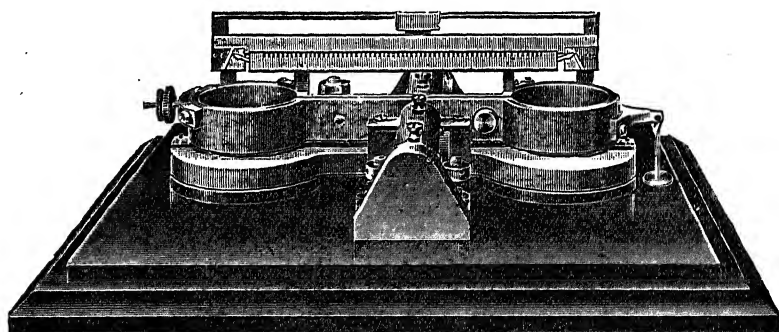
FIG. 18.—Kelvin Kilo-ampere Balance.

minimum force. In order to adapt these balances for alternating currents, a special mode of arranging the conducting circuit has to be adopted. In the balances intended for alternate currents (which may be used also for direct currents) of from five amperes to 250 amperes, the main current through each circle, whether consisting of one turn or of more than one turn, is carried by a wire rope, of which each component strand is insulated by silk covering or otherwise from its neighbour, in order to prevent the inductive action from altering the distribution of the currents across the transverse section of the conductor; whilst to
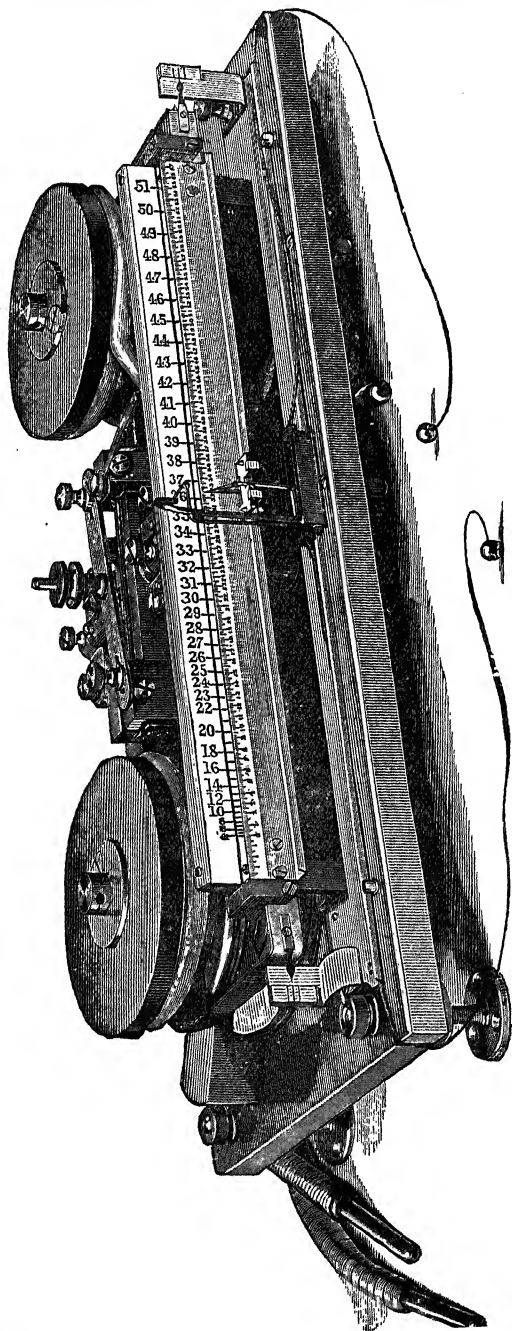
Fig. 19.—Kelvin Hecto-ampere Balance.

avoid induced currents in these parts the coil frames and base-
board are constructed of slate.   The hecto-ampere (*see* Fig. 19)
and kilo-ampere balances are slightly different in arrangement
from the foregoing.   In the last instrument the whole
current to be measured is passed through a single fixed ring
and then divides through the two halves of a movable ring,
which are urged, one up and the other down, by the resulting
attractive and repulsive forces.

For the British Board of Trade Electrical Laboratory Lord
Kelvin designed a special form of ampere balance for
recovering or defining a current of one ampere, of which
the following is the official description :—

CONSTRUCTION AND USE OF THE PRINCIPAL INSTRUMENT FOR
    DETERMINING THE STANDARD OF ELECTRICAL CURRENT.

The complete instrument is shown in perspective in Fig. 20,*
and the details and connections in Figs. 21 and 22.   It is
constructed as follows :—

Balance and Supports.—A sensitive balance, suitable for
the accurate determination of weights up to five kilogrammes
with a beam 16in. in length between the knife edges from
which the scale pans are hung, is fixed on an upper hori-
zontal platform of marble, supported on four marble columns
at a height of 2ft. 3in. above a similar platform fixed on a
suitable stone pier, bedded solidly in a mass of concrete, and
kept clear from contact with the wooden floor of the
laboratory.

Marble Cylinder.—On the lower platform is placed on
three supports, details of which are given below, a cylinder
of white statuary marble, partly hollow (*see* Fig. 20).

The cylinder is thoroughly impregnated with paraffin wax.
A block of ebonite *e* (Fig. 21) is fixed on one side of the
cylinder, and serves as a terminal board.

Fixed Coils.—Two circular grooves are cut completely round
the cylinder, in which are wound coils of No. 18 standard

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* This illustration and many others used in this book are taken from *The
Electrician* by permission of the Proprietors.

wire gauge (0·12cm. diameter), insulated with two coverings
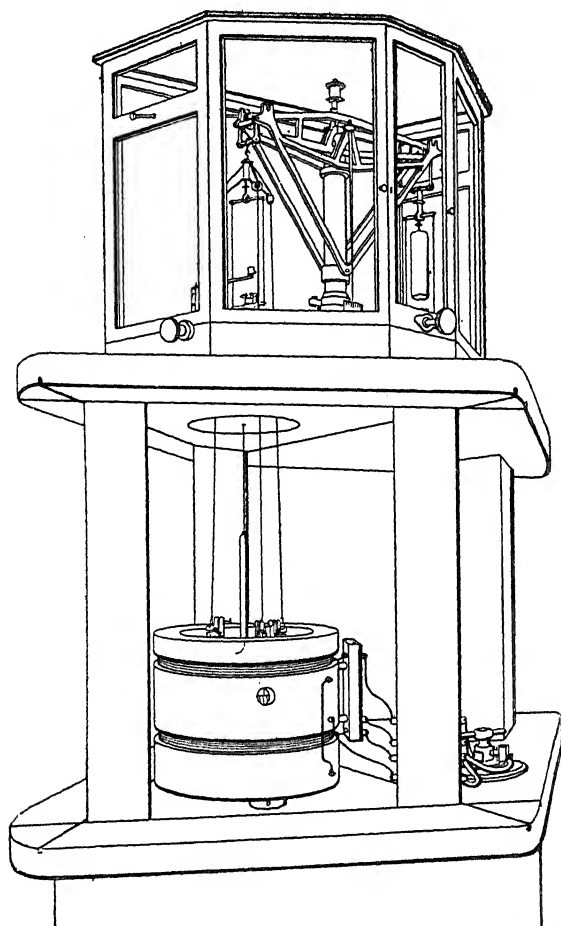of white silk.   The wire was passed through a bath of shellac

Fig. 20.—The Board of Trade Standard Ampere Balance, showing the Weight
raised from the Scale Pan by the Lifting Gear.

and alcohol as it was wound on the marble, and each layer
was well coated with the same material when in its place.
Each coil has 16 turns per layer and 16 layers of wire.

The coils are both wound in the same direction, and the outer ends are strained tight and secured to terminals *t t* (Fig. 21).

The inner ends are passed through the marble and secured to bolts in connection with terminals *i i* (Fig. 21).

Sighting Holes.—Between the grooves on the marble cylinder, at points equi-distant from each other, are bored
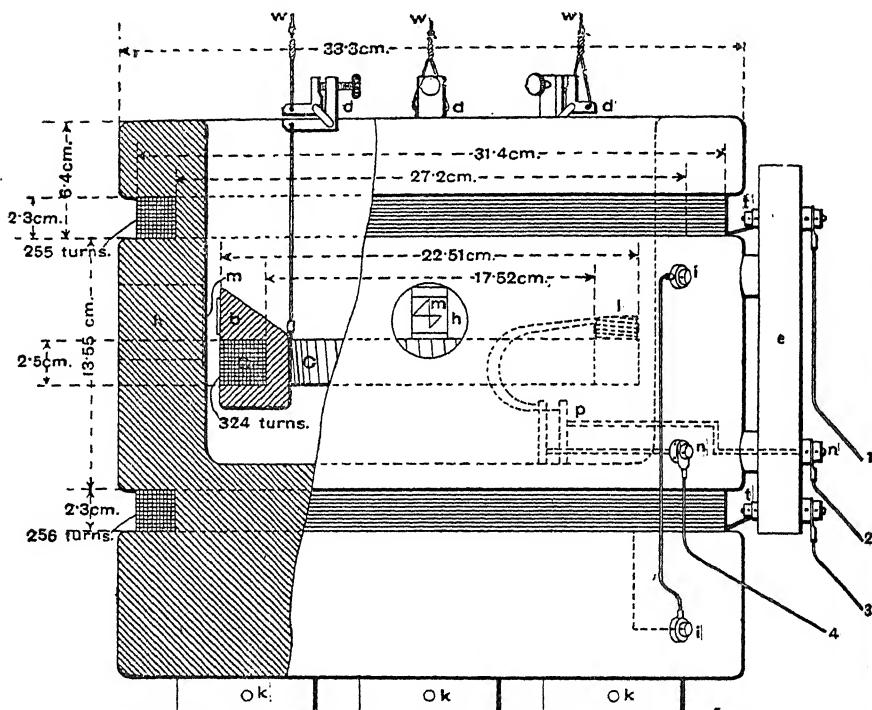

FIG. 21.—Elevation and Part Section of Marble Cylinder of Board of Trade Standard Ampere Balance.

through the marble three sighting holes, one of which is shown at *h* (Fig. 21). These holes are closed on the inside by glass, the exact centres being indicated by the intersection of two straight lines engraved in the glass, and also by the angle of a quadrant of tinfoil affixed to the glass.

Suspended Coil, Suspending Wires, &c.—A circular coil, own in sectional elevation at *c* (Fig. 21), and of the nensions given, is suspended from one end of the beam of balance by means of three gilded phosphor bronze wires 86cm. diameter, *w w w* (Fig. 21). These wires pass ough an aperture in the upper marble platform.

The wire in this coil is the same as in the fixed coils, and imilarly insulated and varnished. There are 18 turns of e in each layer and 18 layers. Each complete layer was wed 24 hours to dry after being coated with shellac nish before the next layer was put on.

he coil is covered with a taping of silk ribbon, closely on to overlap, and varnished with shellac in alcohol, four is, each being allowed to thoroughly dry before applying next one.

he ends of the coil are brought out near each other on upper surface at *l* (Fig. 21). A considerable length of is left for each end, and is formed into a spiral, and then ected horizontally to the centre of the coil, where the wires terminate, and flexible connections are made by silk-covered silver wires, No. 40 S.W.G., from each to posts *p* (Fig. 21), which are connected to terminals *n n*.

iree ebonite blocks *b* (Fig. 21) are fitted over the coil, as n, and secured by a lashing of silk thread, the suspend-vires being attached to brass eyelets screwed into the te.

d d d (Fig. 21) are shown ivory cranks for effecting the al adjustment of the suspended coil.

attachment of the suspending wires to the balance is d by ivory rings, which are hooked on to a three-legged , this being hooked to the stirrup, hung on the knife f the beam (*see* Fig. 21).

scale pan is also hung from this stirrup, as shown in l.

the upper part of each ebonite piece *b*, facing the hole *h*, is fixed a mirror M, the centre being fixed by

the intersection of cross lines and by the angle of a quadrant of tinfoil.

Adjustments.—The proper adjustment of position of the suspended coil is indicated by the coincidence of the angles of the tinfoil quadrants on the mirrors *m*, and the glass back of holes *h*, when the eye is so held that no reflected image can be observed.

Eccentric Movement for Horizontal Adjustment.—A means of fine horizontal adjustment is provided in the supports of the marble cylinder, which are three short slate cylinders, *k k k* (Fig. 21), having projections from their upper and lower surfaces, the upper projection being eccentric and fitting in a hole in the bottom of the marble cylinder, and the lower projection being concentric and fitting in a radial groove in the marble slab.

The Weight.—The iridio-platinum weight shown in Fig. 20 can be lowered into the scale pan, and released from the outside of the case by a system of levers.

The weight is accurately 33·55 grammes.

Enclosure.—The marble cylinder is enclosed by means of three sheets of plate glass, which slide in grooves cut in the marble platforms, and when in position are in contact with the sides of the three marble columns adjacent to the cylinder, these sides being coated with baize, the plan of this enclosure being an equilateral triangle with the axis of the marble cylinder at its centre. The planes of the three glass sheets are respectively parallel with those of the glass at the back of the sighting hole viewed through them.

The balance is enclosed within a case of mahogany and glass (*see* Fig. 20), and arrangements are made as indicated for manipulating the rider weights of the balance and the ampere weight from the outside of this case.

The whole enclosure is fairly dust-tight, and sufficiently air-tight to prevent any disturbance of the weighing operations by currents of air.

Connections.—The connections to the reversing switch Z
(Fig. 22) are brought outside the enclosure by means of rods
passing through the marble column adjacent to the ebonite
block.   Fig. 22 shows these connections, and the direction of
the current through the various parts of the apparatus, the
lower arrows over the fixed coils showing the alteration of
direction caused by operating the switch.
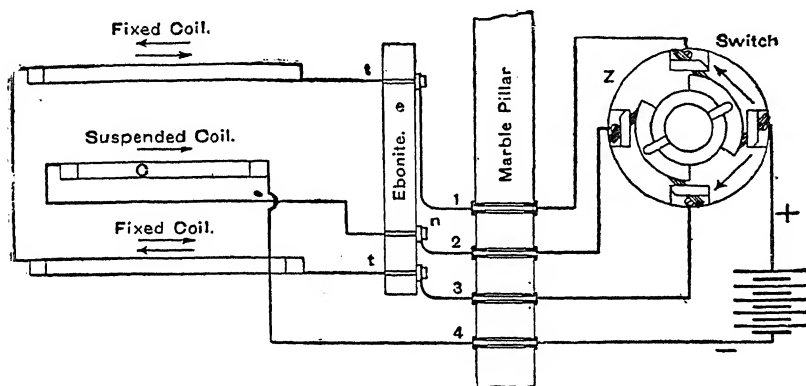
FIG. 22.

Auxiliary Current Balance.

In the construction of the standard ampere balance, the
chief object aimed at has been the securing of the greatest
possible constancy and precision of determination.

On account of the slow period of vibration of the beam,
however, the use of an auxiliary instrument has been found
advantageous.

This is a special form of Lord Kelvin's current balance
instrument arranged to balance with a current of one ampere
passing through the coils when a certain weight is applied to
one end of the beam, equilibrium being also maintained
when, with no current passing, this same weight is applied
to the other end of the beam.

Use of the Instrument.—The standard balance, auxiliary balance, and the instruments to be standardised are connected in circuit with a sufficient number of accumulators and an adjustable resistance, part of which is capable of continuous variation. The whole circuit is arranged so that a current of approximately one ampere will pass on completion of the circuit.

This current is allowed to pass continuously for at least one hour, being directed through the coils of the standard, so as to increase the force of gravity on the suspended coil, and the counterweight in the right hand pan being adjusted from time to time.

At the end of this preliminary period the current is stopped for a short time to take a reading of the zero of the auxiliary balance, and apply a correction by means of the rider weight, if necessary. The current is then again put on, and as rapidly as possible regulated by means of the adjustable resistance to exactly the value of one ampere, as indicated by the auxiliary balance.

The counterweight in the right hand pan standard balance is also exactly adjusted.

The switch Z (Fig. 22) is then operated to reverse the current in the fixed coils, and at the same time the weight is lowered into the left-hand scale pan.

The balance should then still be maintained when the current, after adjustment, if requisite, is exactly one ampere by the auxiliary balance.

The above-described laboratory forms of Kelvin ampere balances are made in two types. They are constructed in one form with a sliding weight and scale so as to enable any constant current to have its ampere value determined, or they are constructed with fixed weights so adjusted that on gradually varying the current the balance tips over at certain fixed standard currents, say, one ampere or ten amperes, &c.

In this last mode of use of the balances there is no sliding weight and tray but the balances have to be employed in

connection with a carbon rheostat or some means for con-
tinuously varying the current strength with extreme precision.
In the case of a primary standard laboratory balance it is
better to have it arranged in the second manner so that by
its means the observer may weigh out, as it were, an ampere
or ten amperes rather than determine the true value of a
non-integral current.

In this last manner the standard ampere balance (*see*
Fig. 23) of the Board of Trade is used, and it is desirable,
where extreme accuracy is required, that the laboratory
should be provided with a one-ampere fixed standard balance
of the above-mentioned description.
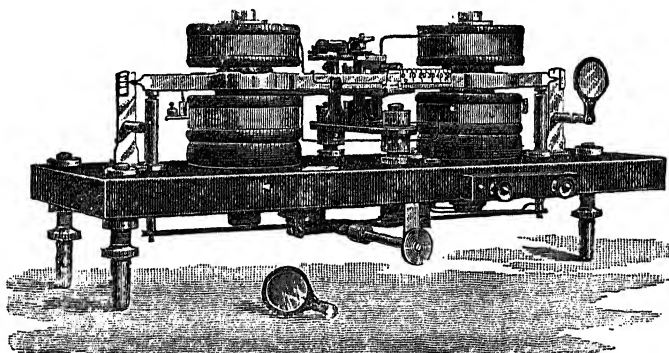
FIG. 23.—Standard One-ampere Kelvin Balance as made for the Board of
Trade Electrical Laboratory.

Deferring, for the present, details of the standardisation of
these ampere balances, we may say that the electrical
laboratory should be provided, at the very least, with two
standard Kelvin ampere balances, the most useful being the
deci-ampere balance weighing from one-tenth to ten amperes,
and a deca-ampere balance weighing from one to a hundred
amperes.

These balances should be carefully set up on level stone
shelves of such height that an observer seated on a stool can
comfortably read the scale and manipulate the rider.

After setting up the deci-ampere balance it should be most carefully standardised by a silver voltameter, using the process described in the Board of Trade specification. Afterwards the balance should not be moved or touched except to regulate the position of the equilibrium flag before each weighing. A comparison should then be made between the standard Clark or Weston cell and the balance as follows :—
Pass through the ampere balance and a tenth of an ohm standard coil a current of about six or seven amperes and regulate this current to perfect steadiness. Then measure with the potentiometer the potential difference between the terminals of the standard resistance and calculate from this the value of the current. The current as read by the ampere balance and the current as determined by the standard cell and resistance should agree within one-tenth of 1 per cent., or to one part in a thousand.

If any refined or careful measurements are to be made the director of the laboratory should spare no pains to make sure that he is in possession of the means of recovering or producing the ohm, the volt and the ampere in his own laboratory, and that these should be in exact agreement as far as possible with the Board of Trade standards and with each other.

It is desirable that the special appliances which form the primary electrical standards of the laboratory—the standard ohm, the standard Cadmium or Weston cell or the standard Clark cell and the standard deci-ampere balance—shall not be used for every-day work, but kept for special verification purposes.

The laboratory being in the above-described manner provided with means for making the three fundamental comparisons of length, mass and time by means of a standard metre, standard gramme or kilogramme weights and chronometer must also be provided in the best possible manner with the appliances for making the three principal electrical measurements of resistance, electromotive force and current

by comparison with the standard ohm, standard cell or voltage and standard current. Before describing the standards of electromotive force it is necessary to mention certain current regulating devices which are essential requisites.

§ 7. **The Regulation of Current.**—One of the most frequently required operations in the electrical laboratory is that of the regulation of a current by means of resistances.

Each working bench or table should be provided with terminals between which a constant potential difference of 100 volts is maintained. From these terminals, however, have to be taken currents of the desired strength. This is best done by providing the laboratory with an outfit of regulating resistances which, when placed in series with any apparatus and across the main terminals, can permit only the desired current to flow.

After many trials and experiments the author devised the following form of working resistance, which is exceedingly useful in a laboratory:*—Each element of the resistance is called a *cage* (*see* Fig. 24), and is made in the following manner. A brass rod about 3ft. long carries on it two porcelain head-pieces or discs having porcelain pin projections on their upper surfaces. One of these is fixed to the rod and the other is movable, but is pushed outwards by a strong spiral spring. To a terminal wire attached to the fixed head-piece is soldered one end of a fine wire of platinoid, reostene or any high-resistance wire. This wire is generally about No. 32 gauge. The movable head-piece is pressed in a little way, and the resistance wire is then laid backwards and forwards over the pin projections on the head-pieces so as to form a zigzag conductor; the end of the resistance wire being then finally connected to another terminal wire passing through the fixed head-piece. When the *cage* is so made the movable head-piece is released and the spiral spring forces it out, and thus keeps the resistance

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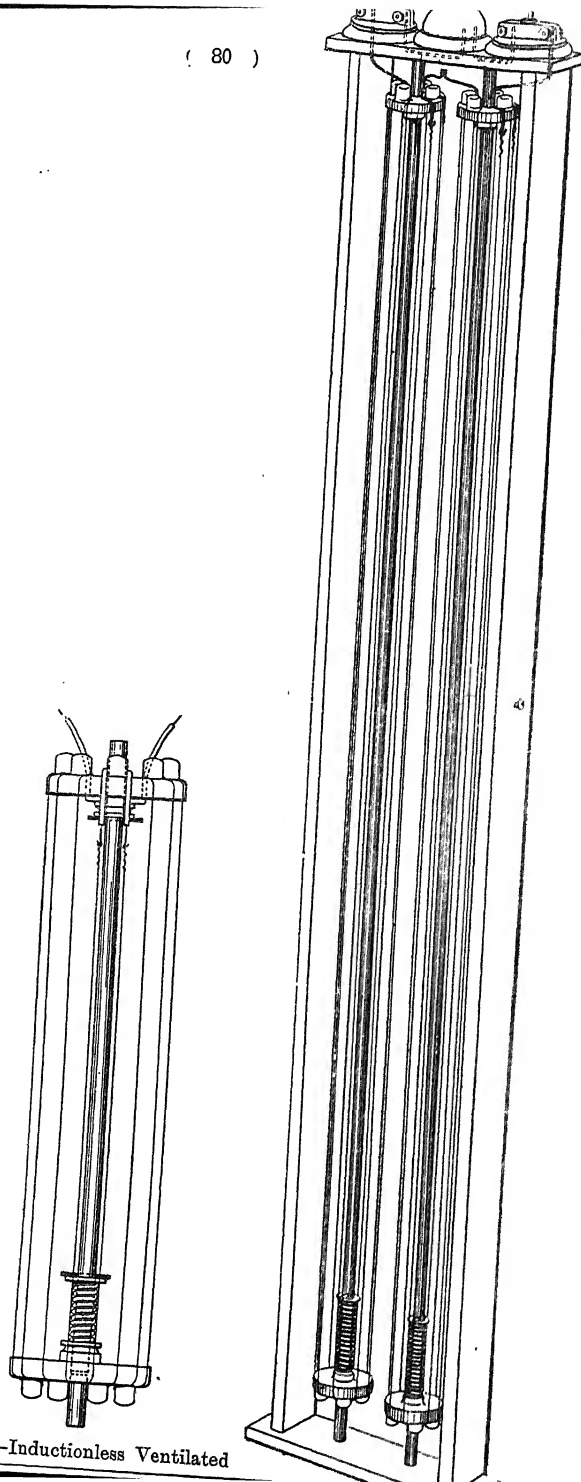* See *The Electrician*, Vol. XXXVI., p. 476.

FIG. 24.—Inductionless Ventilated

wire always tight, even although it expands on heating. By a proper selection of the resistance wire a perfectly ventilated and practically non-inductive resistance may be made in this manner, having a resistance of 100 ohms or more which, will carry safely, without undue heating, a current of one ampere.

A series of these cages can then be fixed in a wood frame placed on castors, with all the several resistances connected in parallel between two omnibus wires; each resistance being provided with its own switch. It will then be seen that the whole resistance frame can be put across the 100-volt terminals, joined in series with a switch and with any apparatus to be traversed by the current, and that the current;
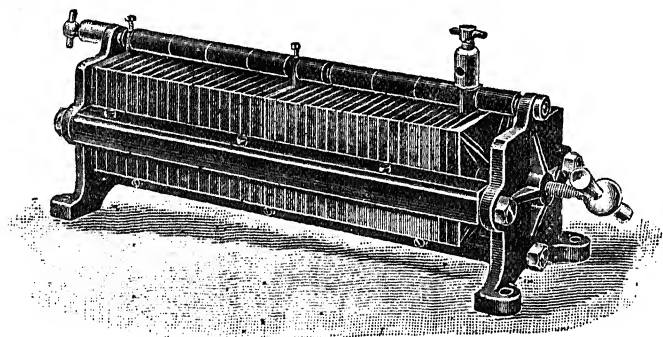
FIG. 25.—Paul's Carbon Plate Rheostat.

which passes through the circuit can then be regulated by the number of cages which are switched on in parallel. When currents of less than one ampere are required these can be obtained by joining cages in *series*. In this case each cage can be coupled with a switch in parallel with it, so that when the switch is *on*, the cage resistance is cut out of circuit.

In addition to a set of the above cages it is necessary to possess several carbon rheostats. One of the most convenient forms of this regulating resistance consists of plates of hard battery carbon. or graphite roughened on the surface and about 3in. square (*see* Fig. 25). A series of about 30 to

50 of these plates are placed in an iron frame and prevented from touching the guide rods by porcelain or fibre insulators. The carbon plates can be more or less squeezed by a strong screw. Metal plates with screw terminals can be slipped in between any carbon plates, and, by means of the pressure applied, a very gradual variation can be made in the interposed carbon resistance. It is essential that the carbon plates shall be hard but not smooth, and the best plates are those *cut* out of gas-oven graphite.
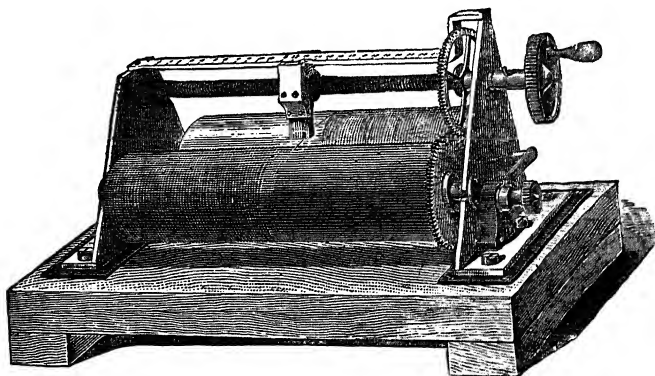
Fig. 26.—Kelvin Wire Rheostat.

The above resistances are a great convenience in the laboratory. They can be used to carry large currents up to 50 or 100 amperes, but they have not a very great range of resistance variation.

For the gradual variation of resistance Lord Kelvin's form of wire rheostat, in which a platinoid wire is wound off a metal cylinder on to a marble or porcelain cylinder, will be found convenient (*see* Fig. 26). In this appliance the resistance wire is kept tight by means of a clock spring in the interior of the metal cylinder, which always acts so as to keep a little tension on the resistance wire.

For many purposes a form of rheostat devised by Mr. Shelford Bidwell is useful. This consists of a slate or

ı cylinder on which a platinoid wire is wound in a
roove, one end of the resistance wire being insulated
other fixed to a metal plate which forms the carrier
ylinder. The cylinder is centred on a long screwed
ₑ screw on which is cut to the same pitch as the
roove on the cylinder. This screw rod works in
earings. A fixed spring contact presses against the
wire, and the other end of the platinoid wire is in
ontact with one screw bearing. As the cylinder is
ound a greater or less length of platinoid wire is
d between this spring terminal and the fixed terminal.
rm of rheostat is substantial and works well if the
contact are kept clean. The contact spring should
with platinoid. It is a good plan to keep these
in boxes with glass lids, the winch handle alone
g, so that dust does not fall on the resistance wire
ts. If it does, the resistance is apt to vary in a
ₑ manner as the handle is turned.
ıe cases incandescent lamps can be used for regu-
istances. It is useful to have some boards on
ₑ mounted half-a-dozen key sockets, wired up in
ₑtween two brass terminal screws. If incandescent
placed in these sockets the resistance between the
can be decreased by steps by switching on lamps.
ıt given out by the lamps is objectionable it can be
ff by dropping over each lamp an asbestos card-
d or cylinder like a canister, the top of which is
th small holes for ventilation. Incandescent lamps
ces, owing to the light and heat given out by them,
ₑarly so convenient as the resistance cages above

regulation of currents for dynamos and motors
ns of rheostat have been introduced. One con-
ırm, as designed by the Author for use in the
aboratory at University College, London, consists in
ₑ from porcelain insulators placed against the wall

of the dynamo room long wires of nickel steel or eureka. These wires are kept tight when expanded by heat by means of a spiral bell spring attached to the extremity of the wire. The wires are electrically connected in parallel as required by means of a parallelising switch (*see* Fig. 27). The length of each wire is adjusted so that the current through it, when the difference of potential of its extremities is 100 volts, does not

Manganese
Steel Wire.

Steel Spring.

Omnibus Bar.

Slate
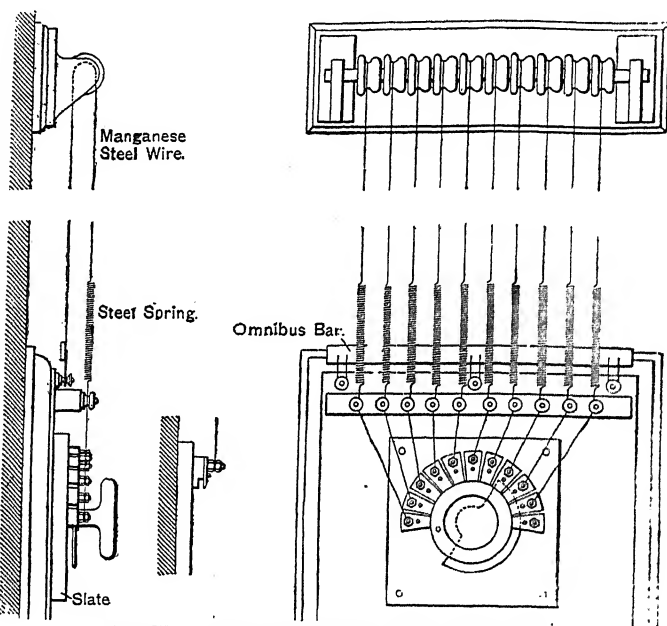
FIG. 27.—University College Dynamo Rheostate.

exceed that value which will make it too hot to touch with the hand. The total resistance between the terminals is then *reduced* step by step by adding resistance wires in parallel. It is much better thus to reduce resistance by equal steps by adding equal resistances in parallel than to increase it by adding equal resistances in series. In the former case the current increases by equal increments per step, and in the latter it decreases in harmonic ratio per step of resistance.

In cases where large currents have to be regulated or varied continuously liquid resistances may be sometimes advantageously employed. One useful form is **Lyon's** Liquid Resistance (*see* **Fig.** 28). This consists of two zinc

Fig. 28.—Lyon Liquid Rheostat.

cones fitting each other, which are immersed in a strong solution of zinc sulphate, and by means of a screw or lever the distance of these metal cones can be varied from perfect metallic contact to a position in which a considerable length

of electrolyte is interposed between them. The resistance can thus be gradually varied. If continuous currents are employed with it, the direction of the current through the resistance should be changed every few minutes by a current reverser, and the current density or amperes per square foot of zinc con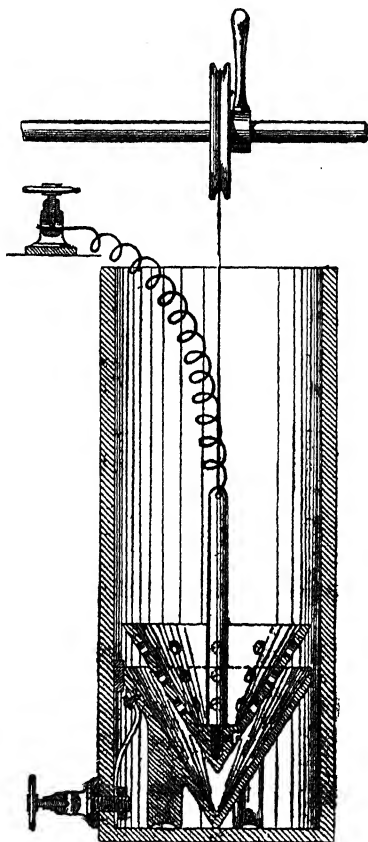e surface should not exceed the limit at which zinc begins to be deposited in irregular crystals or trees on the cathode cone.

The subject of power absorbing resistances will be considered in the chapters dealing with dynamo and transformer testing.*

§ 8. **The Practical Standard of Electromotive Force.**— The practical unit of electromotive force is equal to the terminal difference of potential produced when a current of one ampere flows through a resistance of one ohm. The most satisfactory method of recovering a known difference of potential—say, of one volt—is to pass a current of one ampere, measured by a standard ampere balance, through a suitable resistance of one ohm. There are, however, many cases in which this is not a convenient method of recovering a known difference of potential. Hence attention has been directed to the construction of a standard of electromotive force which shall produce directly a known difference of potential between two terminals. Experience shows that, although the ultimate standard of electromotive force must be recovered by the passage of a known current through a

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* The following references to papers or information on liquid resistances and power absorbing resistances may be found useful :—"Lyon's Liquid Resistances," *The Electrician*, Vol. XXVI., p. 759. "Water Rheostats," G. T. Hanchett, *The Electrician*, Vol. XXXVII., p. 833. "Water Rheostats," *The Electrician*, Vol. XL., p. 696, contains useful facts and figures ; see also *Science Abstracts*, Vol. I., p. 356. "Water Resistance for Alternator Testing," *The Electrician*, Vol. XLI., p. 279. "Motor Starting Resistances," Pochin, *The Electrician*, Vol. XXXIX., p. 38. "Commercial Forms of Electricial Resistances," L. B. Atkinson, *The Electrician*, Vol. XL., p. 863. "Current Carrying Capacity of Wires of Different Materials," L. B. Atkinson, *The Electrician*, Vol. XLI., p. 18. "Dissipation of Electric Energy in Sheets and Wires of Metal," *The Electrician*, Vol. XXXVIII., p. 17. "Construction of Commercial Resistances," D. K. Morris, *The Electrician*, Vol. XXXIII., pp. 605, 627, 667.

known resistance, yet the adoption of a voltaic or electro-chemical standard of electromotive force, by the use of a standard voltaic cell, has considerable practical convenience.

Great attention, therefore, has been given of late years to the minute details of the construction of certain voltaic cell standards of electromotive force. Experience has shown that a very convenient standard of electromotive force is obtained by the use of some modification of the mercury-zinc cell, which was first proposed by Mr. Latimer Clark as a standard of electromotive force, and made in 1873.* The elements of this cell are mercury, mercurous sulphate, zinc sulphate and zinc. Since the above date other forms of voltaic cells have been proposed as standards of electromotive force. The various species and modifications of standard mercury cell in use at present are as follows :—

(*i.*) THE ORIGINAL FORM OF CLARK CELL.
(*ii.*) THE MUIRHEAD FORM OF CLARK CELL.
(*iii.*) THE RAYLEIGH H FORM OF CLARK CELL.
(*iv.*) THE CARHART FORM OF CLARK CELL.
(*v.*) THE BOARD OF TRADE FORM OF CLARK CELL.
(*vi.*) THE REICHSANSTALT FORM OF CLARK CELL.
(*vii.*) THE CALLENDAR FORM OF CLARK CELL.
(*viii.*) THE WESTON CADMIUM CELL.
(*ix.*) THE REICHSANSTALT H FORM OF CADMIUM CELL.
(*x.*) THE HELMHOLTZ CALOMEL CELL.

These cells are not employed as current generators, but merely as standards of electromotive force by using them as subsequently to be described. A few words of description may be given of each of these cells.

(*i.*) *The Original Clark Cell.*—The original form of Clark cell, suggested by Mr. Latimer Clark in 1873 as a standard of electromotive force, consisted of a glass cup or beaker in the bottom of which is placed some pure mercury. Into this mercury a platinum wire dips, which is sheathed throughout its length with glass or gutta percha except at the extreme

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* See *Telegraphic Journal*, Vol. I., p 9.

end where it dips into the mercury. Above the mercury rests a paste formed of mercurous sulphate mixed with zinc sulphate. Above the paste there is a saturated solution of zinc sulphate, and in this latter is dipped a rod of pure zinc. The zinc has a platinum wire soldered to it, and the two wires connecting the zinc and mercury are brought up to insulated terminals (*see* Fig. 29).

(*ii.*) *The Muirhead-Clark Cell.*—In the above original form the Clark cell is not very portable and cannot be turned upside down without producing a change in its
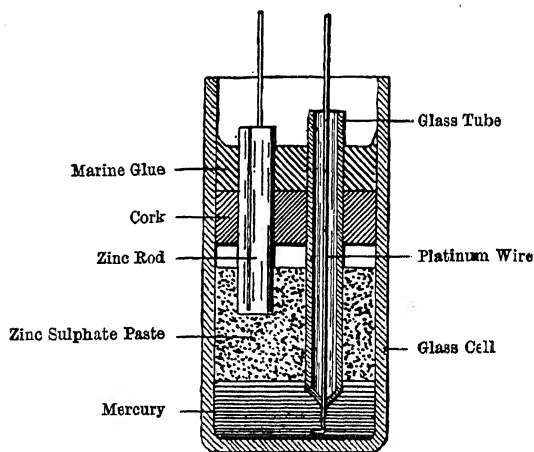
Fig. 29.--Original form of Clark Cell.

electromotive force by bringing the mercury in contact with the zinc and thus contaminating the mercury. An improved form of cell was therefore devised by Dr. Muirhead, in which this difficulty is overcome.

In the Muirhead cell there is no large free mass of fluid mercury. The end of the platinum wire is coiled into a spiral, and this platinum is well amalgamated, and when dipped in mercury it retains a globule of mercury within the spiral by capillary attraction, and this is so adherent that it is not easily detached. The spiral of amalgamated

platinum is buried in the mercurous paste (*see* Fig. 29); above this is placed the saturated solution of zinc sulphate, and in this latter a zinc rod.

A Muirhead cell, if properly sealed, can be sent by parcels post, or otherwise carried about without injury to its electromotive force.

(*vii.*) *The Rayleigh* H *form of Clark Cell.*—Lord Rayleigh investigated in 1885 with great care the causes of the differences in electromotive force between Clark cells set up

Marine Glue

Cork

Zinc Rod

Glass Tube

Zinc Sulphate

Platinum Wire

Mercurous Sulphate

Platinum Spiral
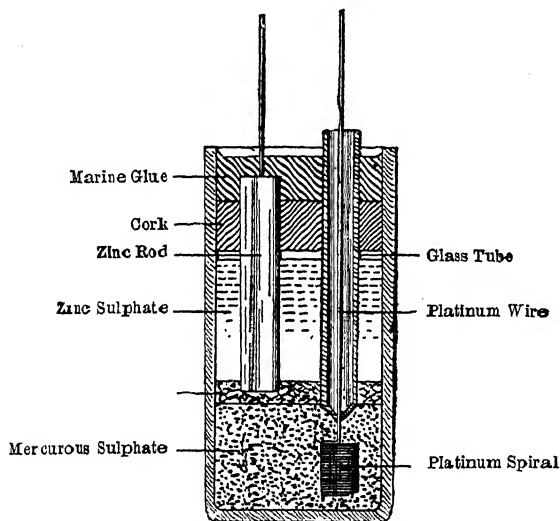
FIG. 30.—Muirhead form of Clark Cell.

by various persons using apparently the same quality of materials and equal care.

He came to the conclusion that the greatest source of error was due to variations in the density of the zinc sulphate solution. The electromotive force of the cell depends upon the concentration of the zinc sulphate solution. If the solution is not a saturated solution the electromotive force of the cell is too high. In some cases the zinc sulphate solution may be *super-saturated*, especially

when the cells have been heated during or after charging. Lord Rayleigh summarises the sources of variation of E.M.F. in Clark cells as follows:—The E.M.F. may be too high (1) because the mercurous paste is acid; (2) because the paste is not saturated with zinc sulphate. The first fault tends to cure itself, and is rarely found after cells are a month old. The second is the usual cause of variation. If the E.M.F. is too low it may be (1) because the cell has become dry, in which case the drop in voltage will be progressive; (2) the solution is super-saturated with zinc sulphate; or (3) the mercury is impure. It follows that to secure the correct or normal E.M.F. the mercury used should
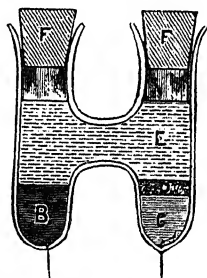
FIG. 31.—Lord Rayleigh's H form of Clark Cell.

B Amalgam of Zinc. C Pure Mercury. D Mercurous Sulphate. E Saturated Solution. of Zinc Sulphate. F Corks.

be free from other metals. It must, therefore, be distilled at a low temperature in vacuo. Next, the zinc sulphate solution must be at all temperatures saturated, and hence solid crystals of zinc sulphate must be present in the cell. Thirdly, the cell must be hermetically sealed with marine glue to prevent evaporation; and, fourthly, the cell must not be heated during manufacture. The zinc sulphate must be rendered neutral by adding a little carbonate of zinc.

Lord Rayleigh devised a convenient form of cell called the H form of cell. This cell consists of a pair of test-tube-shaped glass vessels connected by a horizontal tube or channel (see Fig. 31). In one side tube is placed some pure mercury,

and in the other an amalgam of mercury and zinc. Platinum wires sealed through the glass make contact with the metals. Over the pure mercury is put a paste made of mercurous sulphate and zinc sulphate. The cell is filled up to above the level of the horizontal tube with a saturated solution of zinc sulphate, and the open ends are closed with corks sealed over with marine glue. This H form of cell is very convenient for many experimental purposes.
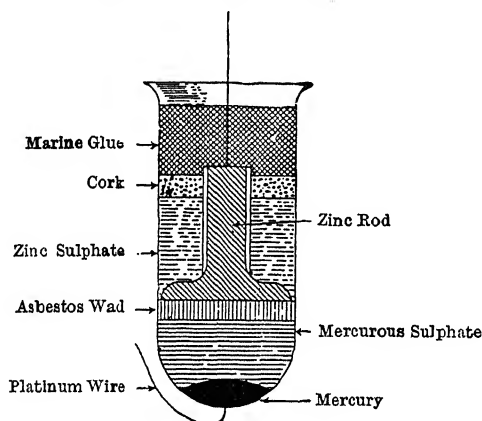
FIG. 32.—Carhart Clark Cell.

Lord Rayleigh found the electromotive force of the H form of cell to be 1·434 volts at 15°C.* and the temperature coefficient to be 0·08 per cent. per degree, or the E.M.F. in volts at t°C.$=E_t$, to be given by the formula—

$$E_t = 1·434 \{1 - 0·00077 (t - 15)\}.$$

A more exact expression for the temperature correction is obtained if the simple coefficient 0·00077 is replaced by the quantity $0·00078 + 0·000017 (t - 15)$. If the electrochemical equivalent of silver is taken as 0·001119 instead of 0·001118, then the E.M.F. of the Clark cell becomes 1·4327 volts at 15°C. and not 1·4342.

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* The electromotive force of the **H** form of Clark cell, as determined by K. Kahle and W. Wien is 1·4488 volts at 0°C., or 1·4322 volts at 15°C. The value now taken is 1·4326 volts at 15°C.,

(*iv.*) *The Carhart-Clark Cell.*—In this cell the zinc is made in the form of a plunger or piston (*see* Fig. 32), and the shank or piston rod is covered with glass. In between the bottom of the zinc and the mercurous paste is interposed a wad of asbestos and the cell is sealed.

(*v.*) *The Board of Trade form of Clark Cell.*—The British Board of Trade Electrical Department have issued a specification for the preparation of standard Clark cells, which is a little elaborate and enters into great detail as to their mode of manufacture. The following is the Board of Trade specification for the preparation of a Clark cell :—

ON THE PREPARATION OF THE CLARK CELL.

Definition of the Cell.

The cell consists of zinc or an amalgam of zinc with mercury and of mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess.

Preparation of the Materials.

1. *The Mercury.*—To secure purity it should be first treated with acid in the usual manner, and subsequently distilled in vacuo.

2. *The Zinc.*—Take a portion of a rod of pure redistilled zinc, solder to one end a piece of copper wire, clean the whole with glass paper or a steel burnisher, carefully removing any loose pieces of the zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

3. *The Mercurous Sulphate.*—Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle ; drain off the water, and repeat the process at least twice. After the last washing drain off as much of the water as possible.

4. *The Zinc Sulphate Solution.*—Prepare a *neutral* saturated solution of pure (" pure re-crystallised ") zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding zinc oxide in the proportion of about 2 per cent. by weight of the zinc sulphate crystals to neutralise any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30°C. Mercurous sulphate treated as described in 3 should be added in the proportion of about 12 per cent. by weight of the zinc sulphate crystals to neutralise any free zinc oxide remaining, and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

5. *The Mercurous Sulphate and Zinc Sulphate Paste.*—Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals

of zinc sulphate from the stock bottle to insure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30°C. Keep the paste for an hour at this temperature, agitating it from time to time, then allow it to cool ; continue to shake it occasionally while it is cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass ; if this is not the case add more crystals from the stock bottle, and repeat the whole process.

This method insures the formation of a saturated solution of zinc and mercurous sulphates in water.

To set up the Cell.

The cell may conveniently be set up in a small test-tube of about 2cm diameter and 4cm. or 5cm. deep (*see* Fig. 33). Place the mercury in the bottom
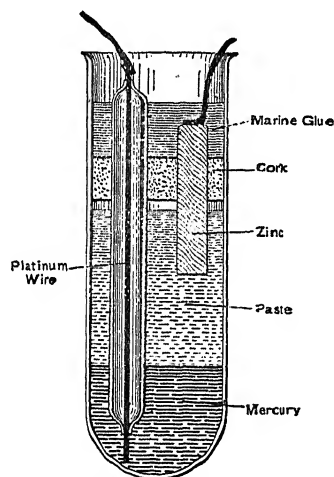
FIG. 33.—Board of Trade form of Clark Cell.

of this tube, filling it to a depth of, say, 0·5cm. Cut a cork about 0·5cm. thick to fit the tube ; at one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire ; at the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube. Wash the cork thoroughly with warm water, and leave it to soak in water for some hours before use. Pass the zinc rod about 1cm. through the cork.

Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube. The ends of the wire project from the ends of the tube ; one end forms the terminal, the other end and a portion of the glass tube dip into the mercury.

Clean the glass tube and platinum wire carefully, then heat the exposed end of the platinum red hot and insert it in the mercury in the test-tube, taking care that the whole of the exposed platinum is covered.

Shake up the paste and introduce it without contact with the upper part of the walls of the test-tube, filling the tube above the mercury to a depth of rather more than 1cm.

Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least 24 hours before sealing, which should be done as follows :—

Melt some marine glue until it is fluid enough to pour by its own weight, and pour it into the test-tube above the cork, using sufficient to cover completely the zinc and soldering. The glass tube containing the platinum wire should project some way above the top of the marine glue.

The cell may be sealed in a more permanent manner by coating the marine glue, when it is set, with a solution of sodium silicate, and leaving it to harden.

The cell thus set up may be mounted in any desirable manner. It is convenient to arrange the mounting so that the cell may be immersed in a water bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

In using the cell sudden variations of temperature should as far as possible be avoided.

The form of the vessel containing the cell may be varied. In the H form the zinc is replaced by an amalgam of 10 parts by weight of zinc to 90 of mercury. The other materials should be prepared as already described. Contact is made with the amalgam in one leg of the cell, and with the mercury in the other, by means of platinum wires sealed through the glass.

The cell resulting from operations carried out in accordance with the above specification has been criticised considerably, and it has been stated that in consequence of *diffusion-lag* there is always some delay in the change of saturation of the zinc sulphate solution when the temperature changes. Hence it is said that it is not possible to obtain the true electromotive force of the cell to a greater accuracy than 0·1 per cent. by applying the ordinary temperature correction.[*]

(*vi.*) *The Reichsanstalt form of Clark Cell.*—A specification for the preparation of a modification of the H form of Clark

[*] *See* Prof. Ayrton and Mr. Cooper, *Proc. Roy. Soc., Lond.*, Dec., 1895. Also Mr. Cooper "On the Permanency of the Board of Trade Clark Cell," *The Electrician*, Vol. XL., p. 748.

cell has been issued in Germany on the basis of the experience gained at the Physikalisch-Technische Reichsanstalt in Berlin. The following are the details of these instructions as given by Dr. K. Kahle* :—

Definition and Properties of the Cell.

The cell contains mercury as the positive electrode, amalgamated zinc as the negative electrode, and as electrolyte a concentrated solution o zinc sulphate and mercurous sulphate. Its E.M.F. is 1·4328 Internationa¹ volts at 15°C. and between 10°C. and 25°C. decreases by 0·00115 volt with ɛn increase of temperature of 1deg.

Construction of the Cell.

The vessel used for the cell consists, as shown in Fig. 34, of two vertical branches closed at the bottom, and joined at the top into a neck closed by a

FIG. 34.—Reichsanstalt form of Clark Cell.

ground-glass stopper. The diameter of the branches should be at least 2cm. and their length 3cm. The neck of the vessel should be 1·5cm. wide, and at least 2cm. long. Platinum wires about 0·4mm. thick are fused into the bottom of both branches.

The vessel is filled in a manner depending upon whether the cell is to be used at the place of construction or is to be transported.

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* See *The Electrician*, Vol. XXXI., p. 265-6.

In the former case pure mercury is poured into one branch, and into the other amalgam of about 90 parts mercury and 10 parts zinc, which is fluid when hot and solidifies on cooling.   The platinum wires must be completely covered by the mercury and the amalgam respectively.   Upon the mercury is poured a layer of paste 1cm. deep, made by rubbing together mercurous sulphate mercury, zinc sulphate crystals, and concentrated solution of zinc sulphate This paste and the amalgam are then both covered with a layer 1cm. deep of zinc sulphate crystals, and finally the whole vessel is filled with concentrated zinc sulphate solution until the stopper on being introduced just touches the surface.   Care should, however, be taken that the vessel contains a small air-bubble, since that prevents it bursting in the case of a great rise of temperature.   At the final closing of the vessel the glass stopper is brushed over at its upper edge with shellac dissolved in alcohol and then firmly inserted.

If the cell is to be portable, a circular electrolytically-amalgamated piece of platinum foil, about 1cm. long and 0.1mm. thick, takes the place of the mercury, and is firmly attached to the platinum wire introduced through the bottom.   Zinc amalgam forms, as before, the negative electrode, and is covered with a layer of zinc sulphate crystals 1cm. deep.   The rest of the vessel is filled up to the stopper with mercurous sulphate paste.   The final closing is affected as already described.

Preparation of the Materials to be Used in the Cell.

Mercury.—All mercury to be used in the cell should be purified by the ordinary processes, and distilled *in vacuo*.

Zinc.—The commercial pure zinc may be employed. To prepare   te amalgam, add one part zinc to nine parts mercury, and keep both in a porcelain dish at 100deg., stirring gently until the zinc is completely dissolved in the mercury.

Sulphate.—Before use, test the commercial zinc sulphate for acid with litmus and for iron with potassium sulpho-cyanide.   If it is sufficiently pure it may be at once re-crystallised in the way detailed below.   If it contains appreciable traces of free acid, equal parts of the zinc sulphate and distilled water are boiled with zinc filings in a suitably formed porcelain dish until no further gas is given off at the zinc, and the solution shows after cooling a white, or, in the presence of ferric hydrate, a brownish precipitate of zinc hydrate.   If the solution is free from iron it may be filtered off after standing for two days. Otherwise it is again heated to 60°C. or 80°C., and electrolysed for six hours by a current not exceeding 0.2 amperes introduced by two pieces of platinum foil of about 50 sq. cm. surface suspended in the liquid.   The liquid having cooled over night, litmus is again employed to test whether any acid has been formed during electrolysis.   In that case the boiling with zinc filings must be repeated and the solution again electrolysed by weak currents.   During this whole treatment care has to be taken that the concentration of the solution remains approximately constant.   It is, therefore, well to cover the vessel containing the solution with a glass plate, so that but little water vapour can escape.   As soon as the solution is sufficiently free from acid and iron it is filtered off.   To each litre of the filtrate about 50gr. of mercurous sulphate, free from acid, are

added and well stirred. The mercury salt will in general assume a yellow colour after long standing. If the solution has stood for a day, and a portion of it, on being shaken up with more mercurous sulphate, does not turn perceptibly yellow, the solution may be filtered off and concentrated in a flat porcelain dish over a water bath. Here we must take care that the crystals do not form at too high a temperature, as otherwise they easily lose a portion of their water of crystallisation. To secure this the flame under the water-bath is extinguished, and the dish left in position covered with a glass plate. If after further cooling no crystals are separated, further concentration is necessary. If the heating was too protracted, and the crystals were formed under unfavourable conditions, a little water must be added and the whole warmed until everything is redissolved. The concentrated solution is poured off, and either further evaporated or kept for future use. The last traces of the solution are removed from the crystals by letting the dish stand for some time in a slanting position. It is not advisable to sharply dry the crystals, as they thereby lose water of crystallisation. For the same reason they must be kept in a closed vessel.

Sulphate of Mercury.—The mercurous sulphate used must not be coloured yellow by a basic salt. If that should be the case, stir up one part of the salt with two parts distilled water, and add, constantly stirring, so much of a solution of one part mercuric sulphate to 1,000 parts of water as is necessary to make the colour disappear. Then pour off the liquid and wash the paste several times with distilled water, but without thereby causing another yellow colouration. If the sulphate is white to begin with, and only shows a faint yellow colouration after considerable time on shaking up with distilled water, it may be used at once. If this colouration is not shown on shaking up with water, the salt must be washed out several times with distilled water until the first traces of yellow colouration appear. If the salt had to be wetted for cleaning, the water should be driven off as much as possible by mechanical means. If dried by heat the yellow colouration will reappear. In order not to have to keep the wet salt only so much salt should be treated by the above process as is necessary for the purpose in hand.

To prepare the paste, two parts of the sulphate should be added to one part of mercury. If the sulphate was dry, it should be stirred up with a paste made of zinc sulphate crystals and concentrated zinc sulphate solution until the whole forms a stiff mass everywhere permeated by zinc sulphate crystals and small globules of mercury. But if the sulphate was wet, only zinc sulphate crystals should be added, taking care, however, that they are in excess, and are not dissolved even after prolonged standing. Here, also, the mercury must permeate the paste in small globules. It is well to crush the zinc sulphate crystals a little before using, so that the paste may be more easily manipulated later on.

Details of Construction.

For the preparation of cells containing mercury as the positive electrode the following details should be attended to :—Before introducing the hot zinc amalgam place the glass vessel, well cleaned and carefully dried, in a hot water bath. Then pass a suitable thin-walled glass tube through the neck

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of the vessel on to the bottom of the branch which is to contain the amalgam. The tube ought to be as wide as is consistent with the dimensions of the vessel. It is intended to protect the rest of the vessel from contamination. The amalgam is introduced by means of a glass tube about 10cm. long drawn out into a point, the other end being provided with an indiarubber tube about 3cm. long closed by a short glass rod. The point of the tube is inserted below the surface of the liquid amalgam heated in a dish, and a portion of the amalgam is sucked into the tube by compressing and releasing the india-rubber. The point is then quickly freed from external impurities derived from the surface of the amalgam, introduced into the cell through the wide tube, and emptied by pressure on the indiarubber. The point ought to be so fine that the amalgam does not issue except on pressing the rubber. The process is continued until the branch contains the required quantity of amalgam. The vessel is then taken out of the water-bath. After cooling the amalgam should adhere firmly to the bottom of the vessel and exhibit a bright metallic surface.

To introduce the mercury and the paste a suitable funnel with a long tube is used. The paste should not touch the upper walls of the vessel, but may be pushed in with a glass rod if too stiff to flow.

Before pouring in the zinc sulphate solution the paste and the zinc amalgam should be covered with zinc sulphate crystals, as these prevent a creeping up of the paste after wetting with the solution. In filling, the zinc sulphate crystals and the paste should not contain large air-bubbles. These may be removed by knocking.

If the cell is to contain amalgamated platinum foil as the positive electrode, the amalgamation may be performed as follows :—The cell vessel is first filled with aqua regia and heated in a sand-bath until a rapid development of gas takes place at the platinum. Then rinse with water, pour mercury into the branch intended to contain the zinc amalgam, and fill the entire vessel with a concentrated solution of mercurous nitrate containing a little nitric acid. Then connect the mercury with the positive and the platinum foil with the negative pole of a battery, and send a current of about 0·5 ampere through the solution until the platinum foil is covered over with firmly-attached mercury globules. The whole process lasts about five minutes. Finally the vessel is thoroughly rinsed with distilled water until not a trace of the nitrate remains.

The zinc amalgam is introduced with the precautions mentioned. After cooling, it is covered with zinc sulphate crystals, to which concentrated zinc sulphate is added after filling until the whole forms a paste. The vessel should stand for two days, so that the crystals are closed up and form a layer impervious to the paste. Then the whole vessel is filled up with the latter.

The following points should be specially borne in mind in the construction :—

1. The mercury intended to serve as positive electrode must be kept rigidly free from contamination by more positive metals. Special care should be taken that no portion of the zinc amalgam comes into contact with the mercury.

2. The cell should always be so arranged that at all temperatures the whole electrically active surface of the electrodes is in contact with zinc sulphate

solution concentrated for the temperature in question. Hence during the process of filling, crystals should be added in such quantity as to ensure their presence in excess even at the highest temperatures which the cell may attain.

3. The zinc sulphate used must not contain free acid. For one thing, the E.M.F. of the cell is affected by it, and, on the other hand, the circuit of the cell may be broken by hydrogen developed at the zinc. For the gas produced cannot escape through the zinc sulphate crystals, but collects underneath them, and finally pushes them up, thereby interrupting the connection between the zinc and the zinc sulphate.

For easy and safe handling the cell is included in a metal case, which may be closed and placed in a petroleum bath. Its lid is provided with two binding screws, each of which is joined to one of the electrodes ; the bottom is perforated, so that the cell is surrounded by the petroleum. To determine the temperature of the cell, a thermometer must be enclosed in the case whose scale can be read from outside. The best plan is to fuse a thermometer into the glass stopper, as shown in the figure, so that the bulb penetrates as far as possible into the cell and the scale projects through the lid.

The great objection which has been raised to the form of Clark cell constructed in accordance with the Board of Trade specification, in which the zinc rod is surrounded throughout the whole or part of its length by clear solution of zinc sulphate, is that there is a source of possible error or uncertainty in its use due to the fact that changes of temperature and consequent changes in the state of saturation of the zinc sulphate are not propagated immediately through the cell. If the temperature of the cell is raised, more zinc sulphate must be dissolved to keep the solution saturated at that temperature, and this saturated solution is not diffused immediately to all parts of the tube. The existence of this diffusion-lag has been proved, by the experiments of Lord Rayleigh, Prof. Carhart, Prof. Ayrton, Mr. Cooper and others, undoubtedly to be a source of error.

The remedy proposed by Lord Rayleigh for this diffusion lag was the H form of cell.

The remedy suggested by Prof. Carhart was to use a cell containing a solution saturated at 0°C.

(*vii.*) *The Callendar-Clark Cell.*—Prof. Callendar and Mr. Barnes have proposed a remedy for diffusion lag as follows : The cell is made up in a rather thin and long test-tube, and has two glass-covered platinum wires or electrodes ; the ends

H 2

of the platinum wires protruding from the sealed ends of the glass tubes (*see* Fig. 35). In the bottom of the tube is placed a small button of a 10 per cent. amalgam of zinc and mercury, and in this is immersed one electrode. Over this a layer of moist crystals of zinc sulphate; on the top of the zinc sulphate a layer of mercurous sulphate paste, in which the end of the second amalgamated platinum wire is buried. The cell is sealed with marine glue in the usual manner. The cell is thus an inverted narrow pattern of the Board of Trade cell.
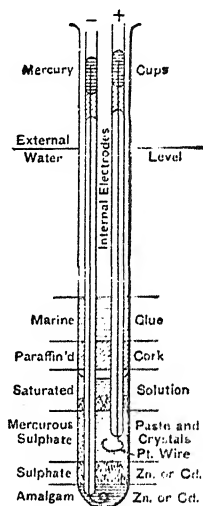
Fig 35.—Callendar form of Clark Cell.

The inventors state that diffusion lag is absent, and that changes of E.M.F. follow immediately all changes of temperature between 0°C. and 40°C.

The large temperature variation of the Clark cell led inventors to search for a modification with a less temperature coefficient. This was found, in 1891, by Mr. Weston in the arrangement known as the cadmium cell. In this cell the elements are mercury, mercurous sulphate, cadmium sulphate and cadmium. The combination has a temperature variation

very much less than that of the corresponding zinc combination. The cadmium sulphate being much more equally soluble at all ordinary temperatures than the zinc sulphate, diffusion-lag does not occur. The only difficulty is that the E.M.F. varies with the proportions of the metals in the cadmium-mercury amalgam used. Hence it is best to employ the H form of cell, in which the cadmium is kept out of contact with the mercurous sulphate. Mr. Weston makes the cell as follows :—

(*viii.*) *The Weston Cadmium Cell.*—The considerable variation in the solubility of zinc sulphate in water at different
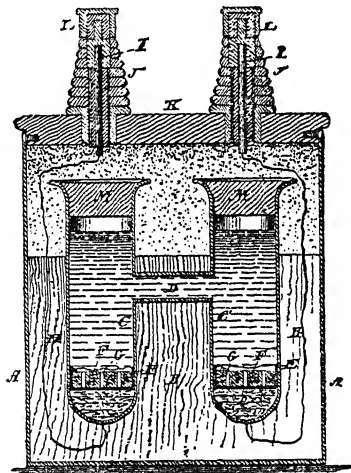
FIG. 36.—Weston Cadmium Standard Cell.

temperatures led Mr. E. Weston, in 1891, to suggest the use of cadmium sulphate instead of zinc sulphate (U.S. Patent No. 22,482 of 1891). The cadmium salt has nearly the same solubility at all temperatures. The cadmium cell is made up in the Rayleigh H-form, as shown in Fig. 36. In one side-vessel is placed a little pure mercury, entirely covered with a layer of mercurous sulphate. In the bottom of the other

limb is placed an amalgam of cadmium, and the tube is filled up with a saturated solution of cadmium sulphate. The cell is then sealed. Platinum wires sealed through the glass at the bottom of the side tubes enable connection to be made between the mercury and the cadmium amalgam and the cell terminals.

Mr. Weston states in his patent specification that the electromotive force of this cell is 1·019 volts, and its temperature coefficient is 0·01 per cent. per degree centigrade.
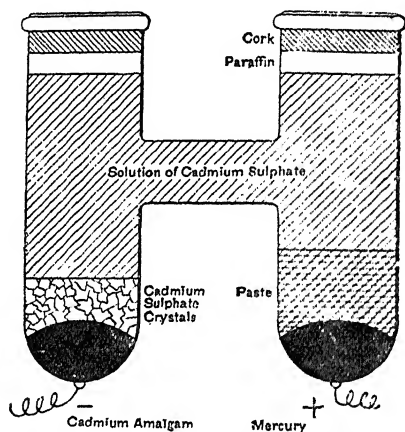
FIG. 37.—Reichsanstalt form of Cadmium Standard Cell.

A valuable feature, therefore, of the cadmium cell is its small temperature coefficient; this may be made by using proper proportions of cadmium and mercury to be as low as 0·004 per cent. per degree centigrade instead of 0 08 as in the case of the Clark cell.

Special attention has been paid to the details of the construction of this cell at the Physikalisch-Technische Reichsanstalt in Berlin, and the following are the instructions for its manufacture as laid down by Prof. W. Jaeger:—

(ix.) *The Reichsanstalt Cadmium Cell.*—This cell is made up in the Rayleigh H-form (*see* Fig. 37). The two legs of

the side vessels are provided with platinum wires sealed through the glass at the lower ends. The negative element consists of a cadmium amalgam—one part of cadmium to six parts of mercury. Over this is placed a layer of pulverised crystals of cadmium sulphate to ensure saturation. The positive element is formed of pure mercury, over which is placed a paste formed by the trituration of mercurous sulphate with metallic mercury and a concentrated solution of cadmium sulphate in which are crystals of the salt. This paste must not be too thin, but must form a stiff pulp. The remainder of the cell is filled up with a saturated solution of cadmium sulphate. The tubes are then closed by a layer of melted paraffin poured on, then a thin washer of cork, and, lastly, the cell is closed with melted sealing wax.

A more portable cell may be made like the Muirhead-Clark cell with an amalgamated platinum spiral instead of liquid mercury. The electromotive force E_t of the cadmium cell in the H-form at $t°$C. is given by the formula,

$$E_t = E_{20} - 0.000038(t-20) - 0.00000065(t-20)^2$$

where E_{20} = the electromotive force at 20°C. in volts and is 1.0185 volts. The above applies to cells in which the amalgam contains from 7 to 14 per cent. of cadmium.

In making the cell the following precautions must be employed:—

Amalgamation of the Platinum Wire.—After the platinum wires are sealed through the glass place a little aqua regia in the cell legs until bubbles of gas arise from the platinum. Then throw this out and replace it by a solution of mercurous nitrate, and, using another piece of platinum as an anode, deposit mercury upon the platinum electrolytically. The platinum may also be amalgamated by making it white hot in a Bunsen flame, and plunging it whilst hot in the mercury.

Preparation of the Cadmium Amalgam.—Dissolve one part of pure cadmium in six parts of pure mercury, and whilst warm and fluid place it in one limb of the H cell and warm it to ensure perfect contact with the platinum.

Cadmium Sulphate Solution.—Digest a saturated solution of cadmium sulphate with cadmium hydroxide to remove free acid, but be careful not to raise the temperature above 70°C. Then digest it still further with mercurous sulphate until no more precipitation occurs. The cadmium sulphate solution must be saturated and have free crystals of the salt in it.

Mercurous Sulphate.—This must be free from acid, and made neutral by triturating with finely divided mercury. In making the paste so much cadmium sulphate must be added that a saturated solution of that salt is formed and is present in the cell.

The cell has the electromotive force above stated if the amalgam of cadmium has from 6 to 13 parts of mercury to 1 of cadmium.

The German investigators seem to have a great preference for the H form of cell, but it is clear that a narrow tubular cell of the Board of Trade form not only more quickly comes to the temperature of the water bath in which it is placed, but is more certain to be wholly at one temperature.

(x.) The Helmholtz Calomel Cell.—It was proposed by Von Helmholtz to employ a cell, the elements of which are mercury, mercurous chloride or calomel, zinc chloride and zinc, as a standard of electromotive force (*see* " Sitzber. der Akad. der Wiss," Berlin, 1882, p. 26).

This cell can be adjusted to have an electromotive force of exactly one volt by the use of a solution of zinc chloride of a certain density, viz., 1·380 at 15°C. Its temperature coefficient of electromotive force is small, being only 1 part in 10,000 per degree centigrade, whereas that of the Clark sulphate cell is 8 parts in 10,000 per degree centigrade. Its electromotive force varies, however, with the state of saturation of the solution of zinc chloride, and as evaporation from the cell tends to increase the density of the zinc chloride solution it is not so definite and permanent as a standard as the zinc sulphate or cadmium sulphate form of cell, unless hermetically and permanently sealed. According to Mr. W. Hibbert (*The Electrician*, Vol. XXXVII., p. 320), the cell rapidly recovers its normal electromotive force if short-circuited.

The Standard Daniell Cell.—In addition to the above-described mercury-zinc or mercury-cadmium standard cells, the Daniell cell or copper, copper-sulphate, zinc sulphate, zinc cell has been used as a standard of electromotive force. Although not a rival in uniformity of electromotive force to the Clark or cadmium cell, it has the advantage that its temperature coefficient within the range of ordinary laboratory temperatures is practically zero. Its electromotive

force is, however, a function of the density of the solutions used.

The Author described, in 1885,* a convenient form of stanpard Daniell cell as follows :—A glass U-tube is prepared
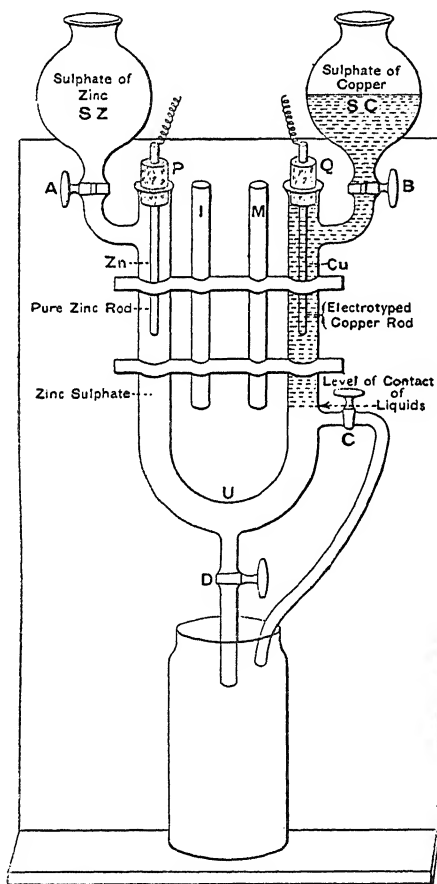

FIG. 38.—Fleming Standard Daniell Cell

having side bulb reservoirs and taps as shown in Fig. 38. The whole apparatus can be made out of glass by a skilful

* See *Phil. Mag.*, August, 1885

glass-blower. It is then fixed up against a board. In this U-tube a Daniell cell is formed by inserting rods of amalgamated zinc and of freshly electrotyped copper in the two limbs, which are respectively filled up with solutions of zinc sulphate and copper sulphate.

The solutions required are made by dissolving the purest re-crystallised sulphate of copper and sulphate of zinc in distilled water.

For the zinc solution, take 55·5 parts by weight of crystals of zinc sulphate ($ZnSO_4$, $7 OH_2$), and dissolve in 44·5 parts by weight of distilled water; and the resulting solution should have a specific gravity of 1·200 at about 20 C. For the sulphate of copper solution, take 16·5 parts by weight of pure crystals of copper sulphate ($CuSO_4$, $5 OH_2$), and dissolve in 83·5 parts by weight of water; and the resulting solution should have a specific gravity of 1·100 at 20°C. If not exact, adjust to these densities precisely.

These solutions should be kept in stock bottles and the reservoirs of the cell filled up when required.

The operation of filling is as follows:—Open the tap A and fill the whole U-tube with the denser zinc sulphate solution; then insert the zinc rod, and fit it tightly by the rubber cork P. On opening the tap C the level of the liquid will begin to fall in the right hand limb, but be retained in the closed one. As the level commences to sink in the right hand limb, by opening the tap B copper sulphate solution can be allowed to flow in gently to replace it; and this operation can be so conducted that the level of demarcation of the two liquids remains quite sharp, and gradually sinks to the level of the tap C. When this is the case, all taps are closed and the copper rod inserted in the right hand limb.

It is impossible to stop diffusion from gradually mixing the liquids at the surface of contact; but whenever the surface of contact ceases to be sharply defined, the mixed liquid at the level of the tap C can be drawn off, and fresh solutions supplied from the reservoirs above.

A freshly electrotyped copper rod is always to be used. The copper surface must have a clean salmon colour free from brown spots of oxide.

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* See *Phil. Mag.*, August, 1885.

The cell freshly prepared has then an electromotive force on open circuit of 1·072 volts.

The use of this Daniell cell enables an approximate recovery to be made of the unit of electromotive force, but it is neither so convenient nor permanent as the improved forms of Clark or cadmium cell.

Every electrical laboratory or testing room should be provided with a number of specimens of the above-described standard cells, either Muirhead-Clark cells, Weston cadmium cells, or the Reichsanstalt pattern of cadmium cells will be found to be most trustworthy. These should be numbered or lettered and careful comparison made at observed temperatures of their relative electromotive forces from time to time, and the values entered under date in the laboratory book. Occasionally comparisons of these with the other volt standards should be made by methods to be described, and with the results of electromotive force determinations by the ampere balance as presently to be discussed. With care and vigilance the laboratory need never be uncertain in its recovery of the standard of electromotive force by more than one part, or at most five parts, in 10,000.

The standard cells should have thermometers placed in the brass cases containing them or, better still, be immersed in water when using them, and in taking careful observations sufficient time should always be allowed to elapse before taking the voltage readings to enable the cell to take the same temperature as that indicated by the thermometer, assuming them to be under circumstances in which the final temperature of both cell and thermometer will ultimately be the same.

§ **9. The Literature of the Mercury Standard Cell.**— As it is impossible to transcribe in these pages the detailed results of all the very numerous researches which have been made during the last fifteen years on the standard Clark, Weston and Helmholtz mercury cells, we shall give here references to some of the principal investigations, and leave

the reader desirous of more information to consult the original papers :—

(a) Investigations on the Absolute Electromotive Force of Standard Cells.

LATIMER CLARK. " On a Voltaic Standard of Electromotive Force." *Telegraphic Journal*, Vol. I., p. 9 ; or *Proc.* Roy. Soc., Lond., 1872.

LORD RAYLEIGH and MRS. SIDGWICK. " On the Absolute Electromotive Force of Clark Cells." *Phil. Trans.* Roy. Soc., Lond., 1884, Part II., p. 411.

LORD RAYLEIGH. " On the Clark Cell as a Standard of Electromotive Force." *Phil. Trans.* Roy. Soc., Lond., 1885, Part II., p. 781.

R. T. GLAZEBROOK and S. SKINNER. " On the Clark Cell as a Standard of Electromotive Force." *Phil. Trans.* Roy. Soc., Lond., 1892, Vol. CLXXXIII., pp. 567·628.

The authors conclude that the E.M.F. is 1·434 volts at 15°C., confirming the value given by Lord Rayleigh.

C. LIMB. " On the Determination of the Electromotive Force of the Clark Cell in Absolute Measure." *Journal de Physique*, 1896 ; also *The Electrician*, Vol. XXXVII., p. 138.

The E.M.F. of the cell was balanced against an E.M.F. produced by the rotation of a magnet inside a coil of wire. The value obtained for the E.M.F. of the cell is 1·4535 volts at 0°C.

H. S. CARHART and K. E. GUTHE. *Physical Review*, Nov., 1899, p. 288 ; also *Science Abstracts*, March, 1900, No. 646.

The E.M.F. of the Kahle form of H-cell was determined by balancing it against a fall of potential down a resistance due to known current. Result found was 1·4363 volts at 15°C.

(b) Modifications of the Clark Cell.

H. S. CARHART. " An Improved Standard Clark Cell with Low Temperature Coefficient." *Phil. Mag.*, 1890 ; *The Electrician*, Vol. XXIV., p. 271.

The author constructed a cell with low temperature coefficient

$$E_t = E_{15}[1 - 0·000387(t - 15) + 0·0000005(t - 15)^2],$$

where E_t = electromotive force in volts at t°C. The above paper was criticised by Lord Rayleigh (*The Electrician*, 1890, Vol. XXIV., p. 285) who, suggests that the low coefficient found by Carhart was due to the zinc sulphate not being saturated. He gives a diagram of his H form of cell.

H. S. CARHART. "A Portable Clark Cell." *Electrical World*, 1895 ; or *The Electrician*, Vol. XXXV., p. 844.

He describes the cell shown in Fig. 32. He uses a solution of sulphate of zinc saturated at 0°C.

E. WESTON. "The Cadmium Standard Cell." *Electrical Engineer* (New York), 1893 ; also *The Electrician*, Vol. XXX., p. 741.

He describes fully his mercury-cadmium sulphate cell.

H. L. CALLENDAR and H. T. BARNES. "On a Simple Modification of the Board of Trade Standard Clark Cell." *Proc. British Assoc.*, 1897, Toronto ; also *The Electrician*, Vol. XXXIX., p. 638 ; also Vol. XL., p. 165.

In this pattern of cell, called the "inverted" cell, the zinc amalgam lies at the bottom of the test-tube, and an amalgamated platinum wire forms the other element.

For correspondence on this form of cell see *The Electrician*, Vol. XXXIX. (K. Kahle), p. 869 (H. L. Callendar), p. 869.

W. JAEGER. "The Reichsanstalt Type of Cadmium Standard Cell." *Electrotechnische Zeitschrift*, October 21, 1897 ; also *The Electrician*, Vol. XL., p. 9.

This is an important paper, and describes in great detail the construction of a standard H form of cadmium cell.

W. HIBBERT. "On the Helmholtz or Calomel Cell." *The Electrician*, Vol. XXXVII., p. 320 ; also *ibid.*, Vol. XXXVIII., p. 177 ; also *ibid*, Vol. XLI., p. 317.

In this cell zinc chloride and mercurous chloride replace the sulphate salts of the Clark cell. It can be made to have an E.M.F. of exactly one volt, and a negligible temperature coefficient.

(c) Specifications for preparing Clark and Weston Cells.

"The British Board of Trade Specification for Clark Cells." *The Electrician*, Vol. XXVII., p. 99 ; also *Ibid.*, Vol. XXXIII., p. 518.

K. KAHLE. "Instructions for Preparing Clark Standard Cells: the Reichsanstalt Specification." *Zeitschrift für Instrumentenkunde*, 1893 ; also *The Electrician*, Vol. XXXI., p. 265.

W. JAEGER. "The Reichsanstalt Specification for Preparing Cadmium Cells." *Electrotechnische Zeitschrift*, October 21, 1897 ; also *The Electrician*, Vol. XL., p. 9.

(d) On Temperature Variations of Standard Cells.

W. E. AYRTON and W. R. COOPER. "The Variation of E.M.F.
of Clark Cells with Temperature." *Proc.* Roy. Soc., Lond.,
1897, Vol. LIX. ; also *The Electrician*, Vol. XXXVIII.,
p. 303.

An exhaustive examination of the effect of temperature on the Board of
Trade Clark cell. The authors give numerous curves.

W. HIBBERT. "The Temperature Coefficient of the Calomel
Cell." *The Electrician*, Vol. XXXVIII., p. 177.

F. S. SPIERS, F. TWYMAN and W. L. WATERS. "Variations in
the Electromotive Force of the H form of Clark Cells with
Temperature." *Proc.* Phys. Soc. Lond., Vol. XVI., p. 38 ;
also *Phil. Mag.*, 1898, Vol. XLV., p. 285.

A. DEARLOVE. "Note on the Temperature Coefficient of the
Cadmium Standard Cell." *The Electrician*, Vol. XXXI.,
p. 645.

A very full and detailed account of experiments on the Weston cell. The
author advocates a cell of Muirhead type made with cadmium salts and
cadmium.

A. CAMPBELL. "A Self-acting Temperature Compensation for
Standard Cells." *Proc.* Phys. Soc., Lond., Vol. XVI., p. 34 ;
also *The Electrician*, Vol. XXXV., p. 601.

W. JAEGER and K. KAHLE. "A Comparison of the Clark and
Weston Cells as regards Temperature Coefficient." See
Wied. Ann., No. 8, 1898 ; also *The Electrician*, Vol. XLI.,
p. 642.

The authors have examined 68 cells (27 Clark cells and 41 Weston cells)
constructed since 1891. They have been tested at intervals of about a year
and the maximum difference observed between the Clark cells was 0·14
millivolt and between the Weston cells 0·18 millivolt, which last in two years
decreased to 0·08 millivolt. The cadmium cells should only be used between
20°C. and 70°C. The E.M.F.'s are—

Clark cells (H form)=1·4328 international volts at 15°C.

Weston cell=1·0186 volts at 20°C.

The ratio of—

Clark at 0°C. to Weston at 20°C.=1·42277

Clark at 15°C. to Weston at 20°C.=1·40663.

The temperature correcting factors are—

for Clark cell$=1 - 0{\cdot}00119\,(t - 15) - 0{\cdot}000007\,(t - 15)^2$

for Weston cell$=1 - 0{\cdot}000038\,(t - 20) - 0{\cdot}00000065\,(t - 20)^2$,

and these factors, by multiplication with the value of the E.M.F. at 15°C. or
20°C. respectively, give the value of the E.M.F. of each cell at $t°$

K. KAHLE. "On the Clark Cell." *The Electrician*, Vol. XXIX., p. 516 ; also *Proc.* Brit. Assoc., 1892, Edinburgh.

This paper contains a valuable table giving the temperature coefficient of different types of H-form Clark cell. The mean value of the temperature coefficient (a) of the H form of cell according to the author is given by

$$a = 0{\cdot}000783 + 0{\cdot}000017(t - 15)$$

and the E.M.F. at $t^{\circ} =$ E.M.F. at $15^{\circ} \times [1 + a(t - 15)]$.

The above author strongly advocates the use of the H form of cell. He says he has set up about 60 H-form cells and has found no difficulty, when using pure materials, in keeping the differences of E.M.F. of the various cells to less than one ten-thousandth of a volt. He gives the following table showing the temperature coefficient (a) for various forms of Clark cell :—

Form of Cell.	Temperature Coefficient.
H-cell set up in Lord Rayleigh's manner	$+ 0{\cdot}000812 + 0{\cdot}000013\,(t - 15)$
H-cell, the paste covering both electrodes	$+ 0{\cdot}000774 + 0{\cdot}000020\,(t - 15)$
The Reichsanstalt form of H-cell, paste covering both electrodes	$+ 0{\cdot}000791 + 0{\cdot}000017\,(t - 15)$

The electromotive force of the H form of cell appears to be about 4 parts in 10,000 less than that of the Board of Trade or original form when taken at 15°C.

(e) Various Investigations.

J. SWINBURNE. "On the Causes of Variation of Clark Cells." *The Electrician*, Vol. XXVII., p. 500; also Brit. Assoc. *Report*, 1891, Cardiff.

S. SKINNER. "The Clark Cell when Producing a Current." *Proc.* Phys. Soc., Lond., Vol. XIII., p. 218; *The Electrician*, Vol. XXXIII., p. 644.

W. R. COOPER. "The Permanency of Board of Trade Clark Cells." *The Electrician*, Vol. XL., p. 748; or *Science Abstracts*, Vol. I., p. 492.

The author has tested a number of Clark cells set up according to the Board of Trade specification, and finds, after $3\frac{1}{2}$ years, the mean differences or mean errors in E.M.F. amount to 1 part in 700 or even 1 in 500. Time introduces a progressive variation in E.M.F., the E.M.F. steadily falling.

W. JAEGER. "On Cadmium Cells." *Ann. Phys. Chem.*, 1898, 65, 1, p. 106; or *Science Abstracts*, Vol. I., p. 498.

T. Wulf. "The Clark Cell on Closed Circuits." *Science Abstracts*, Vol. I., p. 340.

After repeatedly short-circuiting cells through 50 ohms the E.M.F. was not permanently affected.

W. C. Fisher. "The Recovery of Clark Cells after Sending a Current." *The Electrician*, Vol. XXXVI., p. 647.

The author finds repeated short-circuiting produces no permanent injury to the cell.

W. Jaeger. "On Change in the Zinc Sulphate in Clark Cells." *Ann. Phys. Chem.*, 1897, 63, 1, pp. 354-365.

Kohnstamm and Cohen. "The Weston Standard Cell." *The Electrician*, Vol. XLI., p. 381.

J. Henderson. "On Cadmium Standard Cells." *Phil. Mag.*, Vol. 48, July, 1899, p. 152.

For various correspondence on the subject of the Clark cell see *The Electrician*, Vols. XXXIX. and XL., for 1897 and 1898.

§ 10. Mechanical Standards of Electromotive Force. Voltmeters.

In addition to the standard cells or electrochemical standards of electromotive force, an electrical laboratory must be provided with mechanical standards for the measurement of potential difference or electromotive force. These instruments are called voltmeters, and are each constructed to be suitable for a certain range of voltage or potential difference applied to their terminals. These appliances are roughly distinguished into low and high voltage voltmeters, according as they are designed to read over ranges of from 500 volts downwards or from 500 volts upwards. Some of these types of instruments are adapted only for ordinary or not very accurate measurements. These are called *working voltmeters*. Others, for very careful work, are called *standard voltmeters*. If the instrument shows by a scale deflection or pointer indication directly the value in volts of the potential difference of its terminals it is called a *direct-reading voltmeter*.

For the British Board of Trade electrical laboratory a mechanical standard of electromotive force was designed by Lord Kelvin, which, as a standard of voltage, is considered

to be more permanent than an electro-chemical standard consisting of a battery of standard cells. This mechanical standard consists of an idiostatic electrostatic voltmeter, in which the difference of potential between a series of fixed metallic surfaces and a series of metallic plates suspended by a torsion wire, is made to exert a mechanical torque twisting the suspending wire, carrying one set of plates through a certain angle against the torsional rigidity of the wire. The description of this principal Board of Trade 100-volt standard is as follows :—

CONSTRUCTION AND USE OF THE BOARD OF TRADE ONE HUNDRED VOLTS STANDARD OF ELECTROMOTIVE FORCE OR VOLTAGE.

The instrument is shown in plan and sectional elevation on the accompanying diagrams (*see* Figs. 39 and 40). It consists of :—

Suspended Vanes.— (*a*) An arrangement of 10 parallel paddle-shaped vanes, in form similar to the moving portion of Lord Kelvin's Quadrant Electrometer, fitted on an axis passing through the centre of gravity of each vane and separated by distance pieces to a distance of nine millimetres apart.

Concave Mirror.—(*b*) On the same axis is fixed the aluminium frame of a concave mirror 19 millimetres in diameter and about 61 millimetres focal length, which is held in the frame by means of three light phosphor bronze springs.

Suspending Wire.— (*c*) This arrangement is suspended by means of a wire 0·05 millimetre in diameter and 18 centimetres in length, formed of an alloy of 10 parts of iridium to 90 parts of platinum, attached to the end of the axis above the mirror so that the axis hangs in a vertical line.

Horizontal Adjustment of Position of Vanes.—(*d*) The upper extremity of the suspending wire is fixed to the centre·of a circular brass plate having teeth cut in the circumference, into which a tangent screw is geared so as to enable the end of the wire to be rotated in the horizontal plane for the horizontal adjustment of the vanes.

Vertical Adjustment of Vanes.—(*e*) The vertical adjustment of the position of the vanes is accomplished by the raising or lowering of the platform on which the circular brass plate mentioned above is pivoted by means of three screws for raising and three for lowering. These screws regulate the distance of the platform above a similar platform fixed by means of three supports to the framework of the instrument.

Quadrants.— (*f*) The vanes are suspended so as to slightly enter at the zero position of the instrument into the spaces between two sets of 11 thin polished brass plates shaped as quadrants of a circle, which are fixed horizontally one above another to a vertical support. These two sets of plates are in metallic connection, and are carefully insulated from the framework of the instrument.

Vibration Checking Arrangement.—(*g*) In order to reduce the vibration of the suspended system, and the time which must elapse before an accurate
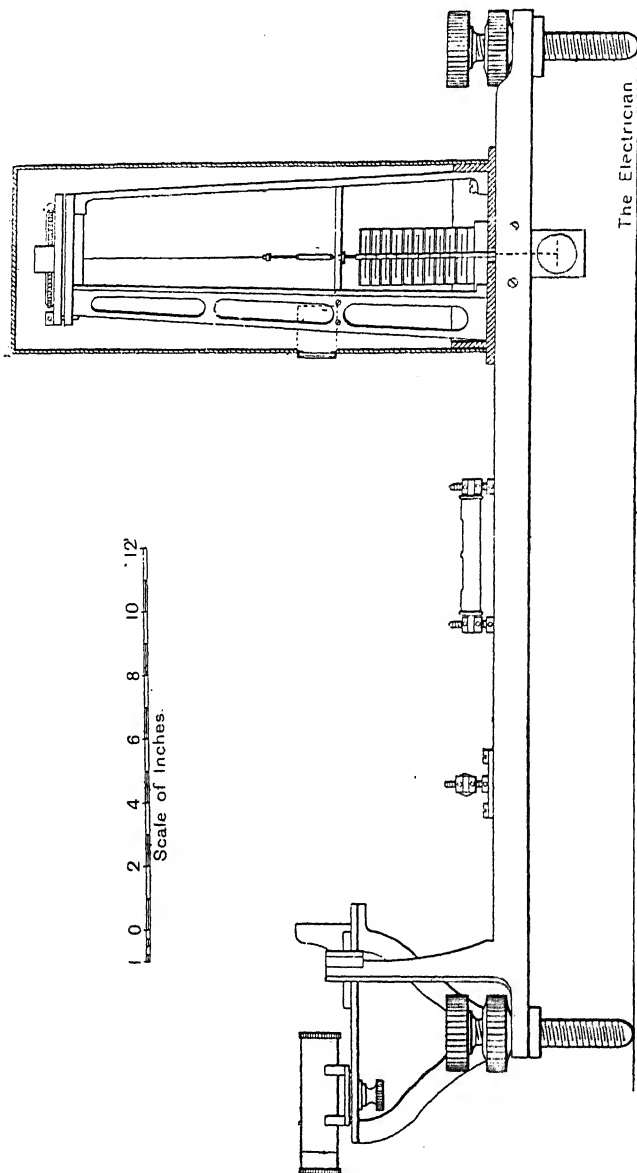
Scale of Inches.

The Electrician

Fig. 39.—Side Elevation of the Volt Standard
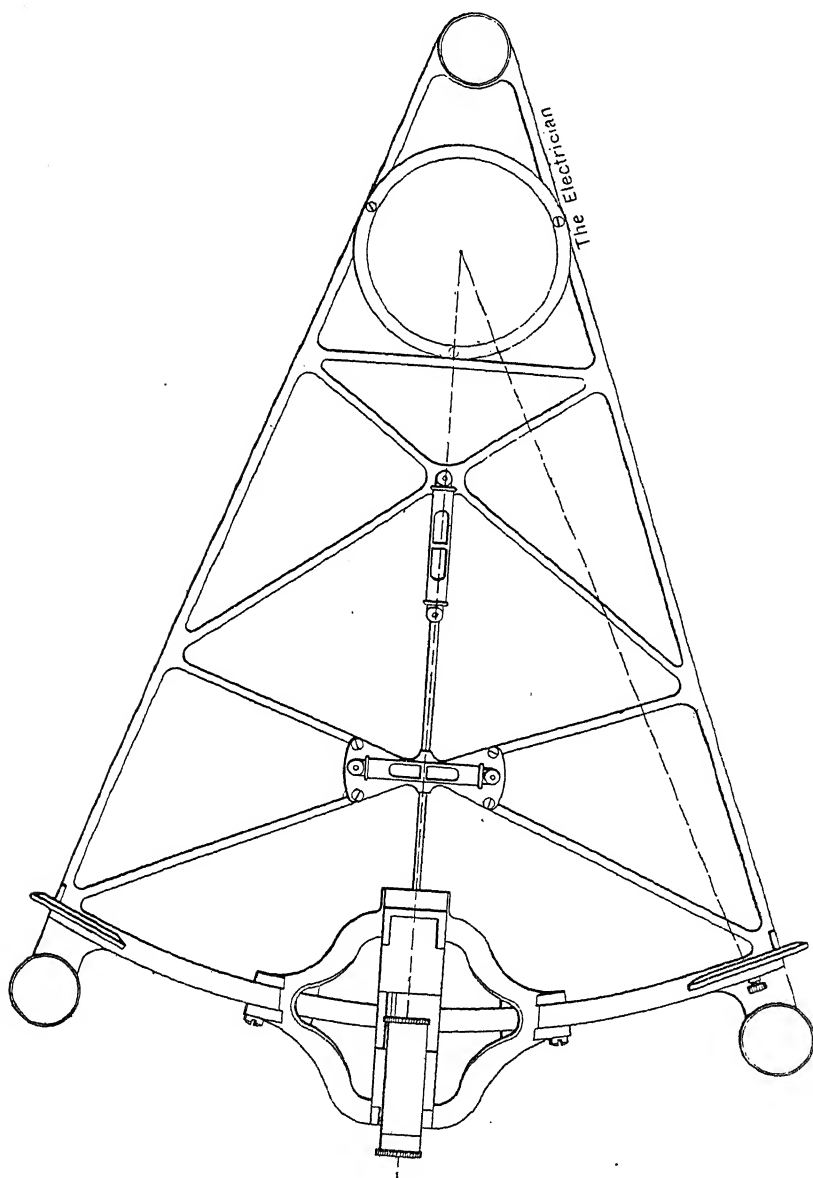
The Electrician

Fig. 40.—Plan of the Volt Standard.

observation can be taken, a thin horizontal brass disc is suspended in refined mineral oil contained within a glass vessel, by means of a wire attached to the lower portion of the axis of suspension of the vanes.

Outer Case.—(*h*) The suspended system and quadrants, with their supports, are enclosed in a brass case having in front of the mirror a rectangular window of parallel-worked glass. A hole is cut in this case for the insertion of a key fitting the squared arbor of the tangent screw (*d*).

Terminals.—(*i*) The terminals, by means of which connection is obtained with the vanes and quadrants, are fixed on a thick piece of polished ebonite projecting beyond the brass cover *h*. One terminal is in connection with the metal framework, the other is entirely insulated ; and a tongue of brass or switch connected by means of a brass rod with the two sets of quadrants can be turned so as to make contact either with the insulated or with the uninsulated terminal.

Framework and Supports.—(*k*) The arrangement described above, forming the electrical portion of the instrument, is mounted near the apex of a framework of brass in the form of a circular arc which rests by means of three feet on a horizontal slab of polished marble supported on a large block of Portland stone on a concrete foundation. A screw thread is cut on the feet for accurate levelling of the framework, to which two spirit levels are attached.

Eyepiece and Cross Wire.—(*l*) On a support attached to the centre of the curved portion of the framework is fixed the observing portion, consisting of a magnifying eyepiece in front and in the focus of which is stretched a vertical copper wire 0·06mm. in diameter.

Fiducial Marks.—(*m*) At each end of the arc is erected a vertical support, to which is fixed a tablet of brass with the surface facing towards the mirror platinised. On each of these faces is engraved a vertical line. These lines form the fiducial marks ; that on the left of the observer giving the zero position, and that on the right the correct position for 100 volts pressure. The distances from the mirror (*b*) of these tablets and of the sighting wire (*l*) is adjusted so that the image of the line on the tablet, when the mirror is at the proper angle, coincides with the sighting wire.

Use of the Instrument.

Adjustments.—The instrument must be in accurate adjustment as regards level and position of the suspended system. The vertical adjustment of the latter is obtained by trial. The vanes should be at equal distances from the quadrants above and below, this position giving minimum sensibility. The horizontal adjustment of the suspended system is obtained when the image of the zero fiducial mark produced by the mirror (*b*) exactly coincides with the sighting wire (*l*), the switch (*i*) being turned so as to connect the quadrants with the metal framework of the instrument. (The levelling of the framework and vertical adjustment of the suspended portion of the instrument were carefully attended to when the instrument was first set up, and these adjustments have since that time remained constant. The horizontal adjustment of the suspended system requires occasional attention from time to time.)

Readings.—Arrangements should be made for obtaining a pressure which can be continuously varied from about 98 to 102 volts. This pressure is

1 to the 100-volt standard, and to the instruments to be compared with
. is adjusted until the image of the fiducial mark indicating 100 volts
re exactly coincides with the sighting wire (l) by observation through
epiece. This coincidence is maintained, by adjustment of the pressure,
essary, until the expiration of five minutes from the time of first
1g the pressure, when, if there is no visible vibration of the mirror, the
re is exactly 100 volts.

rspective view of this absolute 100-volt standard is
u in Fig. 41.
struments of the same design can be made for any
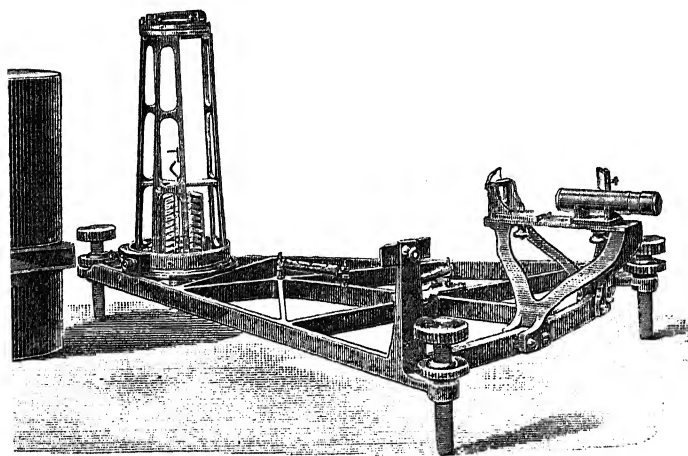   of electromotive force, and a set of six standard

FIG. 41.—Board of Trade S andard Kelvin Voltmeter.

ters of the above described form, covering a range
0 to 3,200 volts, has been made for the British Board
.de Electrical Standardising Laboratory. With these
nents an accuracy of standardisation can be attained
1 part in 3,000.
possession of an absolute 100-volt standard of the
sind is a great convenience when much standardising
mercial voltmeters has to be carried out.
general purposes, and as a secondary standard, a
ient form of voltmeter is Lord Kelvin's Multicellular

Electrostatic Voltmeter (*see* Fig. 42). In this instrument there are a series of connected fixed and movable metal plates as in the standard voltmeter, but the movements of the movable system are indicated by the displacement of a needle over a scale. The scale of the instrument is divided so as to read directly in volts. If the plates forming the fixed portion of the instrument and the movable plates are not made of the same metal there will be a small difference or discrepancy amounting to a fraction of a volt between the
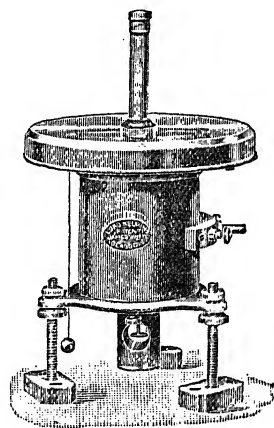
Fig. 42.—Lord Kelvin's Multicellular Electrostatic Voltmeter.

readings of the instrument taken with continuous potentials, when the fixed plates are positive and the movable negative, and when they are reversed in sign of potential. This is due to the contact difference of potential of the metals. In this case any statement of the instrumental reading must be accompanied by a statement as to the nature of the relative sign of electrification of the movable plates.

§ 11. **The Instrumental Outfit of an Electrical Laboratory.**—The majority of the measurements made in the

electrical laboratory resolve themselves ultimately into one or more of the following five measurements :—

(i.) The measurement of the value or strength of an electric current, or else proving the absence of a current, in some circuit.

(ii.) The measurement of a potential difference or of an electromotive force, or else proving the absence of a potential difference.

(iii.) The measurement of a resistance or its reciprocal conductance.

(iv.) The measurement of the time integral of a current or of an electric quantity.

(v.) The measurement of the rate of expenditure or dissipation of electric energy in a circuit, or of electric power taken up in it.

To describe all the instruments in detail which have beer invented for the above measurements would be to transfer to these pages the contents of a library of trade catalogues ano circulars.   We shall limit ourselves here to mentioning the general results of experience as to the most trustworthy and convenient appliances for performing the above electrical measurements which should be provided in a well-equipped electrical laboratory or testing room.

§ 12. **Current - Measuring Instruments**.—The most frequently recurring and fundamental of all electrical measuring processes is the operation of determining the presence, or proving the absence, of an electric current in a circuit.   If present, its strength or magnitude generally has to be determined in terms of the standard or unit current called the ampere.   An instrument which merely shows the presence or absence of a current in a circuit is called a *detector* or *galvanoscope*.   If it gives, in addition, a means of comparing the relative value of two currents, it is called a *galvanometer*. If it shows by its indications the ampere-value of the current it is called an *amperemeter* or *ammeter*.

By far the most numerous class of instruments in use as galvanoscopes, galvanometers and ammeters depend for their operation on the fact that a magnetic flux exists round a conductor, conveying, as we say, an electric current. This flux can be detected and measured by the mechanical force or torque acting either upon a magnet placed near the conductor, or upon a small mass of iron ; or upon another movable conductor traversed by the same current. Another class of instruments depends upon the heating effect produced by a current in a conductor and the measurement of the linear expansion of the conductor due to this heating.

We may, then, classify the current measuring or detecting instruments required in the laboratory as follows, depending upon the principle employed in their construction :—

I.—Galvanoscopes or Detectors.

(*a*) Simple linesman's or laboratory detector, or magnetic needle suspended in a coil of wire.

(*b*) Pole-testing paper or solution.

II.—Galvanometers or Current Measurers.

(*a*) Movable needle galvanometers with coil fixed and suspended magnetic needle, either simple or astatic, *e.g.*, Kelvin mirror galvanometer, or ordinary needle or mirror instruments of high or low resistance such as Wiedemann's galvanometer.

(*b*) Movable coil galvanometers, in which a coil traversed by the current to be measured is suspended between the poles of a strong fixed magnet, *e.g.*, Kelvin recorder pattern galvanometer or d'Arsonval galvanometer, as modified by Holden, Ayrton and Mather, and Crompton.

(*c*) Tangent galvanometers, in which the suspended magnetic needle has a magnetic length very small compared with the diameter of a large fixed coil or coils, *e.g.*, the Post Office pattern, or ordinary single-coil tangent galvanometer ; the Helmholtz, or two-coil tangent galvanometer ; the three-coil tangent galvanometer.

III.—Amperemeters

(*a*) *Electrodynamic* instruments, in which the forces acting between conductors conveying currents are utilised as an ammeter principle, *e.g.*, Kelvin ampere balances, Siemens dynamometer, Weber's electrodynamometer, as modified by Siemens ; Pellat's absolute electrodynamometer, &c.

(*b*) *Electromagnetic* instruments, in which the mechanical force between a magnet and a conductor conveying a current, or between a

mass of soft iron and a conductor conveying a current is utilised as an ammeter principle. These are variously designed as follows :—

(i.) Movable coil instruments, *e.g.* : Kelvin recorder pattern of ammeter, d'Arsonval and Weston ammeters, and similar instruments.

(ii.) Movable soft iron instruments, *e.g.*: Ayrton and Perry, Nalder, Evershed, Dobrowolsky, Thomson, and most ordinary trade ammeters.

(*c*) *Electrothermal* instruments, in which the heating property of the current is utilised, *e.g.*: Hartmann and Braun ammeters, Cardew, Holden and other hot-wire instruments.

(*d*) *Electrochemical* instruments, in which the time-average of a current is measured by an operation of electrolysis. This method of current measurement is the ultimate process of determination, and defines the current quantity in terms of the unit current by the operation on which the official definition of unit current is based. Instruments for electrochemical measurement of current quantity are called *voltameters*. The chief electrolytic processes are those in which a solution of a silver salt, or of a copper salt, or else dilute sulphuric acid, are employed as electrolytes.

(*e*) *Electro-optic* instruments, by which a current can be measured by measuring the optical rotation produced in the plane of polarised light by its magnetic field acting on a standard substance of which the Verdet constant is known. If a tube with glass ends, and filled with bisulphide of carbon, is placed in the interior of a solenoid, a current passing through the helix exerts a magneto-rotary effect on a ray of plane polarised light passing along the tube in the direction of the axis of the helix. Such an arrangement may be calibrated as an amperemeter, but it is only suitable for a very limited class of work.

(*f*) *Electrostatic* instruments. A current can be measured most accurately by the measurement of the electrostatic fall in potential down a conductor conveying the current. If a current is passed through a known resistance, and if an electrostatic voltmeter or electrometer is applied to measure the fall of potential down it, we have at once a measure given of the strength of the current.

This last method in its various modifications is by far the most practical and useful method of current measurement.

The resistance through which the current to be measured flows is not necessarily placed in the immediate neighbourhood of the potential-measuring apparatus. This last may be either an electrostatic voltmeter, as described in the next section, or else an electromagnetic voltmeter effecting the same purpose.

Further details will be given in the sections devoted to current measurement.

In addition to simple detectors, now and then required, by far the most convenient form of galvanometer for all ordinary work in the testing room and electrical laboratory are the *movable coil galvanometers.* These consist of a fixed permanent magnet having suspended between its poles a small light coil of insulated wire. The current to be detected or measured
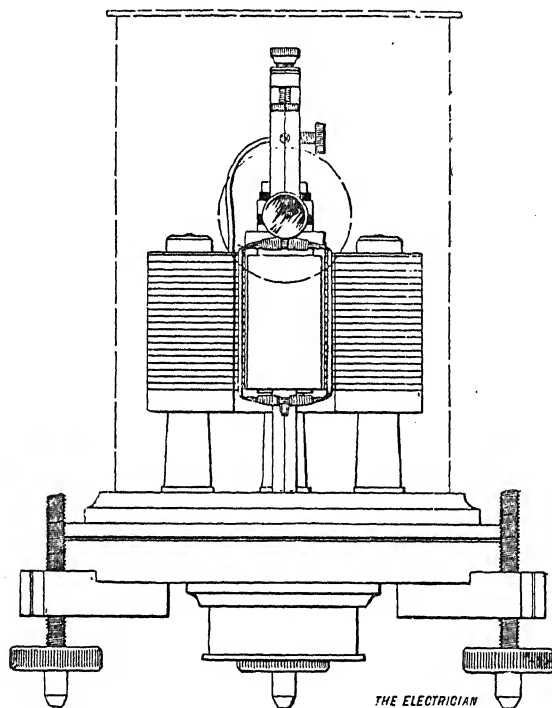

FIG. 43.—Holden-Pitkin Movable Coil Galvanometer.

passes through the coil, entering and leaving, through the suspending wires which may be arranged either bifilarly, as in the Crompton pattern of galvanometer, or attached to the top and bottom of the coil, as in the Holden-Pitkin pattern. The movable coil galvanometer has two great advantages in use: First, it is not much affected by the

passage of currents in neighbouring wires or stray magnetic fields, and hence may be used in places where an ordinary
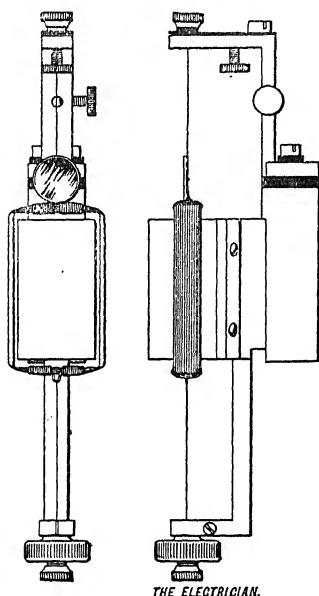

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Fig. 44.—Coil and Core of Holden-Pitkin Galvanometer.
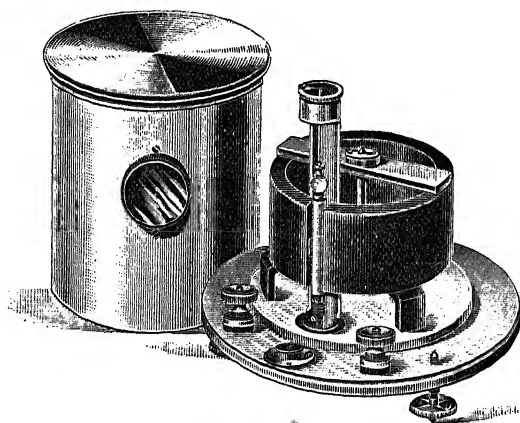
Fig. 45.—Ayrton-Mather Movable Coil Galvanometer.

movable needle galvanometer would be useless; secondly, the coil can be very quickly brought to rest by short-circuiting the terminals. Hence the galvanometer is or may be made very dead-beat or damped. In the form designed
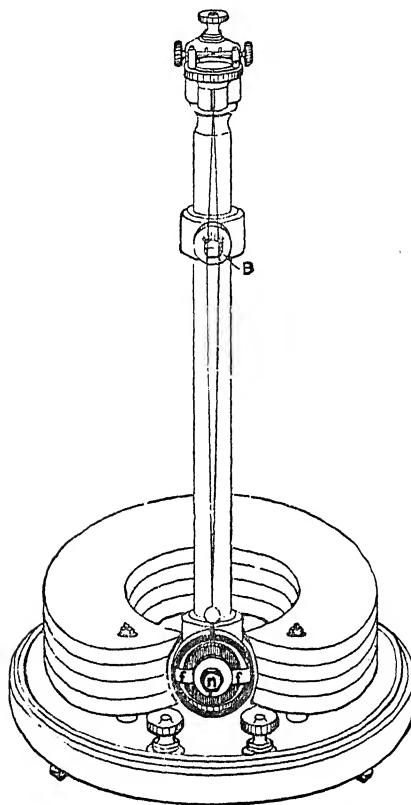
Fɪɢ. 46.—Crompton Movable Coil Galvanometer.

by Holden and made by Pitkin (*see* Figs. 43 and 44), or that designed by Ayrton and Mather and made by Paul (*see* Fig. 45), or that designed and made by Crompton (*see* Figs. 46, 47, and 48), the movable coil galvanometer is

by far the most practical and useful form of galvanometer
for an electrical testing laboratory. It was originally designed

Fig. 47.

by Lord Kelvin for cable signalling although now often
called the d'Arsonval form of galvanometer.

Fig. 48.

The varieties of electromagnetic galvanometer which have
been designed are innumerable. In all of them the current

in a conductor is measured by the measurement of the mechanical action of the associated magnetic field either on

(*a*) Another magnet,
(*b*) A mass of soft iron,
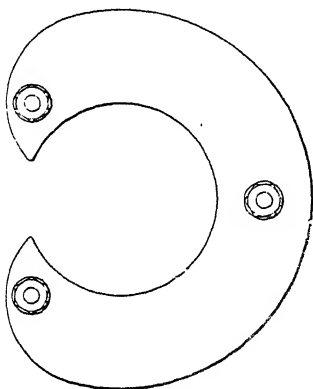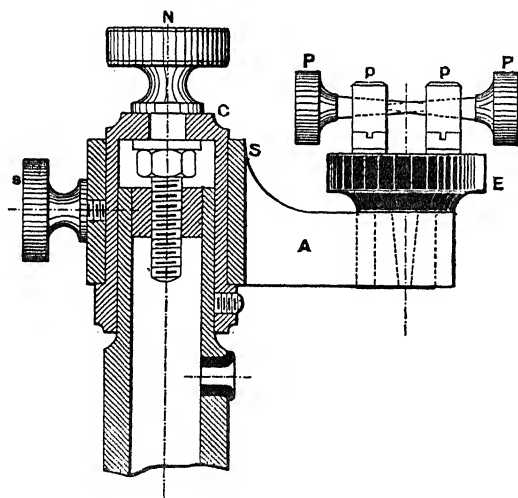(*c*) Another movable conductor carrying the same current.

The magnetic force may produce, in either (*a*), (*b*) or (*c*), a rotation or torque, or a forcive or bodily displacement. In the first case a uniform field is required, in the second a non-uniform field. Hence we may classify the fields as :—

(A) Uniform for rotational deflection,
(B) Non-uniform for translational displacement.

Then the principal currents or current to be measured may be in the fixed or the movable part of the instrument. Hence we have a further classification into :—

(α) Fixed coil galvanometers.
(β) Movable coil galvanometers.

Lastly, some arrangement, or *control*, is necessary to bring back the displaced portion, whether magnet, soft iron or coil, to its original zero position when the current is stopped. This may be achieved by the aid of another magnetic field, or the elasticity of a spring or torsion of a wire, or by the weight or inertia of the moved mass. Hence we classify according to the *control* into :—

I. Magnetic control,
II. Elastic control,
III. Gravity control,
IV. Inertia control.

We can, therefore, symbolise any given type of galvanometer by an expression of four symbols denoting the classes in which it is placed. Thus the ordinary mirror Kelvin galvanometer is represented by the symbol (αAαI.). The d'Arsonval or recorder type of galvanometer by the symbol

(αAβII.). The above method of classification of galvano-
meters is that due to Prof. G. F. FitzGerald.*

In appraising a galvanometer, or evaluating its applica-
bility for any purpose, we have to take into account five
qualities which, when numerically expressed, may be called
the five specific constants of the galvanometer. These quali-
ties are :—

    (i.) The periodic time of the movable system, or time
        of one complete small oscillation when the
        movable part is disturbed and then left to
        itself.

    (ii.) The logarithmic decrement corresponding to different,
        or some known, amplitudes of swing, *i.e.*, the
        logarithm (Napierian) of the ratio of the amplitude
        of one excursion on one side to the next one on
        the other side of the zero point.

    (iii.) The sensitiveness, both ballistic and deflectional.

    (iv.) The internal resistance, or coil resistance.

    (v.) The zero-keeping quality, or degree of precision,
        with which the movable part returns exactly to
        the original zero position when disturbed and left
        to itself again.

If the needle or coil of a galvanometer is disturbed, it
comes to rest after one or more vibrations. The time of one
complete vibration, or interval between passing the zero
point in the same direction, is called the *periodic time*. These
swings of the needle or coil are resisted by the air or other
causes of friction, and the amplitude of the excursions
gradually diminishes. If the amplitude of each swing is
measured, it will generally be found that successive ampli-
tudes decrease nearly in a geometric progression. Hence, in
this case, the logarithm of the ratio of one excursion to the
next one in the same direction is constant. The constant
value is called the *logarithmic decrement* of the galvanometer
The logarithms taken are Napierian. The logarithmic

* See *The Electrician*, Vol. XXXVIII., p. 715.

decrement usually varies with the amplitude of the swing, owing to the fact that the retardation experienced by the coil or needle depends to some extent on the velocity with which it leaves its zero position.

If a small constant current, say a micro-ampere, is passed through the galvanometer coil, it will cause a certain steady deflection of the movable needle or coil. These deflections are nearly always read by means of a mirror and scale. A concave mirror attached to the galvanometer needle, or coil, as the case may be, has a ray of light thrown upon it and a sharp image of an illuminated wire or incandescent lamp filament is by it thrown upon a scale. The scale is generally placed one metre or 1,000 millimetres from the mirror.

The *sensitiveness* or *deflectional constant* of a galvanometer may be defined as the scale deflection in millimetres produced by a current of one micro-ampere passing through the galvanometer coil, the scale being at one metre distance from the mirror. It may also be stated in terms of the potential difference in micro-volts, which must be applied to the terminals of the galvanometer to produce this same unit deflection, or as the deflection in millimetres, at metre distance of scale, per micro-volt on the terminals. The sensitiveness of a galvanometer must always be controlled by the possession of good *zero-keeping quality.* By this is meant that when the current is stopped the galvanometer needle or coil returns again to a fixed and constant zero position. It is easy to give a galvanometer a spurious sensitiveness, but if the zero position of the movable portion is not constant, the value of the galvanometer for quantitative purposes is very small.

A galvanometer with a very large logarithmic decrement is called a *dead-beat* galvanometer. One with a very small logarithmic decrement is called a *ballistic galvanometer.* The *ballistic constant* of a galvanometer is defined as the reciprocal of the "throw" or excursion of the needle or coil when one micro-coulomb of electric quantity is discharged through it.

The electrical laboratory should be provided with a collection of movable coil dead-beat, and movable coil ballistic galvanometers of various resistances, some large, 500 or 1,000 ohms or more, and some of low resistances, such as 0·5, 5 or 10 ohms. It is desirable also to have at least one very sensitive Kelvin mirror astatic movable needle galvanometer of 6,000 to 10,000 ohms resistance.

Generally speaking, tangent galvanometers and needle instruments are more suitable for a physical laboratory than for an electro-technical laboratory, the disturbances caused by the presence of large electric currents rendering it most difficult to use the movable needle deflectional instruments. As regards ammeters, for ordinary working purposes the most convenient are those which, like the Weston ammeters, are movable-coil instruments. These instruments are very dead-beat, and have equal or nearly equal scale divisions per ampere or per milliampere. These instruments are, however, only available for continuous currents.

For use with alternating currents, some form of hot-wire ammeter, such as that of Hartmann and Braun, is very useful. Its indications are entirely independent of the frequency of the alternations, and the instruments are also very dead-beat.

For many purposes the Siemens dynamometer is an invaluable laboratory instrument (*see* Fig. 49). It consists of a fixed coil or coils, and a suspended coil which embraces the fixed coil, and the normal position of which is with its plane or axis at right angles to the plane or axis of the fixed coil. The movable coil is hung by a few fibres of floss silk, and its position is controlled by a spiral spring, the lower end of which is attached to the movable coil and the upper end to a torsion-head. The current is led into and out of the movable coil by means of mercury cups. The wires forming the fixed and movable coils should always be brought to separate terminals. The instrument can then

be used for several purposes. If the coils are joined in series, and a current, either alternating or continuous, sent through them, forces are brought into existence which cause the movable coil to be acted upon by a couple or torque, and to be twisted round. If, then, a contrary twist is given to the torsion-head and the movable coil brought back to its zero position, the angular twist required to effect this is proportional to the square of the strength of the current. The dynamometer can thus be calibrated
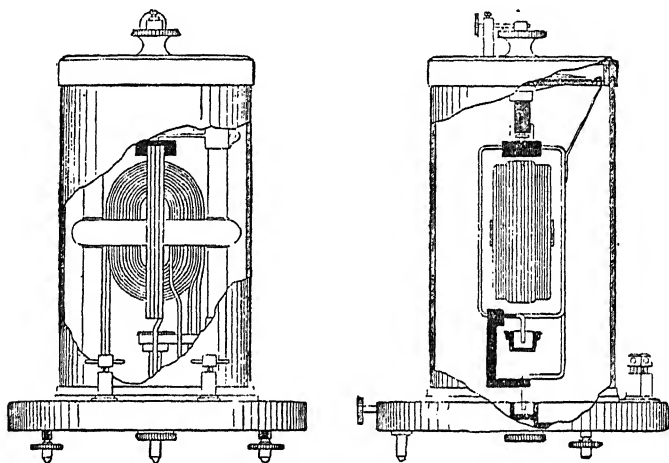
FIG. 49.—Siemens Electrodynamometer.

to measure current strength. If the current is unvarying, the square root of the torsion or scale reading gives, when multiplied by a constant, the current strength. If the current is alternating and periodic it gives the square root of the mean of the squares of the equi-distant instantaneous values during the period or the root-mean-square (R.M.S.) value of the current, provided that the periodic time of the current is small compared with the periodic time of a free oscillation of the movable coil. If

different currents are sent through the two coils the instrumental readings can be made to give the mean value of the product of their maximum values multiplied by their power-factor or cosine of the angle of phase difference, if the currents are simply periodic.

§ **13. Voltage Measuring Instruments.** — Any of the above galvanometers or amperemeters, if wound with wire of sufficiently high resistance, becomes an instrument which may be used for the determination of the steady potential difference between two points. For the current through any current-measuring instrument is proportional to the steady potential difference between its terminals, and inversely as the resistance of the instrument. Hence, if the resistance of the instrument is so high (say, 1,000 to 20,000 ohms) that the current it takes off, when placed as a shunt circuit between two points on any other circuit, open or closed, does not disturb sensibly the potential difference between those points, the indications of the instrument are proportional to the original potential difference between those points.

For many laboratory purposes electromagnetic voltmeters, as they are called, of the above type will be found convenient.

There are, however, many measurements in which it is desirable that no current shall be taken off between the points, the potential difference (P.D.) of which is required. Also it is often necessary to use the same instrument for continuous and for alternating currents. In these cases the best voltage-measuring instruments to employ are the electrostatic voltmeters. These are now made to measure voltages varying from 1 volt up to 40,000 volts or more. Their general principle is as follows :—

Let there be two sets of plates or metallic surfaces, one fixed and the other suspended, so as to be capable of moving in between the fixed surfaces. Let the normal position of the movable plate be just outside the fixed one. The two sets

of plates are insulated from each other, and thus form a condenser or Leyden jar having a certain capacity If we produce a certain potential difference between these plates, this P.D. brings into existence forces of attraction between the plates proportional to the square of the difference of potential,
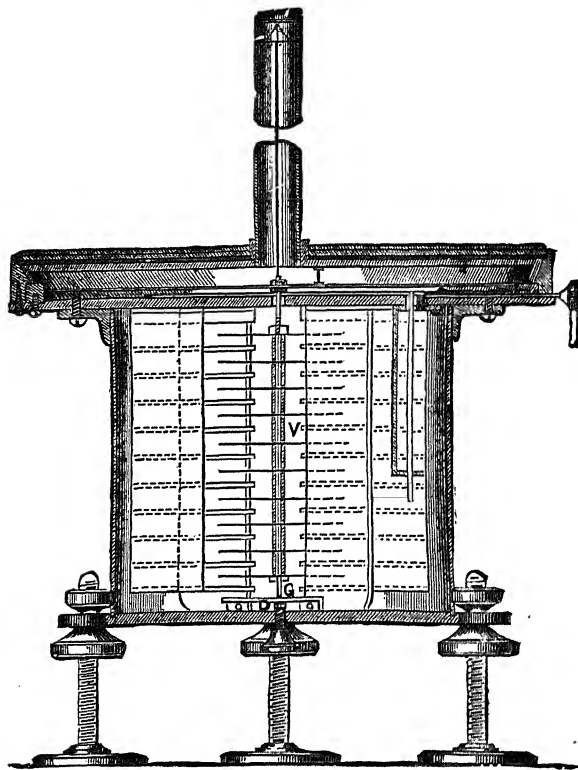

FIG. 50.—Kelvin Multicellular Electrostatic Voltmeter.

and therefore independent of its sign.   This force draws the movable plates in between the fixed ones, so as to increase the capacity of the condenser formed by the two plates or sets of plates.   If this force is resisted, either by the torsion of a suspending wire or by gravity, we have

an arrangement called an *electrostatic voltmeter*, which can be calibrated to show difference of potential, either the steady value or the root-mean-square value, according as the pressure is uniform or periodic.

Voltmeters of this form have been designed by Lord Kelvin, Prof. Ayrton, and others. Of these the most

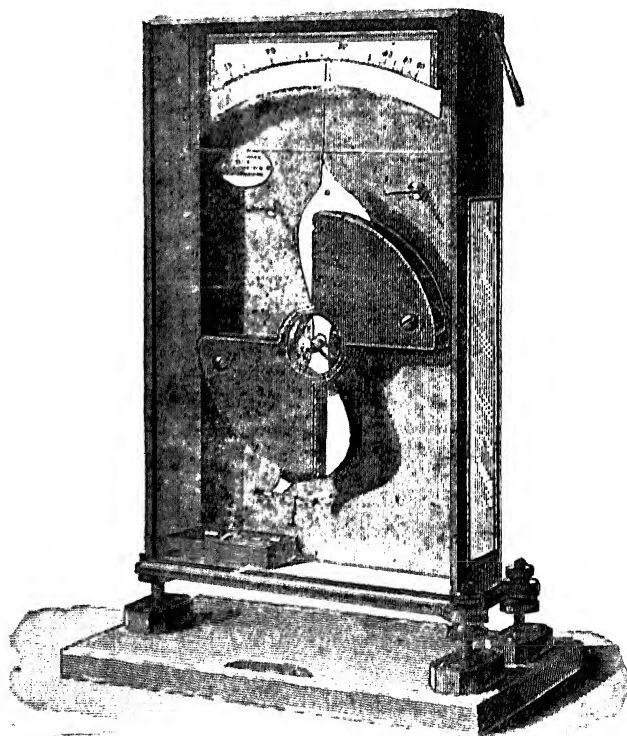Fig. 51.—Kelvin Electrostatic Voltmeter.

convenient form for the electrical engineering laboratory are the horizontal and vertical pattern of multicellular electrostatic voltmeters, as designed by Lord Kelvin for measuring electrical potential difference from 50 to 150 or 250 volts (*see* Fig. 50); high-pressure electrostatic voltmeters, as designed by Lord Kelvin (*see* Fig. 51) and by Prof. Ayrton for high

pressures from 1,000 to 2,500 volts; and the low-pressure electrostatic voltmeters of Prof. Ayrton, for pressures from 2 to 50 volts, are also exceedingly useful instruments.

These electrostatic voltmeters are also available as current-measuring instruments. If a current is sent through a resistance, and the fall of potential down the resistance is measured by a correct electrostatic voltmeter, the current through the resistance becomes known. This method of current-measuring is convenient in measuring small alternating currents, such as the magnetising current of a transformer.

Electrothermal voltmeters, such as those of Cardew, Holden, and Hartmann and Braun, are at times required when dealing with alternating currents. They are, however, less generally useful than the electrostatic instruments because they take up much more power to operate them.

As regards the method of standardising all the above instruments, and obtaining measurements of current and potential difference by means of the potentiometer method, the details of these processes will be discussed in a later section of this treatise.

The instrument called the potentiometer is of the greatest utility in the electrical laboratory, and a good potentiometer is one of the fundamental requisites of an electrical testing room. In its simplest form it consists of a uniform wire stretched over a scale divided into 2,000 parts. This wire has its extremities connected to a couple of secondary cells (*see* Fig. 52), and an interpolated resistance inserted in the circuit. By varying this resistance it is possible to adjust the current in the wire so that the fall of potential down a length of the wire equal to 2,000 scale divisions is just 2 volts. This is achieved by placing a Clark cell and a sensitive galvanometer as a shunt on the wire, and making contact at two scale divisions on the wire the interval between which corresponds numerically to the electromotive force value of the Clark cell. Thus, suppose the cell has an

electromotive force of 1·434 volts at 15°C., then two sliding contacts on the galvanometer and standard cell circuit are set to make contact at the zero and at the 1,434th scale division. When this is done the resistance in series with the two-cell secondary battery is altered until the galvanometer indicates no current. In order that it shall be possible to do this, the positive pole of the Clark cell must be connected with that end of the potentiometer wire to which is joined the positive pole of the secondary cells, called the working battery. When this adjustment is made, the potentiometer is said to be *set*. If, then, it is desired to measure any other potential difference, say that due to the passage of a current
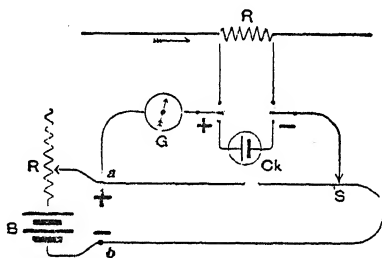
Fig. 52.

through a low resistance, wires called potential wires are brought from the ends of this resistance, and in one of these circuits a galvanometer is inserted. One potential wire—viz., that attached to the highest potential point—is joined to that end of the slide wire in connection with the positive pole of the working battery. The other potential wire is attached to a slider making a variable contact on the slide wire. The slider is then moved until the galvanometer indicates no current. This can be always done if the fall in potential down the resistance is less than that down the slide wire. The reading on the slide wire, as shown by the position of the slider, gives at once the potential difference in volts between the potential wires.

FIG. 53.—External View of Crompton's Potentiometer.

The practical potentiometer takes one of two forms. It may be, as described, a simple slide wire potentiometer, or the slide wire may be in part or in whole replaced by coils of wire arranged in series, as in the potentiometers of Crompton, Fleming, Nalder, Wulf, and others.
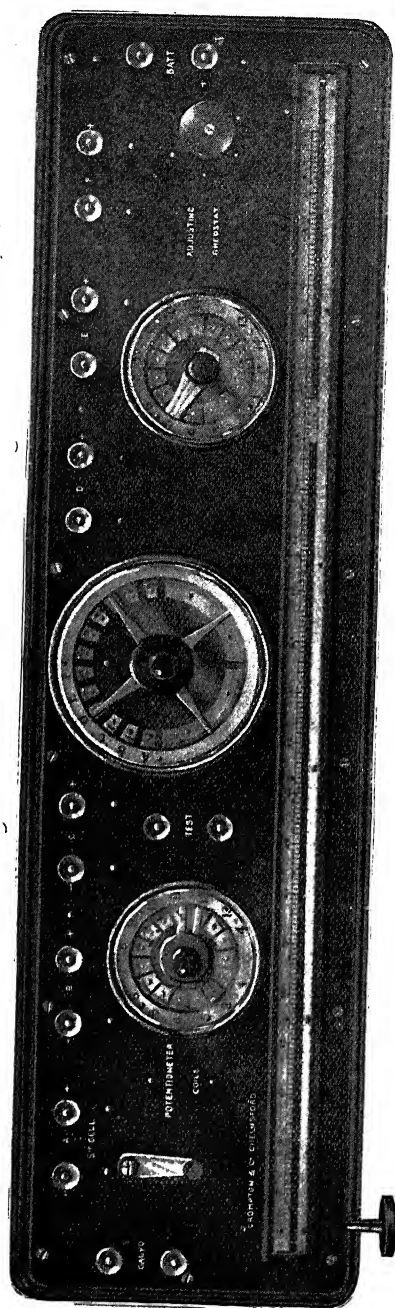
The electrical laboratory should be provided with one or two potentiometers.* One should be a simple slide wire instrument for rough work, the other a standard instrument for exact work, made entirely with carefully adjusted coils of wire.
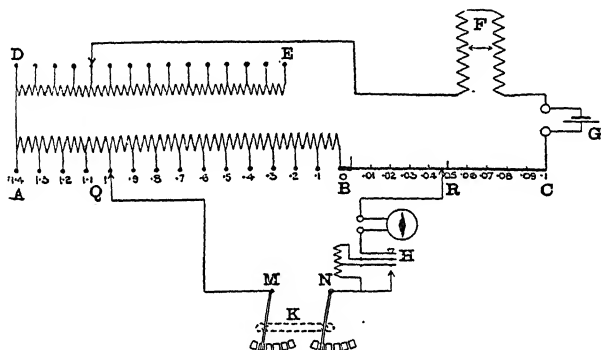
FIG. 53A.—Scheme of Connections of Crompton Potentiometer.

The Crompton potentiometer is shown in Fig. 53, and the connections in Fig. 53A. The external appearance of the Nalder potentiometer is shown in Fig. 54. In the former (the Crompton) instrument the slide wire consists of 14 coils placed inside the case of the instrument, and a slide wire equal in resistance to one of these coils stretched over a scale outside the box. In series with these coils is a circular rheostat for adjusting the potentiometer current. There is a double pole six-way switch, which enables any one of six

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* The arrangement now called the potentiometer was first described by Poggendorff, *Poggendorff's Annalen*, Vol. LIV., page 161, 1841. The method of making the potentiometer direct reading, so as to give the potential difference without calculation, is due to the Author, and was first described in *Industries*, July and August, 1886.

test circuits to be placed in series with a galvanometer and in shunt with any portion of the slide wire. The galvanometer circuit has one end attached to a sliding contact which moves over the slide wire. The other end can be attached to the junction between any one of the 14 coils in series with it. When the potentiometer is "set," the current through the slide wire is such as to make the fall in potential down the whole wire 1·5 volts. The scale is divided into
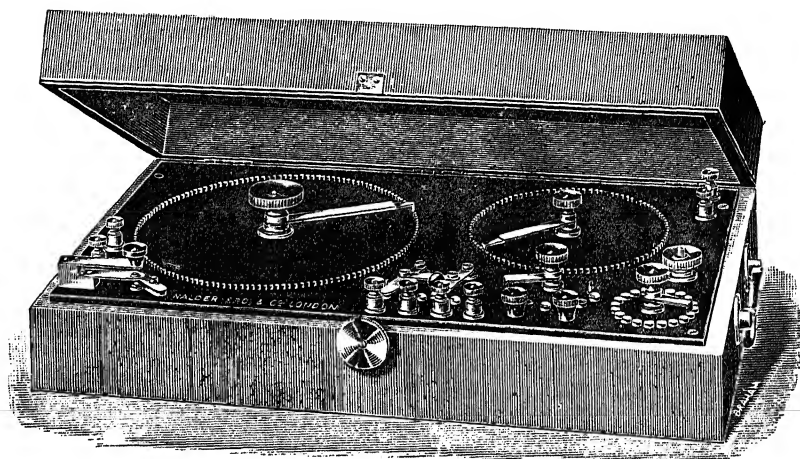
FIG. 54.—Nalder's Potentiometer.

1,000 parts, and each slide wire unit represents one-ten-thousandth of a volt.

In the Nalder instrument the whole of the slide wire consists of coils placed within the box containing the instrument, and contact is made with the junction between these coils by means of a circular revolving arm.

In Fig. 55 is shown a diagram of the connections of the instrument as depicted in Fig. 54. Referring to the second figure, it may be seen that a secondary battery is joined on to the terminals F, and sends a current
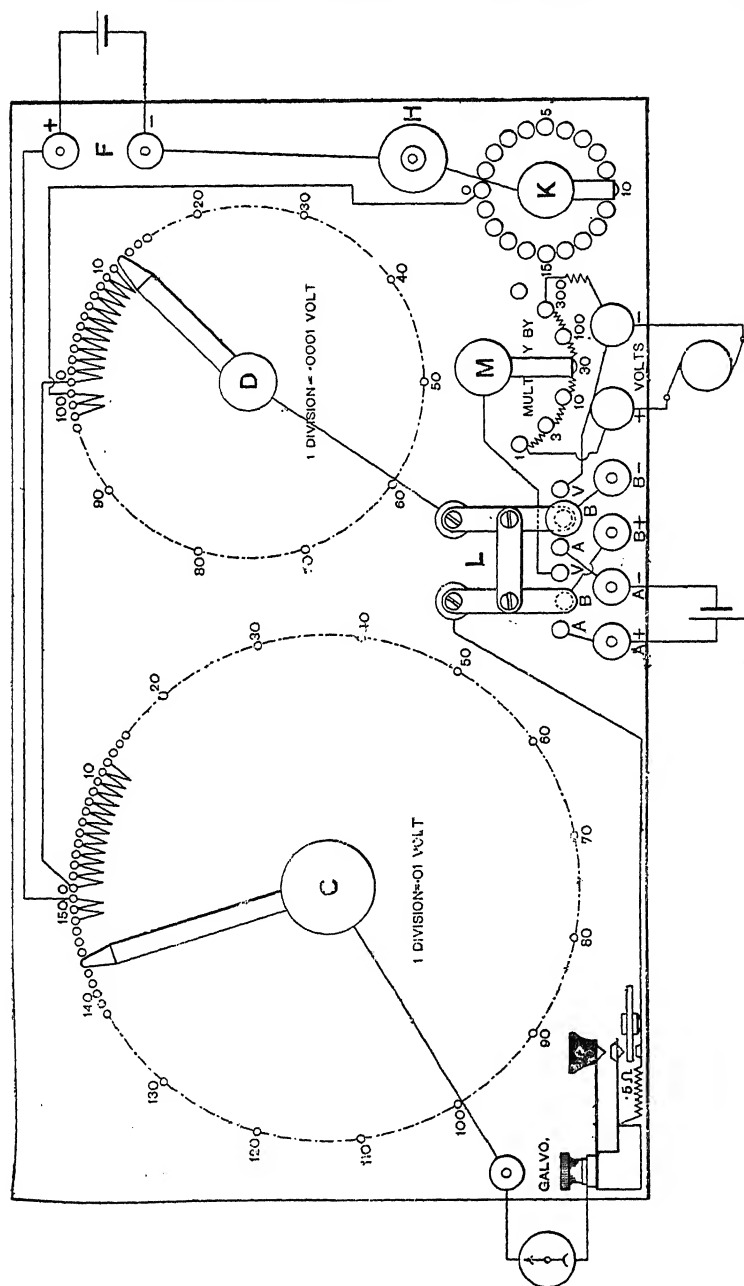
Fig. 55.—Connections of Nalder Potentiometer.

through the resistances connected with the dials C and D, and then through adjusting resistances K and H. The dial D has 100 equal coil resistances in it, the whole set being equal to the resistance of one coil in C. The dial K contains 19 equal small wire resistances, and the resistance H is a carbon resistance for fine adjustment. A standard cell is joined up to A A, and a galvanometer to the terminals marked *galv.* If the cell has a voltage 1·4412 volts, the arm of dial C is set to 144 and that of D to 12. The resistances H and K are then adjusted so that the galvanometer shows no current, and the potentiometer is then " set." The voltage to be measured is connected to the terminals B B if below 1·6 volts, and if above that to the terminals marked volts, in Fig. 55, which are bridged by a wire of high resistance. A fraction of this potential difference, either $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{300}$, is then measured, according as the switch M is on the contact marked 3, 10, 100, or 300,

In practice it is found that potentiometers in which the contact with the galvanometer circuit is made either by rubbing or pin contacts, or by a slide contact moving over a wire, give trouble owing to imperfect contacts due to the deposition of dust on the contact surfaces and on the wire, and time is wasted in getting the necessary adjustments made. These defects led the Author to design a form of potentiometer in which there is no slide wire and no rubbing or pin contacts, but in which the whole of the potentiometer wire or resistance is represented by coils of wire contained in a case. The galvanometer circuit containing the electromotive force to be measured is brought to two terminals, which are in connection with two bars or rings of brass, by means of plugs fitting into holes bored out partly in these bars or rings and partly in adjacent metal blocks. It is possible to insert any required portion of the whole potentiometer wire resistance, within limits, in between the galvanometer terminals, whilst at the

same time resistance is added or subtracted from a main current circuit, on which the galvanometer circuit is a shunt, in such fashion that a constant current flows through the main potentiometer circuit. This compensating resistance is added or removed by plugs, as in the case of a plug pattern Wheatstone bridge. Fuller details of the use of the instrument will be given in a later section of this work.

Generally speaking, the most convenient ammeters for laboratory use are those which depend upon the measurement of the fall of potential down a low resistance. The ammeter then consists of two parts :—

(i.) A suitable resistance, which carries practically the whole of the current, and which may even be removed a considerable distance from the remainder of the instrument.

(ii.) A potential measuring part, which consists of some form of movable coil galvanometer of high resistance, or for special tests may be a potentiometer.

The galvanometer will generally take the form of a permanent and well-aged magnet for producing a field in which is suspended a light high resistance coil, with pointer attached, moving on jewelled centres, and also a steel spiral controlling spring to keep the coil in a normal position, and against which the electromagnetic force acts (*see* Fig. 56). These instruments have the advantage that the scale divisions are usually very nearly equidistant. There is no blank part or non-readable portion of the scale, and they are also very dead-beat. In Fig. 56 is shown part of the mechanism of a Siemens and Halske voltmeter constructed on the above principle. Part of the magnet is removed to show the coil. The coil consists of an insulated wire wound on a copper frame to damp the movements. The coil has attached to it an index needle, and its displacement is resisted by a steel spiral spring. The

resistance traversed by the current to be measured may be
contained in the case of the instrument, or it may be entirely
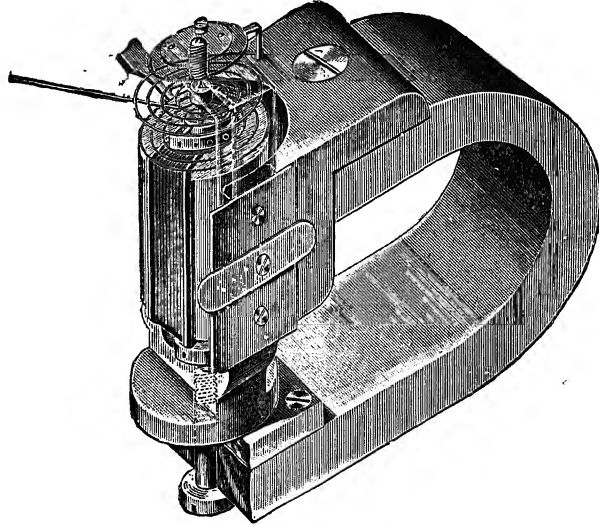
Fig. 56.

separate and used outside. In Figs. 57 and 58 are shown the
details and mode of use of an Arnoux-Chauvin dead-beat
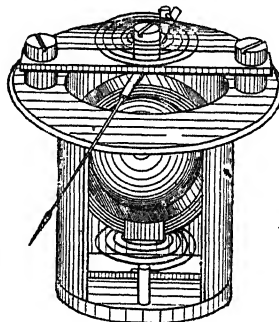
Fig. 57.

ammeter and shunt constructed on the above principle,
consisting of a high resistance movable-coil galvanometer

with equi-divisional scale used as an ammeter by measuring the fall in potential down a low resistance shunt of known value.

Instruments of the above-described type are much employed in the laboratory as ammeters and voltmeters for the measurements of continuous currents and potential differences. A series of very convenient and portable
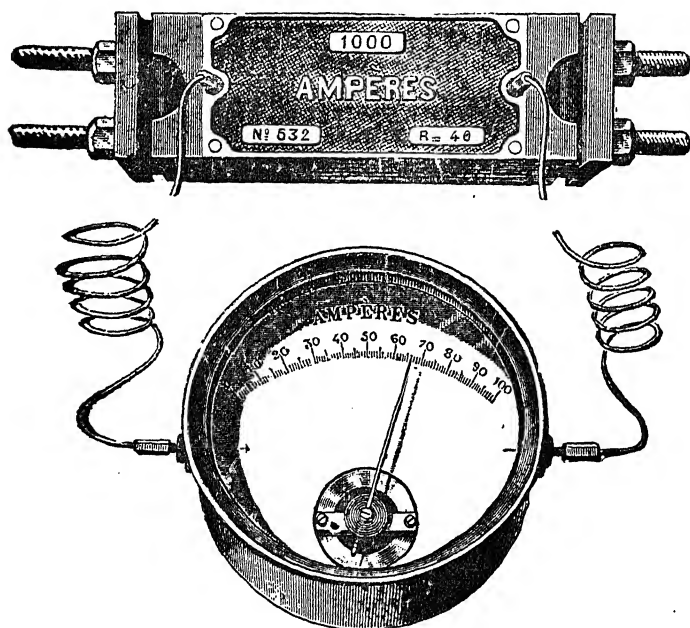
Fig 58

instruments of the above kind have been devised by Mr. Weston. These are remarkable for the almost exact equality of distance of the scale divisions. They are made for various ranges of work, such as ammeters to measure from 0 to 15 amperes, 0·0 to 100 amperes, milliamperemeters and voltmeters.

The three most convenient Weston instruments for general laboratory work are the ammeter, reading from zero to 15 amperes ; the milliammeter, reading from zero to 1·5 amperes by hundredths of an ampere ; and the voltmeter, which reads from zero to 150 volts on one pair of terminals and from zero to 15 volts on another pair.

Instruments of a similar kind are made by many other manufacturers.

§ 13A. **Resistance-Measuring Instruments.**— The comparison of conductors in regard to the quality called their electrical resistance is generally conducted by means of
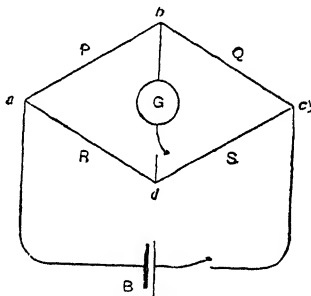
FIG. 59.—Wheatstone's Bridge, arrangement of Circuits.

instruments technically termed *bridges* or *Wheatstone's bridges,* provided the resistances to be compared are neither of them very large or very small in magnitude. In its simplest form the arrangement called a *Wheatstone bridge* consists of six conductors joining four points. In one of these circuits is placed a source of electromotive force such as a battery, and in one other circuit, called the conjugate circuit, is placed a galvanoscope for detecting the presence or absence of a current.

The circuit arrangement is depicted in Fig. 59. When the resistance of the circuits is so adjusted that closing the battery circuit (B) does not produce any permanent current in the conjugate galvanometer circuit (G), the " bridge " is said to be

balanced. To effect the balancing, two of the resistances (P and Q) are adjusted to a certain ratio, and these are called the *ratio arms* of the bridge. Of the other pair of resistances or *arms* of the bridge, one is a standard of resistance (S) and the other (R) is the resistance to be compared. The balance is obtained either by altering the ratio arms or by altering the value of the resistance of the standard or measuring arm.

The bridge may be employed in one of two ways :—

(i.) To obtain the ratio of two resistances.

(ii.) To obtain the difference of two resistances.

The first method is the one generally employed in ordinary work, and may be called the Wheatstone method. The second is that used in very exact work, and is due to Prof. G. Carey Foster.

The ordinary bridge, as a laboratory instrument, takes one of two forms :—

(**i.**) The Slide Wire Bridge, in which the ratio arms can be continuously varied, and the standard of resistance generally remains unchanged.

(**ii.**) The fixed coil pattern, or Plug Bridge, in which the ratio arms can only be given certain decimal ratios, but the standard or comparison resistance can be varied within wide limits by adding fixed resistances in series.

The complete theory of the bridge will be considered in Chapter II. under the head of RESISTANCE MEASUREMENT; but meanwhile we may state the simple principles of the bridge as follows :—

First, let it be assumed that the *bridge is balanced.* Then it is clear that, starting from point a, the voltage or fall in potential down P and R is the same, because the points b and d are, by supposition, at equal potentials, as they must be if no current flows through the galvanometer. Similarly the potential fall down Q and S is equal. Call these potential falls V_P, V_Q, V_R, V_S. Then also, since no current flows through the $b\ d$ circuit or galvanometer circuit, we must have equality

L

in the currents in P and Q and in those in R and S.   Call these currents C_P, C_Q, C_R, C_S, and the resistances of the arms P, Q, R, S.   Then we have

$$V_P = V_R \text{ and } V_Q = V_S;$$

also

$$C_P = C_Q \text{ and } C_R = C_S.$$

But, by Ohm's law, $\dfrac{V_P}{C_P} = P, \dfrac{V_Q}{C_Q} = Q, \dfrac{V_R}{C_R} = R, \dfrac{V_S}{C_S} = S.$

Hence

$$\frac{P}{Q} = \frac{R}{S} \text{ or } R = S\frac{P}{Q}.$$

If the bridge is not balanced—that is, if the current through the galvanometer circuit is not zero, but has a value C_g—then it will be subsequently shown that the galvanometer current can be calculated from the expression

$$C_g = E\frac{QR - PS}{\Delta},$$

where E is the electromotive force of the battery and

$$\Delta = Br(P + Q + R + S) + B(\overline{P + Q} \,.\, \overline{R + S}) + r(\overline{P + R}\,\overline{Q + S})$$
$$+ PQRS\left(\frac{1}{P} + \frac{1}{Q} + \frac{1}{R} + \frac{1}{S}\right),$$

B being the resistance of the battery circuit, and r that of the galvanometer circuit.*

The practical forms of bridge required in the electrical laboratory are as follows:—

I. A standard slide wire bridge, for the careful comparison of standard coils with other nearly equal resistance coils.

II. A simple form of plug bridge, for approximate measurements of resistance.

III. A well-constructed form of dial plug bridge, for accurate work.

IV. A differential bridge, for exact comparisons between standard coils.

V. A low-resistance bridge, for very small resistances.

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* For the method of dealing with problems of networks of conductors see § 2. Chap. II., where the reader is referred to a Paper by the Author in the *Phil. Mag.*, August, 1885 ; also *Proc.* Phys. Soc. London, Vol. VII., 1885.

, simplest form of (L) or slide wire bridge consists of a
mahogany board, on which is strained a gilt manganin
tinum-silver wire, the wire being uniformly stretched
en heavy terminal pieces of copper. The ends of this
re united by a parallel bar of copper in which are two
one to be closed by the standard resistance (S) and the
by the resistance (R) to be measured. These resistances
erted either by means of good terminal screws or by
e of mercury cups. In the practical construction of a
vire bridge one of the difficulties consists in keeping
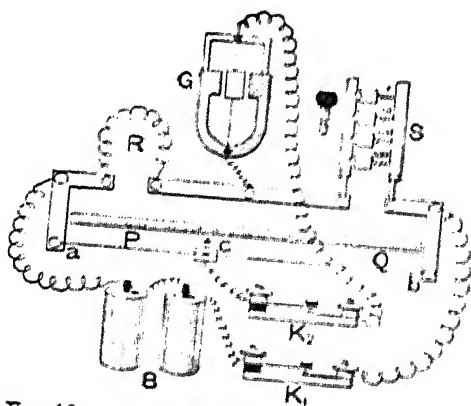
FIG. 60.— Simple Slide Wire Resistance Bridge.

e wire tight and yet in allowing it to expand or
The arrangement of the simple slide wire bridge
in Fig. 60. The slide wire is attached to thick end
of copper drawn back by springs which keep the
t tight enough without overstraining it. By cali-
the wire, as explained in the next section, the
e of these copper terminal blocks can be deter-
terms of the resistance of a centimetre of length of
Over the slide wire moves a slider (r) having a knife-
tact and key, and parallel with the slide wire is a
cale equal in length to the slide wire. The battery.

consisting of two or three dry cells, is connected to the ends of the slide wire through a contact key, and the galvanometer is connected to a terminal between the standard resistance and resistance to be measured and the knife contact of the slides. The galvanometer used with the bridge is preferably one of the movable coil dead-beat type. The battery may consist of three dry cells. The operation of measurement consists in finding the point at which the slider must make contact with the wire so that no current passes through the galvanometer. The ratio arms of the bridge are then the sections into which the slider divides the slide wire, and the value of the resistance (R) being tested is equal to
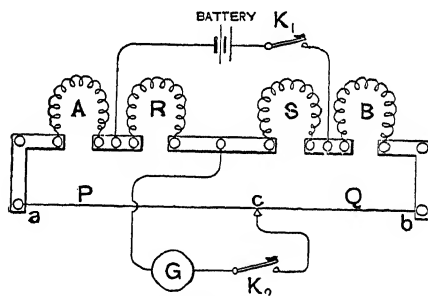
FIG. 61.—Double-gap Slide Wire Bridge.

the comparison resistance (S) multiplied by the ratio (P : Q) of the slide wire sections. Increased sensitiveness is given to this bridge by making double gaps in the copper bar and adding two more coils or resistances to the ends of the slide wire. These coils are then extensions of the slide wire, and the slide wire becomes only a portion of the ratio arm resistance.

Thus, let the resistances of these added coils (*see* Fig. 58) be A and B, and let x and y be the lengths of the sections into which the slide wire is divided by the contact piece (c) when the balance is obtained. Then, if R is the resistance being measured and S is the standard resistance, we have

$$R:S = (A+x):(B+y).$$

A displacement of the slide of one millimetre along the slide wire corresponds then to a much less difference between R and S than would be the case if the coils A and B were not present. This extension enables us to compare together very accurately two coils by measuring the difference between the resistances of the two coils by a method due to Prof. G. Carey Foster, and also to calibrate the wire or determine the resistance of the slide wire per unit of length.

Let the bridge be first balanced with coils A and B (as shown in Fig. 61).

Then let the position of the coils A and B be interchanged and a fresh balancing position be found at a distance x' divisions of the slide wire from the left hand end.

Let ρ be the resistance of the length of x divisions of the slide wire, and let W be the resistance of the whole slide wire a b.

Let ρ' be the resistance of the length of the x' divisions of the slide wire.

Then corresponding to the two cases we have resistanc equations

$$R:S=A+\rho:B+W-\rho,$$

$$R:S=B+\rho':A+W-\rho'.$$

Hence
$$\frac{A+\rho}{B+\rho'}=\frac{B+W-\rho}{A+W-\rho'},$$

or
$$\frac{A+\rho}{B+W-\rho}=\frac{B+\rho'}{A+W-\rho'},$$

Therefore it follows that

$$\frac{A-B-W+2\rho}{A+B+W}=\frac{B-A-W+2\rho'}{B+A+W},$$

or
$$A-B=\rho'-\rho.$$

In other words, the difference of the two resistances A and B is equal to the resistance of that length of the slide wire included between the x' and x divisions at which the balance is found on the slide wire in the two experiments.

If, then, the wire is of uniform resistance per unit of length, and if this resistance per centimetre is known, the difference between the resistances A and B is known.

The slide wire can be calibrated or tested for uniformity as follows:—

In place of the resistances R and S substitute another slide wire and another contact slider piece, so that the ratio arms of the bridge can be varied at pleasure. Then by moving the second slider the balancing position on the slide wire $a\,b$ can be brought to any required position for any two given coils A and B. For A and B select two coils of wire having a very small difference in resistance, and proceed as above described to find the length $x' - x$ of the bridge wire which has a resistance equal to $A - B$. Then move the second slider a little, so as to change the ratio of R : S, and find again in another place on the slide wire $a\,b$ the length $x' - x$ equal in resistance to $A - B$. If this length $x' - x$ is not equal to the same number of slide wire divisions in all parts of the wire, the wire is non-uniform in resistance. It is possible, by moving the second slider, to regularly inspect and measure the resistance per centimetre of the slide wire; but the process of applying the necessary correction for non-uniformity of slide wire is so troublesome in practice that it is better to reject the slide wire if non-uniform and obtain another and better wire.

If, then, we obtain a slide wire the resistance of which per centimetre is uniform and is known, we can at once determine the difference in resistance between any resistance coil and a standard resistance not differing from it in resistance by a greater amount than the resistance of the whole slide wire.

The Author designed in 1880 a special form of slide wire bridge for making such difference measurements very quickly, and this has been found especially convenient in comparing together standard coils and others intended for standards.*

* This bridge and the B.A. Standard Coils are now deposited in the National Physical Laboratory, Teddington.

Following is a description of this form of standard comparison bridge :—

Description of the Resistance Balance.—A circular disc of mahogany 18in. in diameter and about 1in. thick (*f*, Figs. 62 and 63) stands upon three short feet, L. Upon this, and concentric with it, is screwed down a disc of ebonite 14in. in diameter and ¾in. thick (*e*). This ebonite disc has a semicircular groove turned in its circumference. The circular wooden base extends on one side into a narrow rectangle, *j*, 4in. wide and of the same thickness as the disc. To this are connected two other rectangular pieces *h*, *i*, which are joined together by slotted brass bars *g*, (Fig. 62) underneath, in such a manner as to permit the two intervals to be made wider or narrower at pleasure. This promontory is of wood, of the same material and thickness as the disc *f*, and is supported and levelled by three levelling-screws, *n*, *n'*, *n''*. Through the centre of the ebonite disc passes a brass centre-pin D D' (Fig. 62), on which is centred a brass arm, H H', capable of revolving just clear of the disc. Beneath the arm, and soldered to it, is a short brass spring *x*, which depends vertically downwards. This spring carries at its extremity a small prism of platinum-iridium with one edge vertical and turned inwards. In the groove turned in the disc *e* is stretched a platinum-iridium wire about $\frac{1}{32}$in. in diameter. The wire extends round about $\frac{24}{27}$ of the circumference, and is about 39in. long, and the groove is of such a size that the wire lies with exactly half its thickness embedded in it. This wire is represented by the thick black line A C A' in Fig. 62. The ends of this wire are soldered to copper strips *k*, *k*. On the wood rectangles *j*, *h*, *i* is fastened an arrangement of longitudinal copper strips *k*, *k*, which connect together eight transverse square copper bars in the manner shown in Fig. 62. On the ends of these transverse bars are fixed vertical copper pins $\frac{3}{16}$in. in diameter and ¾in. high. On these pins are slipped short lengths of india-rubber tube, which extend beyond the pins so that they form
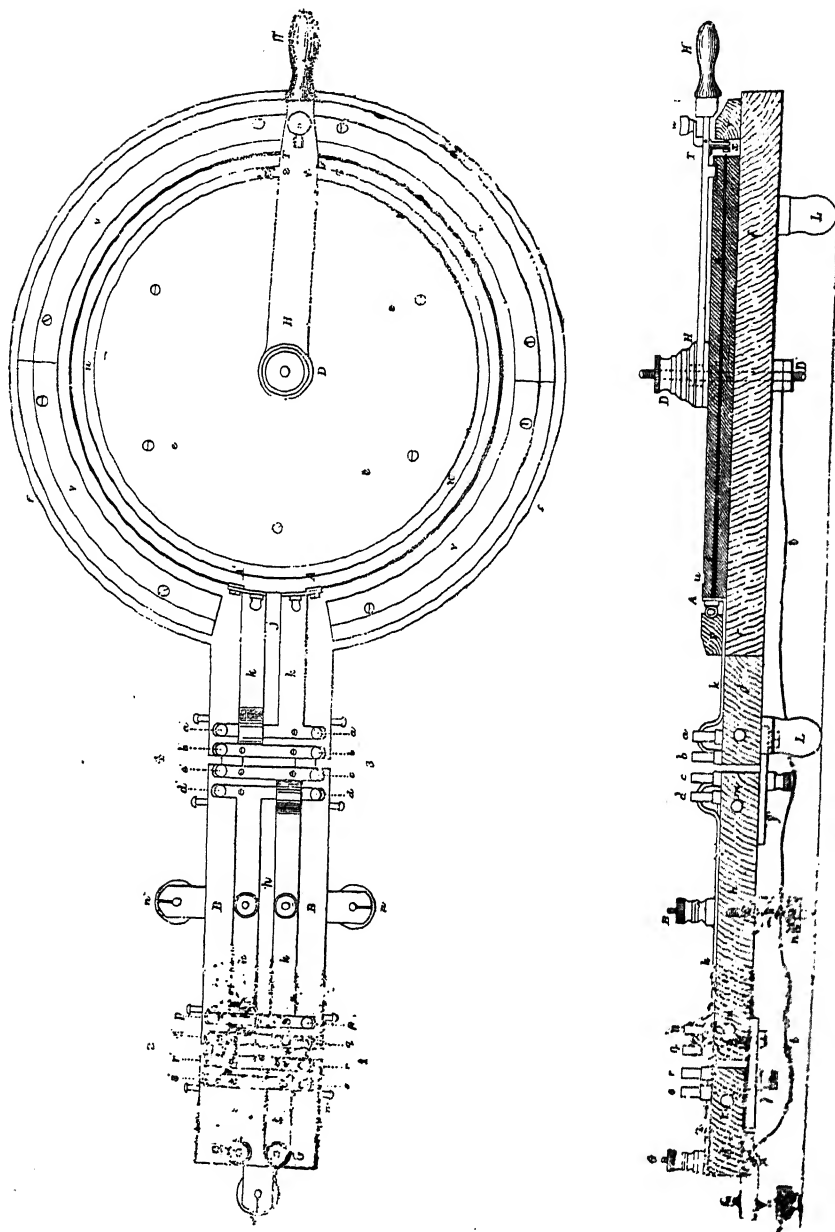
Fig. 62.—Fleming Circular Slide Wire Bridge.

small cups about 1in. deep, p' (*see* Fig. 62). The top of the copper pin is well amalgamated with mercury, and forms the bottom of the cup. These cups are filled about a quarter full of mercury. On the longitudinal strips of copper are fixed three binding screws B, B', G; and a fourth (G') simply goes through the wood, and is connected by a wire t underneath the baseboard with the centre-pin D, and is therefore in metallic connection with the spring x. The battery is connected with the terminals B, B', and the galvanometer with the terminals G, G'. To the arm H H' is adapted a trigger, T, of such shape that when the button w, which is of ebonite,
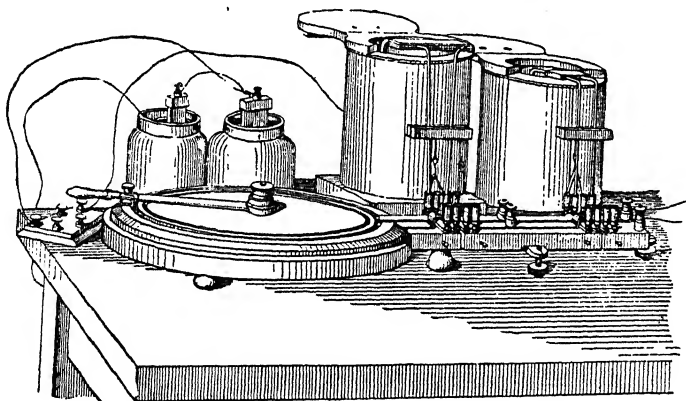
Fig. 63.

is pressed down, the spring x, carrying the platinum-iridium knife-edge, is bent inwards until it touches the wire strained round the circumference of e. The arm caries a vernier, N,, which travels round sunk in a shallow groove in the face of the ebonite disc; and the ebonite is graduated on the face on the margin of the groove. The graduations are cut into the ebonite, and then rubbed over with powered chalk mixed with gum and water. This gives a graduation very legible and pleasant to look at. The length of the wire is just one thousand divisions; and the vernier enables these to be divided into tenths. The zero of graduation is so

placed that, when the pointer of the vernier reads zero, the knife-edge on the spring x is exactly opposite the extremity of the platinum-iridium wire.

It is thus clear that the revolving arm carrying its knife-edge can be moved round so that, on pressing the trigger button w, the knife-edge makes contact at any point of this wire, and thus connects this point with the terminal G'.

This part of the arrangement answers to the sliding block and piston-contact piece of the ordinary divided-metre bridge.

Method of using the Balance.—Let now two resistance coils of about equal resistance be provided, and let the coil terminals of one coil be placed in the mercury cups p and r, and those of the other be placed in q' and s'. And let two more coils be taken of not very unequal resistance which it is desired to compare with each other; let the terminals of one be placed in the mercury cups a and c, and those of the other in b' and d'. It will then be seen that, if a battery be connected with B B' and a galvanometer with G G', that we have the usual Wheatstone bridge arrangements (*see* Fig. 64 for a diagram of the connections). Two quart Leclanché cells are best suited for ordinary use. If a more powerful battery is used, there is danger of heating the platinum-iridium wire, and so expanding it that it may slip down out of its groove.

The coils in the intervals between the cups p and r and q' and s' form two branches; and the coil in the interval between a and c, together with the resistance of the platinum-iridium wire round to the place where the spring x touches it, forms the third branch, whilst the coil in the interval b' d', together with the remainder of the wire, forms the fourth. The "bridge" wire consists of the arm H H' and the wire under the baseboard, together with the galvanometer inserted between G and G'. By moving round the arm H H' and pressing the button w we can find a position where there is no current through the galvanometer. The copper strips k k are made of copper so thick that their resistance is practically nothing. Having established a balance between

the conductors and read the vernier, the next operation is to lift up the legs of the coil which were inserted in the cups a and c and drop them into the cups b and d. Likewise a similar change is effected on the other side; the terminals of the coil inserted in b' and d' are changed to a' and c'. An examination of the connections as shown in Fig. 64 will show that the result of the operation is as if the coils had *changed place*s whilst preserving their former connection. Now let the arm be moved round and a fresh position of equilibrium found by pressing the trigger and reading the vernier. A little consideration will show that the difference
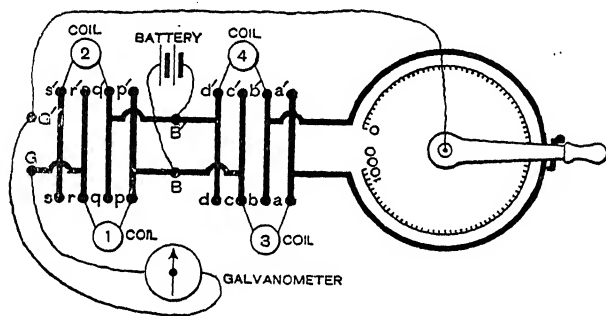
Fig. 64.

of these readings gives the difference between the resistances of the coils in terms of a length of the bridge wire; for the amount by which one coil exceeds the other in resistance is equal to the resistance of that part of the bridge wire included between the two readings.* In order to render this method of determining the difference of the two coils practicable, the platinum-iridium wire must be exceedingly uniform in

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* This method of obtaining the difference of two resistances in terms of a length of the calibrated bridge wire was suggested by Prof. G. C. Foster, F.R.S., in a Paper read before the Society of Telegraph Engineers, May 8, 1872. In this Paper is given an account of the method of calibrating a wire. It is obvious, without any further proof, that if the coil placed in a and c exceeds in resistance that placed in b and d, then on exchanging them, since the united resistance of coils and bridge wire remains the same, that the contact knife-edge must be moved back along the bridge wire by a length exactly equal in resistance to the excess of one coil over the other.

resistance, or else a table of calibration will have to be made. Great pains were taken to procure a length of wire as uniform in size and resistance as possible ; and considerable care was taken, in laying the wire in its groove, not to strain it in any way. It lies evenly in its groove, just sufficient tension being put upon it to keep it in its place. The whole resistance of the wire from end to end is not far from $\frac{1}{20}$ of an ohm at about 15°C.

The wire was carefully calibrated by measuring the difference in the resistance of two pieces of thick brass wire of such lengths that the difference of their resistances was about equal to that of thirty divisions of the bridge wire ; and this difference was measured at about a hundred different equidistant positions all along the bridge wire, and found to be so nearly the same that no table of calibration was deemed requisite. To protect the bridge wire from injuries, as well as to preserve it from being heated by radiation from surrounding bodies, a wooden ring $v\,v$ is fastened down on the baseboard. The ring is $1\frac{1}{4}$in. wide and $\frac{3}{4}$in. deep, and its internal diameter is 1in. greater than that of the ebonite disc. The wire, therefore, lies hidden away on the side of a square-sectioned circular tube ; and, furthermore, a shield of cardboard faced with tinfoil lies upon the face of the disc e, extending just beyond the ring. An aperture is cut in this shield to permit the passage of the trigger, as well as to allow the vernier to be read. By this means the wire is not only out of sight, but out of reach of all radiation as well as mechanical injury.

Arrangements for determining the Temperature Variation Coefficients of Coils.—To determine the temperature variation coefficient of any given coil we proceed as follows :—Three other coils are provided, two of them nearly equal in resistance, which we will call 1 and 2. A third coil, 3, must be taken, whose resistance is nearly equal to that of 4, the coil whose variation coefficient is desired (*see* Fig. 62). The terminals of 3 are inserted in the mercury cups a and c, those of 4 in b' and

d', those of 1 in p and r, and those of 2 in q' and s'. Now the operation to be conducted is to keep the coils 1, 2, and 3 at a fixed temperature, and to keep 4 successively at two known temperatures differing by about 15°C., and to obtain the difference of the resistances of 3 and 4 at these two temperatures. The difference of these differences, divided by the difference of the temperatures, is the mean coefficient of variation of resistance between these temperatures. The chief difficulty to be contended with is that of keeping the temperature of the coils constant during the operation, and of ascertaining what that temperature is; for, as Prof. Chrystal has remarked in his report (Brit. Assoc. *Report*, 1876), it is not easy to tell whether the temperature of the water in which the coil rests is identically the same as that of the wire, since the latter is embedded in a mass of slowly conducting paraffin. To reduce as far as possible the difficulty of keeping the coils at a constant temperature, they are placed in water vessels made of zinc (*see* Fig. 65). These water boxes are composed of two cylindrical vessels—an outer case 9in. high and 8in. in diameter, and an inner one of lesser size; the two are connected at the top, so that they form a sort of jar with hollow sides and double bottom. This interspace forms an air-jacket. Around the inside vessel near the top is a row of small holes; and two tubes communicate at the bottom—one with the inner vessel and the other with the annular interspace. The top is closed by a wooden lid with apertures for thermometer and stirrer. Water can be made to flow from the supply pipes into the inner vessel : it rises up and overflows through the holes, and drains away down the interspace and out by the other pipe. The bodies of the four coils are placed in four water boxes of this description; and water from the town mains being sent in a continuous stream through all four water boxes, the coils are rapidly brought to and maintained at a known temperature. Any desired temperature can be given to one coil by leading warm water from a cistern into its vessel. The annular air-filled space renders the rate of cooling very slow.

Hence the coils, once at the desired temperature, can easily be kept there. Fig. 63 gives a sketch of the arrangement, two of the water boxes being removed to show the connections.

The advantage of the somewhat complicated arrangement of copper bars will now be seen. We can, without withdrawing the coils 3 and 4 from their water boxes, and without in any way disturbing the other arrangements, *reverse* the position of the coils 3 and 4 on the bridge, by simply lifting up the legs
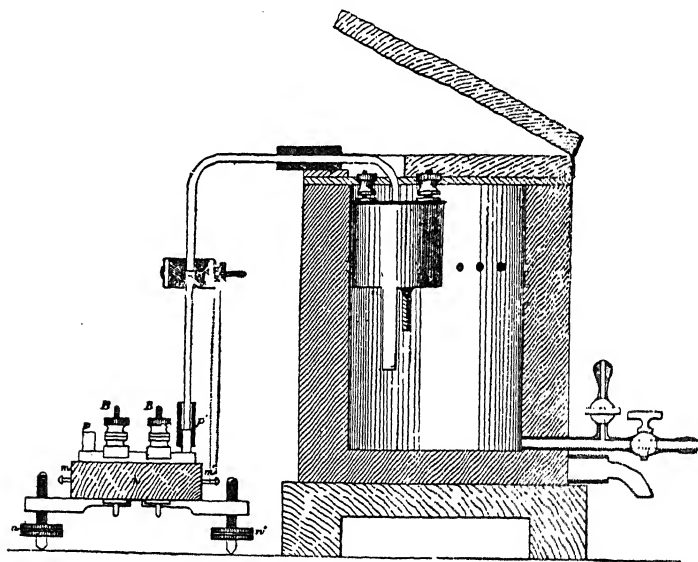
Fig 65.

half an inch and changing the mercury cups into which they dip. Thus the legs of coil 3 are changed from cups a and c to b and d, and those of coil 4 from b' and d' to a' and c'. This exchange does not occupy more than a few seconds; and hence we can obtain the two readings necessary to give the difference of the resistance of the coils 3 and 4 when they are at different temperatures in a very short time. During this short time the temperatures of the two coils will not change perceptibly, protected as they are by an air-jacket.

In the ordinary form of straight bridge there is considerable trouble in exchanging the coils, because the water vessels have to be moved and the mercury cups readjusted ; and all this time the coils are cooling, so that the two readings are never made under the same circumstances as regards temperature. Beginning, then, with all four coils at the same temperature, we take the difference between 3 and 4. To get them all at the same temperature, water from the town mains is allowed to circulate through the system for half an hour. At the end of this time the difference of 3 and 4 is taken ; and several readings are taken at small intervals of time to see if the temperatures are constant. This being done, the temperature of coil 4 is raised by the introduction of warm water until it is about 15°C. above that of coil 3. It is best to raise the temperature about 20°C. above the other at first, and keep it there for 20 minutes, and then let it fall very slowly. In this way coil and water cool together, and an equilibrium of temperature is established between them. The difference between 3 and 4 is again taken ; and from these two readings we have, as seen above, the mean variation coefficient between the two temperatures. Another method, which would probably be a more accurate one for obtaining the mean coefficient of variation between 0°C. and 15°C., would be to wait until the temperature of the water in the town mains was about 15°C., and then to keep three of the coils at that temperature, and to cool the fourth by means of ice to zero. If then all four were kept at 15°C. and the observations repeated, we should have the means of finding the variation coefficient of the fourth coil between 0°C. and 15°C. Prof. Chrystal in his report threw out the suggestion that resistance coils should have a thermoelectric couple attached to them, one junction being buried in the heart of the paraffin surrounding the wire, and the other outside. This has been tried in some coils recently made, and proves a satisfactory method of ascertaining the equilibrium of temperature between the wire and the water.

Another source of error in the ordinary methods arises from uncertain or variable resistances at the mercury cups It is important that the copper legs of the coil terminals should press very firmly against the tops of the copper pins on which the india-rubber tube cups are fixed. To ensure this, the plan adopted is to fasten on the coil legs an ebonite clamp. Along the edge of the wooden promontory *j h i* (Fig. 62) are put brass pins, *m ;* and by means of steel spiral springs fixed to these and attached to the clamps the coil legs are pressed down very firmly (*see* Fig. 63). The ends of the pins which carry the india-rubber cups and the ends of the coil legs being well amalgamated, we get, when they are thus firmly pressed in contact, a very good joint, and one whose resistance is small and constant. If the clamps are not used, then one leg may get lifted up a little, and thus a short length of mercury interposed, which leads to an error in a reading.

Example of a Determination of the Variation Coefficient of a Coil.—In the bridge constructed on the above plan for the Cavendish Laboratory, Cambridge, the whole resistance of the platinum-iridium wire is very nearly 0·0512 of an ohm, or not far from $\frac{1}{20}$ of an ohm, at about 15°C. As the whole length can be divided by the vernier into 10,000 parts, this gives as the value of $\frac{1}{10}$ of a division $\frac{1}{200000}$ of an ohm.

The unit in the following example is $\frac{1}{10}$ of a division. To secure the greatest accuracy of measurements a sensitive low-resistance galvanometer must be used. The image of a wire strained across a slit is reflected on a scale in the usual way, and read at a distance by means of a telescope. The galvanometer should give an indication, when used with precautions, due to a difference of one-tenth of a division when comparing two ohm coils. But as the temperature can hardly be measured with certainty to within less than $\frac{1}{20}$ of a degree, this alone renders such refinement of reading nugatory, in the absence of better methods of ascertaining with certainty the real temperature of the wire.

An example may be here given of the use of this bridge in comparing two resistance coils. Call them F and K. Let K be the coil whose variation coefficient is required.

I. *Difference of Resistance of Coils* F *and* K *at* 11°C.

	Bridge Readings.		Difference.
Exp. i.	5000	4955	45
Exp. ii.	5000	4954	46
Exp. iii...............	5000	4955	45

The first column gives the number of experiment, the second the reading with the coils F and K in one position on the bridge, the third when F and K are reversed or have exchanged places on the balance; and the fourth gives the difference of their resistance at 11°C. in units of the bridge wire.

II. *Difference of Resistance of Coils* F *and* K *at* 28·2°C.

	Bridge Readings.		Difference.
Exp. i.	5439	4492	947
Exp. ii.	5442	4497	945
Exp. iii...............	5440	4490	950

As before, the fourth column gives the difference of F and K at 28·2°C. Taking the mean difference at 28·2°C. to be 947 units, and that at 11°C. to be 45 units, we have

$$\frac{947 - 45}{28·2 - 11} = 52·4 \text{ units}$$

as the mean variation coefficients between 11°C. and 28°C. in units of bridge wire. Since the coils F and K are approximately ohm coils, this gives as the variation coefficient of the coil K ·0262 per cent. This coil was of platinum-silver wire. These three determinations occupied about an hour and a half, during which time many more readings were taken, all closely agreeing with the above. The actual measurement of the differences requires but a few moments to effect, the principal

expenditure of time being that required to bring the coils to the same temperature as the water.

In practical laboratory or testing room measurements of resistance the *slide wire* form of bridge is not much used. It is more convenient to employ one of the forms of *plug bridge*. In this arrangement the resistances, which form three arms of the bridge, consist of coils of insulated wire wound on bobbins and contained in a box. These resistance coils are connected in series as required. There are two principal modes of effecting this junction. The top surface of the box generally consists of a slab of ebonite, on which are fixed brass blocks. In the series pattern or Post Office pattern
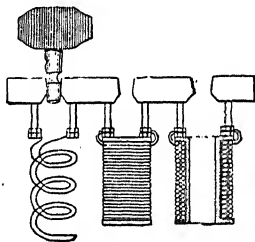
FIG. 66.—Arrangement of Resistance Coils, Blocks and Plugs in a Series Pattern Bridge.

of bridge the brass blocks are arranged in rows and the coils connected between them, as shown in Fig. 66. Each block is bored out in such a manner that a brass plug fitting into a conical hole drilled partly in one block and partly in the adjacent one metallically connects the blocks and short-circuits the coil joined in between them. The plug is carefully ground in, so that the interconnection offers only a negligible resistance. Hence, in the case of such an arrangement of coils interconnected by blocks, the insertion or withdrawal of plugs cuts out or adds resistance into the circuit in which these coils are connected.

In the series form of plug bridge the ratio arms generally consist of five coils having resistances respectively of 1, 10, 100, 1,000, and 10,000 ohms. The third or measuring arm

consists of a series of sixteen coils having resistances of 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500, 1,000, 2,000, 2,000, 5,000 ohms. The box is usually provided with two contact keys and terminals for the galvanometer, battery, and resistance to be measured. The chief objections to the series pattern of plug bridge are that the withdrawal of any plug from a hole tends to loosen all the rest and so creates bad contacts. Hence it is necessary to be continually going over the plugs and tightening them up Moreover, plugs withdrawn are not in
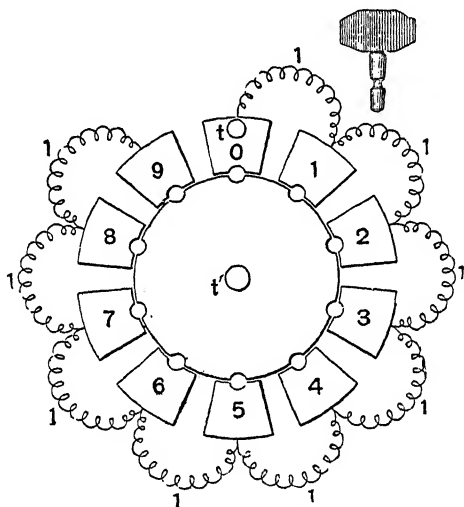
FIG. 67.—Arrangement of Coils in Dial Pattern Wheatstone Bridge.

use, and are therefore in danger of being oxidised or spoilt by being laid about on the table or held in a damp hand.

These objections are avoided in the form commonly called the *dial pattern* of bridge. In this form the measuring arm of the bridge consists of a series of decimal coils—viz., nine 1-ohm coils, nine 10-ohm coils, nine 100-ohm coils, &c. These coils are connected between ten blocks fixed on the upper surface of the box. These blocks are arranged round a central block (*see* Fig. 67). Ten conical holes are bored out in between the

central block and the outside blocks, and only a single plug is
employed. As the plug is moved from hole to hole it puts in
series between the first outside block and the central block any
resistance from zero to 9 ohms, zero to 90 ohms, and so on.
A set of three, four, or five of these coils and *dials* are arranged
in and on the box, and serve to set in series units, tens,
hundreds, &c., of ohms, the different dials being intercon-
nected, the central block of one dial being connected to the
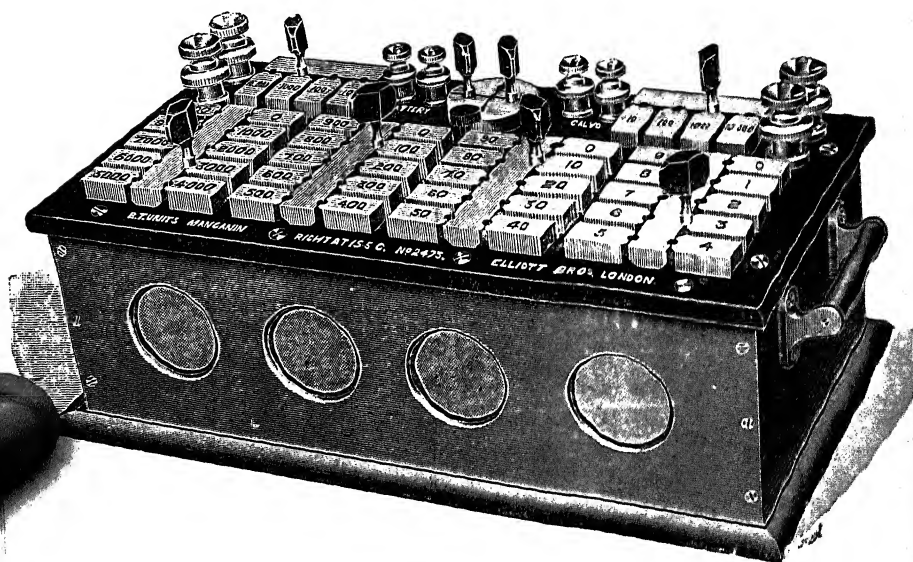
Fig. 68.— Standard Bar pattern Wheatstone Bridge.

first outside block of the next. Manufacturers often arrange
the blocks in a rectangular form, as in Fig. 68. In Fig. 69 is
shown a diagrammatic scheme of the arrangement of coils in
a standard Wheatstone bridge of the above kind.

In an electrical laboratory one table should be devoted to
a dial or bar pattern resistance bridge. The bridge should
be contained in a wooden box, which can be closed when
the apparatus is not in use to keep light and dust from

the ebonite slab. The battery should consist of three or four dry cells of the Leclanché type. The galvanometer should be a movable coil galvanometer, having a resistance of 500 to 1,000 ohms. Thick flexible copper cable connectors should be brought from the terminals of that arm of the bridge in which the resistance to be measured is placed to two mercury cups fixed on a wooden stand. In these cups can be inserted the extremities of any coil or wire the resistance of which is required, and this can be determined in a few moments of time. Fuller details of the processes of measurements and necessary precautions are given in Chapter II. of this volume dealing with the measurement of resistance.
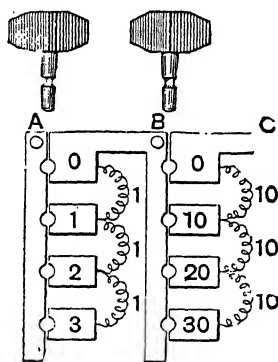
FIG. 69.—Arrangement of Resistance Coils, Plugs and Block in Bar Pattern Bridge.

The laboratory equipment must include some form of slide wire bridge suitable for rough measurements, and also a standard slide wire bridge for the comparison of coils by the Carey Foster method.

A compact form of bridge for this latter purpose is that devised by F. H. Nalder, as shown in Figs. 70 and 71.* The ratio arms of the bridge consist of two coils, generally 1 ohm, 10 ohms, 100 ohms, or 1,000 ohms, wound on one bobbin.

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* See *The Electrician*, Vol. XXXI, p. 241, or *Proc.* Phys. Soc. Lond., June 1893.

The bridge consists of massive bars of copper mounted on an ebonite slab. In these bars are formed mercury cups, into which the copper terminals of the coils are dipped. The ratio arm coils are connected in between the cups $A A_1$, $B B_1$ (*see* Fig. 71). The coils to be compared are connected to the cups $I I_1$, $J J_1$. In the space between the bars is a circular disc of ebonite which carries certain copper connecting bars. When these bars are placed in one position they connect the coil placed in cups $A A_1$ in series with the coil placed in cups $I I_1$ and that placed in cups $B B_1$ with
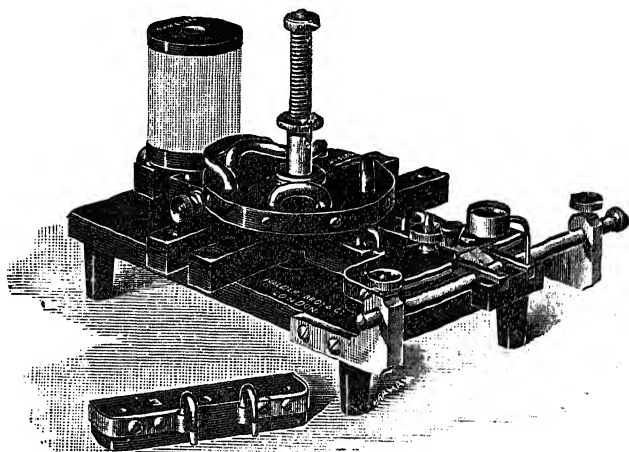
Fig. 70.—Nalder Differential Resistance Balance.

that placed in cups $J J_1$. When however the copper interconnectors are lifted up and replaced in a different position they exchange the places of the coils in $I I_1$ and $J J_1$. The slide wire of the bridge is a very short platinoid wire, G, and the instrument is provided with a number of these slide wires of suitable resistance per centimetre for various comparisons.

Bridges are also constructed for quick measurement, where great accuracy is not required, in which radial arms moving round dials and making contact with studs are made to

throw the necessary resistance coils into the measuring arm of the bridge. A bridge of this kind is useful in rapidly taking the preliminary reading of the resistance of a coil to be subsequently more carefully measured in a standard bridge. It then answers the same purpose as the "finder" on an astronomical telescope.

The special forms of low-resistance bridge will be dealt with in the section on resistance measurement.
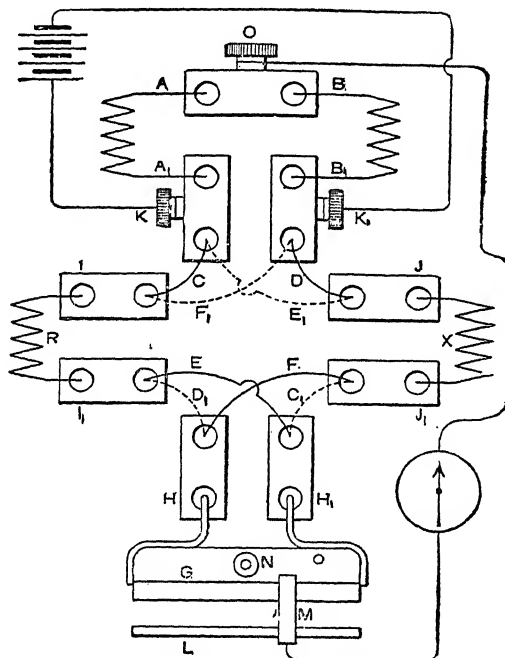
Fig. 71.—Connections of Nalder Resistance Balance.

Owing to the action of light and dust in deteriorating the surface insulation of ebonite, the ordinary form of plug Wheatstone bridge with lacquered brass blocks and ebonite slab is not a good one to employ in the workshop. The Author has therefore designed a form of workshop resistance balance in which there is no exposed ebonite or metal parts.

The box is made of oak, and the lid or upper surface presents only a set of rows of holes (*see* Fig 72). Underneath the lid and attached to it are a series of ebonite rings, on which are arranged a row of brass blocks and a brass disc. Resistance coils of equal value—viz., units, tens, or hundreds of ohms—are joined in between the blocks as in the dial arrangement, and plugs shaped like a bradawl, placed in one of the holes bored out partly in the blocks and partly in the central disc, serve to throw in any required resistance between the first block and the disc. The resistances forming the ratio arms are in the same manner arranged to be used in series with one plug only for each ratio arm. The ratio arm coils are therefore 0·1, 0·9, 9, 90, and 900 ohms, and these, when joined in series, give 1, 10, 100, and 1,000 ohms as required.
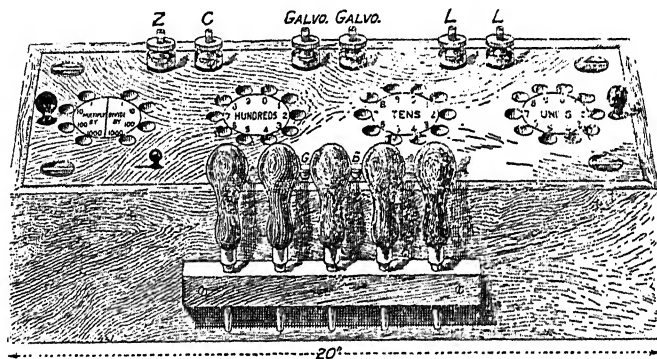
FIG. 72.—General View of the Fleming Workshop Bridge.

The resistance bobbins on which the coils are wound are formed in the following manner :—

Two hollow half-cylinders of thin sheet copper (Fig. 73) are constructed, each having a lug, and two of these half-cylinders are put together with a thin separating piece of ebonite and bound together with a thin silk tape. This constitutes the bobbin. The resistance wires of manganin are then cut the proper length, and the two ends of the wire are soldered to the two lugs of the copper half-cylinders (Fig. 74), the

remainder of the wire being wound non-inductively on the
bobbin and tied in position by a thin silk tape. The wire is
therefore so arranged that it quickly gets rid of any heat, and
no paraffin wax is used to overlay the wire or coils.

The measuring arm of the bridge consists of three sets of
coils—units, tens, and hundreds—each set consisting of nine
coils interposed between 10 brass blocks, which are arranged
in a circle and carried in a sort of ebonite tray fixed on the
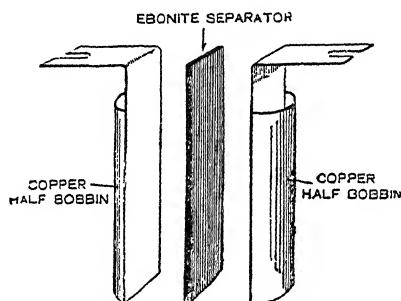
FIG. 73.—Construction of the Resistance Bobbins.
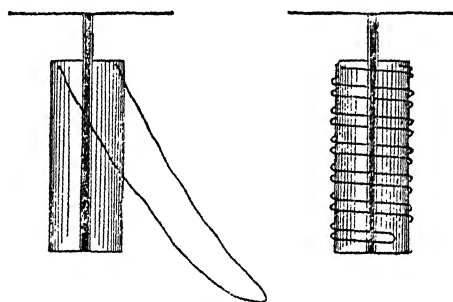
FIG. 74.—Method of Winding.

underneath side of the teak lid of the box. These blocks are
interconnected as required with a central brass ring by means
of a travelling plug. A diagram of the bridge is given in
Fig. 75, and a section of the dials in Fig. 76. The plugs
resemble bradawls, except in their lower extremities, and
have a substantial handle; they are kept when not required
in a sort of umbrella stand attached to the box, seen in the

general view of the instrument, Fig. 72.   These plugs are
inserted into interconnecting holes between the brass blocks,
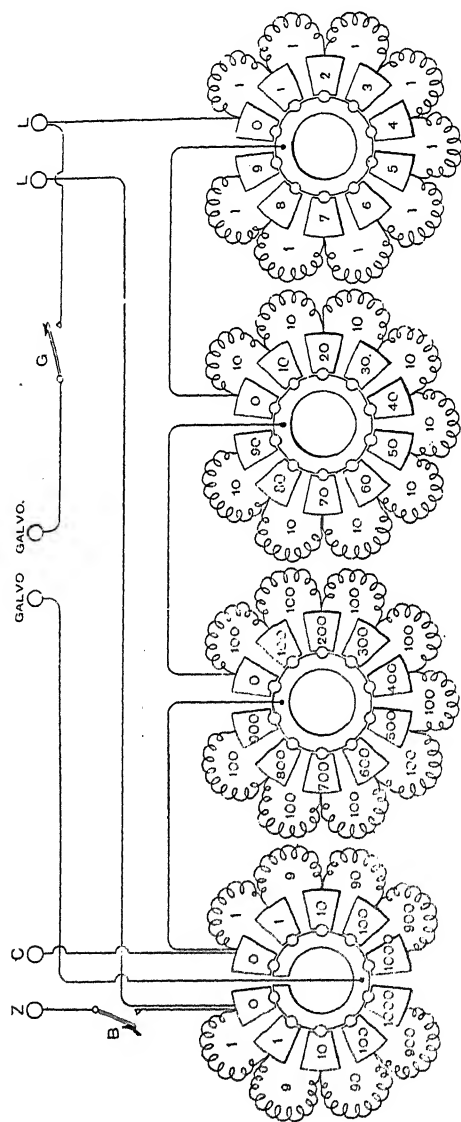
Fig. 75. — Diagram of Connections of Fleming Workshop Bridge.

and the common ring through small holes bored into the lid of the box. Hence the outer surface of the box exhibits only sets of small holes arranged in circles and which are marked respectively units, tens, hundreds.

In order to prevent dust getting into the holes between the brass blocks, there is a contrivance in the form of a self-closing shutter. On the underneath side of the lid are a series of circular discs of thin ebonite, which are perforated by holes corresponding with the holes for the plugs in the lid of the box. These circular shields move round a central pivot, and by means of a spring are kept so turned that the holes in the shield do not normally coincide with the holes in the box lid. Hence all these box lid holes are closed as it were by a small shutter, but when once opened and a plug inserted in the hole the plug prevents the shutter from closing. The circular shutters can be turned round by means of a
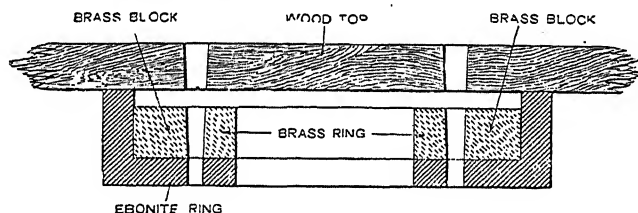
Fig. 76.—Section of Dial.

common bar and finger pin protruding through the lid, so that, in order to insert the first plug when commencing his test, the user presses back the pin and thereby opens all the shutters. But the moment all the plugs are withdrawn from their hole, the shutter under the lid springs back and closes all the holes belonging to that dial.

The ratio arm of the bridge is formed of a series of coils which are arranged dial fashion, and which are therefore not 1, 10, 100, 1000 ohms as usual, but 1, 9, 90, 900. Two plugs are therefore provided for the ratio arm dials, and the ratio arm dial is furthermore marked in such a manner that the user has no difficulty in discovering the number by which he must multiply and the number by which he must divide the ratio arm resistance to get the resistance of the circuit measured. The shutter closing the holes of the ratio dial is

independent of the others, and is not provided with a finger pin for opening. The spring which keeps the shutter of this dial normally closed is not very strong, and the shutter is easily pushed back by the point of the first plug as it is inserted.

The whole of the connections are made under the lid to three pairs of terminals, marked on the outside as in Fig. 72, and in addition two keys are fixed under the lid, which are manipulated by two little ivory buttons, like electric bell pushes, protruding through the lid. There is therefore absolutely no exposed ebonite to deteriorate in insulation, and no lacquered brass, with the exception of the aforesaid six terminals, the handles by which the bridge is carried about, and the plugs. The terminals are constructed with a peculiar kind of lip, which renders it easy to grip in them either a single large wire like a No. 10 wire or a single small wire like a No. 40.

One other point is worthy of notice. In the ordinary patterns of plug bridges the frequent use of the plugs wears them down into a shoulder, so that they no longer fit tightly into the plug holes. In this workshop form of bridge the plug has a semi-circular groove turned in it at such a height in the plug that when in place the top of the block is just on a level with the middle of the groove. This device is an effective cure for "shouldering" in the plugs.

In the selection of a Wheatstone bridge for very accurate work in resistance measurement it is well to have in view the objections which can be raised against the ordinary form of plug and coil bridge. These objections may be summarised as follows :—The bridge, as supplied by most makers, consists of a wooden box, with an ebonite slab forming the top, on which are fixed the brass blocks with conical brass plug connectors. The resistance coils are in the box, and are connected between the plug blocks. If platinum-silver wire, or any material not having a zero temperature coefficient, is used for the manufacture of the coils, then there is very considerable difficulty in ascertaining the true temperature of the coils when the resistance measurement is actually being made. A thermometer placed with its bulb in the box

merely gives us the temperature of the air in the box ; that of the coils, perhaps embedded in paraffin or shellac, may be very different, and moreover, the temperature of each coil which has current passing through it may not be the same.

In the next place, very great labour is involved in checking the relative value of the coils, and, owing to dirt or slight want of fitting of the conical plugs, the value of the resistance added or removed by withdrawing or inserting a certain plug may not be at all times precisely identical. In the series coil or Post Office pattern of plug bridge the putting in of one plug slightly displaces all the brass blocks, and the expansion of the ebonite top prevents perfect fitting of the coned plugs.

The dial pattern of bridge is therefore preferable to the series coil pattern, because each of the plug blocks is independent. Furthermore, the coils, as usually made, cannot be annealed after being wound, and hence are liable to secular changes in resistance due to strain.*

Finally, the last adjustment of balance cannot always be made with integer coils, and the last decimal place in the resistance measurement has to be estimated from the galvanometer deflection (as described in the section on resistance measurement).

To meet the above objections a form of standard bridge has been designed by Messrs. Callendar and Griffiths, of which a description is given in Chapter II.†

§ 14. Electric Quantity-Measuring Instruments. —

Ballistic Galvanometers.—The passage of a certain electric discharge or quantity of electricity through a circuit,

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* For a good summary of all that can be said against the ordinary form of plug resistance bridge the reader is referred to a series of articles on "The Electrical Measurement of Temperature," by Mr. G. M. Clark, in *The Electrician*, Vol. XXXVIII., p. 274, 1897.

† See also *The Electrician*, Vol. XXXVIII., p. 747, 1897.

reckoned as the time-integral of a varying electric current having finite limits, is measured practically either by a *ballistic galvanometer*, if the whole discharge is over in a second or less, or, if the discharge endures for a very considerable time, it is measured by some form of ampere-hour meter. For a certain class of magnetic work the *ballistic galvanometer* is a necessary appliance. The most practical and useful form of this instrument is a *movable coil* fixed magnet galvanometer. The movable coil may be either a narrow, shuttle-shaped coil, as in the instruments designed by Ayrton and Mather, or a circular, ring-shaped coil, as in the ballistic galvanometer of Crompton.

Deferring at present a detailed discussion of the principles of the ballistic galvanometer, we may here merely state that the elementary theory of this instrument is as follows :—If through the coil of the galvanometer an electric discharge or very brief current is permitted to take place, the magnetic field due to the discharge causes an impulsive torque or couple to act upon the movable portion of the instrument, whether the latter is the magnet or the coil. *If the discharge is all concluded before the movable portion has been sensibly displaced from its zero position*, then the following conditions hold good :—

Let I denote the moment of inertia of the movable portion of the galvanometer.

Let $\mu\theta$ denote the control or torque brought into existence to restore that part to its original position when displaced through an angle θ.

Let ω be the angular velocity of the moving portion at any time t after the displacement begins.

If, in the first place, we neglect the retarding effect of friction, we may say that the equation of motion of the moving system at the time t is expressed by the equation

$$I\frac{d\omega}{dt} = -\mu\theta,$$

which is the analytical statement of the fact that the rate

at which the angular momentum of the moving system is being destroyed at any instant is proportional to the angular displacement of the movable portion.

Since
$$\omega = \frac{d\theta}{dt},$$

we have
$$I\frac{d^2\theta}{dt^2} + \mu\theta = 0$$

as the equation of motion of the coil when we neglect frictional retardation of all kinds.

From the above equation it is easily shown that the duration (T) of one complete oscillation of the movable system is given by the equation

$$T = 2\pi\sqrt{\frac{I}{\mu}}.$$

Suppose the coil at rest in the field of the controlling magnet, and let a discharge be made through the coil, which is all completed before the impulse sensibly overcomes the inertia of the coil and gives it an angular displacement or "throw." If B is the induction density or field strength due to the fixed magnet at the place where the coil is situated, and if i is the coil current at any time t after the beginning of the discharge, then, assuming the coil has not sensibly moved from its zero position, and neglecting as small the reaction of the coil current on the fixed magnetic field, we may say that the coil is experiencing a torque or couple represented by CBi, where C is some constant depending on the form of the coil.

Hence, as above, if ω is the angular momentum of the coil at that instant,

$$I\frac{d\omega}{dt} = CBi,$$

or
$$Id\omega = CBidt,$$

where I is the moment of inertia of the coil. The product idt is numerically equal to the small quantity of electricity

dq which has passed through the coil in the element of time dt

Hence $\qquad\qquad\qquad I\,d\omega = CB\,dq.$

Suppose time reckoned from the instant when the discharge begins, and that the discharge is complete before the coil has experienced any appreciable change in position. The result of the passage of the whole quantity Q of the discharge through the coil will be to apply to it an impulsive torque which will cause it to leave its zero position with a definite velocity Ω. Under the above conditions we can integrate the last equation, and omitting the constant of integration, write

$$I\Omega = CBQ. \qquad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The angular energy with which the coil leaves its zero position is equal to $\frac{1}{2}I\Omega^2$.

If the impulsive torque gives the coil a twist through an angle θ, and if μ is the torque due to the suspension, whether bifilar or unifilar, per unit angle, then $\mu\theta$ is the restoring or opposing torque due to the suspension brought into existence by the displacement through an angle θ. Hence the potential energy of the coil system at the end of its swing is equal to $\frac{1}{2}\mu\theta^2$, or to half the product of the torque $\mu\theta$ and angular displacement θ. Accordingly, we must have an equation between the kinetic energy imparted to the coil and its potential energy when at the position of final displacement, or

$$\tfrac{1}{2}I\Omega^2 = \tfrac{1}{2}\mu\theta^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Combining together equations (1) and (2), we arrive at the equation

$$Q = \frac{\sqrt{\mu I}}{CB}\theta,$$

or, $\qquad\qquad\qquad Q = G\theta.$

The above equation shows that, under the limitation assumed the angular excursion of the coil is proportional to the whole quantity of electricity which has passed through the galvanometer.

If to the coil is attached a mirror in the usual manner, and the lamp and scale is placed at a distance of say one or two metres so that the angular deflection of the coil does not exceed 5 deg. or 10 deg., the corresponding excursion or displacement of the spot of light upon the scale will be proportional to the total quantity of electricity which has passed through the coil.

The *ballistic constant* (G) of the galvanometer is the number by which the scale deflection in millimetres or centimetres must be multiplied in order to obtain the total quantity of electricity in microcoulombs which produced that observed deflection.

In the chapter on the Measurement of " Electric Quantity " instructions will be given for the proper standardisation of the galvanometer. In the meantime, a few of the qualities which should be possessed by a ballistic galvanometer may be discussed.

The movable magnetic needle ballistic galvanometer is a most troublesome instrument to use, owing to the needle disturbances created by outside currents and magnetic fields. The most practical form of ballistic galvanometer for the electrical testing room or laboratory is a movable coil galvanometer with fixed magnet. This galvanometer must have its coil wound on a non-conducting frame, and may be either a narrow coil—*i.e.*, a coil of long or shuttle-shaped form, as in the Ayrton-Mather galvanometer shown in Fig. 45, page 123 —or a circular bifilar suspended coil, as in the Crompton-d'Arsonval galvanometer shown in Fig. 46, page 124. In either case, if the galvanometer is to be used for ballistic purposes, as little "damping" must be present as possible. Hence the coil must not be wound on a closed metallic frame, and preferably not surrounding a soft iron core. On the other hand, when used simply as a deflectional instrument it is desirable to secure as much damping of the coil as possible. In this last case winding the coil on a metallic frame is an advantage, because the eddy currents set up in the frame by its movement in a strong field retard the movement and bring

the coil to rest without unnecessary vibrations when the current through it is stopped.

When, however, the galvanometer is to be used for ballistic purposes, then it should be as little damped as possible. The free period of vibration of the coil in this case, when disturbed, should not be less than five seconds, and for many purposes preferably 15 seconds at least. It is quite useless to employ for many experiments a ballistic galvanometer having a smaller free period of vibration, since then the fundamental requirement for ballistic work is not fulfilled. The galvanometer should, however, have a damping arrangement by which a small, properly-timed current may be sent through the galvanometer coil to bring it to rest when its excursion has been made, and thus save unnecessary loss of time in experiments. This may be achieved by the use of a shunt circuit (consisting of a small dry cell), a very high resistance (consisting of a slip of vulcanised fibre rubbed over with plumbago), and a contact key, all joined in across the galvanometer terminals. By making suitably timed taps on the key the galvanometer coil can have small currents sent through it which will bring it at once to rest. In the case of low resistance ballistic galvanometers, merely short-circuiting the terminals of the galvanometer by closing a key in that circuit is sufficient to destroy at once the vibration of the coil and bring it to rest.

Each ballistic galvanometer used in the laboratory should be set up on a very steady stone shelf or pillar, and the usual lamp and scale adapted to it. In this connection it may be pointed out that by far the most convenient scales to use for galvanometer purposes are the semi-transparent celluloid scales. These are divided into millimetres, and should be set up on a firm stand at a distance of one metre or 1,000 millimetres from the mirror. The source of light should be an incandescent lamp with a simple horseshoe filament. Over this may be placed an asbestos hood with a slit in it, so that only the image of one side of the filament is thrown upon the scale. The galvanometer should be

provided with a concave mirror of one metre focus. It is then easy to so place the incandescent lamp that the sharp image of one leg of the filament is thrown upon the semi-transparent scale. The scale must have a lateral movement, so that it can be moved parallel to itself and perpendicular to the direction of the ray of light reflected from the mirror when the galvanometer coil is in its zero, or undeflected, position. The scale deflection of the sharp line of light upon the screen, measured in millimetres and divided by 1,000, gives very approximately the tangent of twice the angle of deflection of the galvanometer coil; and this may be taken to be proportional to the coil deflection when that angle does not exceed 10 deg.

The testing room should be provided with at least two ballistic galvanometers, one having a resistance of about 20 ohms and the other a resistance of 500 to 1,000 ohms or more. This latter galvanometer should have a time of free vibration of its movable coil of at least 12 to 15 seconds.

The correction to be applied to the excursion of the coil to allow for air friction or other retarding forces which tend to diminish the deflection, as well as other precautions in the use of the ballistic galvanometer, will be given in a later chapter.

In connection with the standardisation of this instrument it is useful to possess an apparatus called a graded condenser as well as a standard half-microfarad condenser. These last instruments are virtually Leyden jars, consisting of insulating sheets of mica or of paraffined paper coated on both sides with tinfoil, but so as to leave a margin of uncoated surface. A collection of these elements is assembled together, so that all the tinfoil coatings on one set of sides are in metallic connection and all the corresponding opposite side coatings also in connection.

The construction of a standard condenser is a matter requiring some experience, and it need only here be said that it is not worth while to purchase one from any except a

manufacturer of repute.   A standard half-microfarad condenser is, however, a necessary article.   Two or three other
condensers should be also obtained, having a capacity of
about one microfarad each.   Also a graded condenser,
divided into microfarads and fractions of microfarads such as
0·1, 0·2, 0·2, 0·5, 1·0, 2·0, 2·0, and 5·0, is a very useful appliance.
All these condensers should have been tested with 1,000 volts
on their terminals by the maker, so that no risk may be
incurred in using them on 100-volt circuits.   The proper
charge and discharge keys for use with the condensers can
be procured as required.

In connection with the measurement of electric quantity,
in those cases in which the duration of the discharge is
considerable (as in secondary battery tests) it is exceedingly
desirable to have at hand an ampere-hour meter which will
automatically record the total quantity which has passed
through it in coulombs or microcoulombs, and at the same
time show the variations of current.   Of these instruments
the most useful is the graphical recording ampere-hour
meter.   In this instrument a drum covered with paper is
rotated by a clock train at a uniform rate, say once in 24
hours.   A pen moving over the surface of the drum is
displaced by some mechanism which acts as an ampere meter.
In the Holden ampere-hour meter this current-measuring
part is constructed by utilising the expansion produced in a
series of fine wires placed in parallel.   A lever arrangement
enables the "sag" of these wires, when heated, to displace
the pen over the drum by an amount depending on the
current through the wires.   If, then, the pen is variably
displaced as the drum revolves, we have a curve described on
the paper which shows, by the value of its ordinates at each
moment, the ampere value of the current.   If the displacement of the pen is exactly proportional to the current, and if
the speed of the drum is uniform, then the area of the curve
described by the pen is proportional to the whole quantity in
ampere-hours which has passed through the instrument.   In

secondary cell testing the possession of a correct graphic recording ampere-hour meter of the above kind saves much time and many tedious observations.

Remarks on the subject of ampere-hour meters generally for use in the commercial supply of electric current will be reserved for the section dealing with meter testing.

§ 15. Instruments for the Measurement of Electric Power. Wattmeters.—In many pieces of testing work, particularly in certain alternating-current measurements, it is necessary to be able to measure at one operation the power, or mean power, given to an electrical circuit, which may be called the *power circuit.* This measurement is made by means of a wattmeter.

In its most general form a wattmeter consists of two coils of wire, one of which is fixed and is called the *current coil* and the other of which is movable and is called the *pressure coil.* The circuit in which the power to be taken up is to be measured is joined up with the wattmeter, so that the current passing through the circuit is that passing through the current coil. The pressure coil is then joined up so (i.) that one terminal is connected to the entrance end of the current coil and the other to the exit end of the power circuit, or (ii.) so that the pressure coil is connected to the ends of the power circuit.

It will be seen that, if the pressure coil is joined up in the first manner, the voltage on the pressure coil is that due to the fall in volts down the current coil as well as that in the power circuit. If, on the other hand, it is joined up in the second way, then the current in the current coil is not simply that in the power circuit, but takes account also of the current in the pressure coil.

In dealing with the particular measurements in which the wattmeter is employed we shall point out how it should be used. Meanwhile, it may here be stated that if the pressure coil is held in a position either with its plane parallel to that

of the current coil or embracing it, and with its plane at right angles to it, then, when connected up with the power circuit there will be found to be a mechanical force or torque between the coils. The restraining force required to hold the coils in any definite relative position in which the electro-dynamic force between them is not zero is proportional to the mean product of the values of the currents passing through the coils. If one current is the current through the power circuit, and the other current is proportional to the potential difference between the ends of the power circuit, then their mean product is proportional to the mean power taken up in the power circuit. The controlling condition, however, is that the free time of vibration of the movable coil of the wattmeter must be large compared with the periodic time of the currents, if these are periodic.

The wattmeters used in the testing laboratory generally take one or two forms. In the Siemens form the movable coil embraces the fixed coil (*see* Fig. 77). The movable coil is suspended by a few fibres of floss silk, and has attached to it one end of a spiral steel torsion spring. The other end of this spring is fixed to a torsion head, with an indicating arm moving over a circular divided scale. The movable coil carries a pointer, by which it can be brought into a recognised and fixed position. The arm attached to the torsion head can be moved independently of the head, and clamped when desired by a clamping screw. The current enters and leaves the movable pressure circuit by means of mercury cups, into which dip the amalgamated ends of the movable coil. The instrument is provided with four terminals, two of which are the extremities of the fixed coil and two are the terminals of the movable coil.

The other form of wattmeter is that of Lord Kelvin, which is similar in general construction to an ampère balance. The circuit formed by the coils of the suspended or balance arm is, however, brought to a separate pair of terminals. The circuit formed by the fixed coils is separate from that of
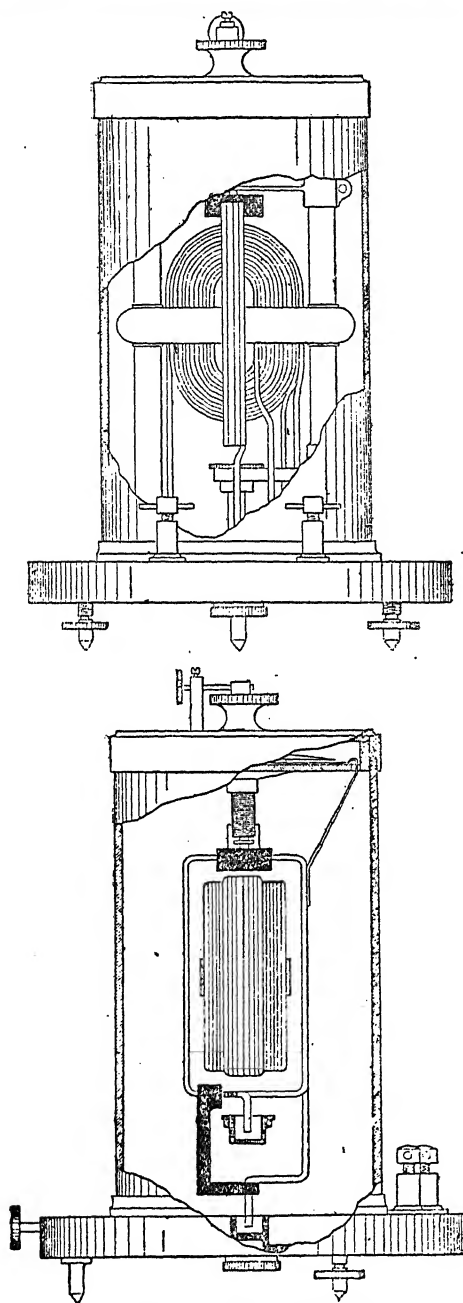
Fig. 77.— Siemens Wattmeter.

the movable ones.   One of these forms the pressure circuit and the other the current circuit of the wattmeter.

Another form of wattmeter is that in which the pressure coil is suspended by a bifilar wire suspension.   In this case the current enters and leaves the movable coil by the suspension wires.   This form is well adapted for a deflectional instrument in which the deflections of the movable coil are read by a mirror and scale.

In the selection of a wattmeter for use in the testing laboratory, especially if it is to be used with alternating currents, it is necessary to be guided by the following facts :—

For use with alternating currents there must be no metal work near the fixed or movable coils.   Instrument makers are far too fond of lacquered brass.   They put brass covers and shields round wattmeters, and carry the coils on brass pillars and supports.   The result is that eddy currents are set up in these metal portions, which react upon the currents in the movable coil and destroy the correctness of the indications of the instrument.   An alternating-current wattmeter should, as far as possible, be constructed of hard, well-seasoned wood and ebonite, with the exception of wire circuits and terminals.

A useful form of the instrument is made by providing with separate terminals to its two circuits the ordinary and cheap form of Siemens electro-dynamometer.

If the wattmeter is being used with continuous currents, then it is necessary to so place it that the magnetic axis of the movable coil is in the direction of the earth's horizontal magnetic field at the place where it is used.   If this is not done, the terrestrial force will exercise a deflecting action on that coil.   Hence the wattmeter should be used on a rotating turn-table, which enables it to be rotated in azimuth without disturbing the level of the instrument.

In dealing with the special uses of the instrument the various precautions attending its use will be pointed out.

§ 16. General Hints on the Outfit of a Testing Laboratory.—In concluding this chapter a few general hints may be given as to the equipment of an electrical testing laboratory.

In most cases where this has to be done by the inexperienced, the general desire seems to be to provide a number of glass door apparatus cases, and to stock the shelves as far as possible with the beautiful creations in ebonite and lacquered brass of the electrical instrument maker. Hence it is that in so many colleges and technical institutions we find a large collection of expensive apparatus, very little of which is of real use in research or commercial work.

The guiding principle in equipping an electrical testing room should be to buy at first as little as possible, unless and until the purpose of the laboratory is very clearly defined. The instruments that are bought as standard instruments should be very carefully selected, and, as far as possible, made to careful specifications. An odd lot of galvanometers bridges, resistance coils, keys, &c., should not be bought.

The main purpose of the laboratory having been defined, whether for teaching, research, commercial testing, or standardising, the first provision should be in the conveniences for generating and distributing the currents. Round the laboratory should run several circuits, with means for bringing the potential difference at any place to the standard voltmeters and potentiometer. The principal resistance bridge, the principal potentiometer, the ballistic galvanometers, the ampere balances, standard voltmeters, and low resistance bridge should each be set up complete on its own table, with everything required for that measurement screwed down to the table. There should be no moving about of galvanometers and keys from one place to the other. The small local currents required should be obtained from dry cells of Leclanché type, or from small 2-cell secondary batteries.

The set of apparatus on each table, when not in use, may be kept free from dust by having a black cloth thrown over

it. It stands there, however, ready for use, and the resistance of a bit of wire or coil or the checking of a voltmeter can be carried out at a moment's notice without loss of time.

In many laboratories there is an enormous waste of time in collecting together out of cases, setting up, and connecting apparatus for the simplest measurement. There is also a great amount of capital sunk in apparatus pretty to look at but perfectly useless for real work.

Ample provision should be made, by wire resistances and carbon resistances, for regulating currents. Galvanometers should, as far as possible, be movable coil galvanometers, so that they are not disturbed by the presence of currents in neighbouring wires. These may be kept covered over when not in use with cardboard boxes or hoods.

No attempt has been made in the sections of the present chapter to indicate all the apparatus necessary, as that must depend on the purpose of the laboratory. In the following chapters instructions will be given for carrying out the chief electric and magnetic measurements.

As an illustration of the equipment required in an electrical standardising laboratory, we may conclude this chapter by a brief description of the arrangements in the British Board of Trade Electrical Laboratory at No. 8, Richmond-terrace, Whitehall, London.* This laboratory was established in consequence of a deputation, comprising most of the prominent members of the electrical profession, to the Board of Trade in 1889, for the purpose of urging the necessity of the establishment of such an institution. A scheme was submitted by this deputation for the establishment and working of a standardising laboratory, and this scheme has been kept in view as far as was consistent with the amount of money procurable from the Treasury :—

The first suggestion for the establishment of a Government electrical standardising laboratory was given in a Paper read by the Author in November, 1885, to the Institution of Electrical Engineers (then called the

* See *The Electrician*, October 5 and 12, 1894, Vol. XXXIII., pp. 665, 693.

Society of Telegraph Engineers and Electricians. This Paper was entitled "On the Necessity for a Standardising Laboratory for Electrical Test Instruments." The discussion on the Paper led to the formation of a committee to further this object, and four years later the suggestions of the Author were realised.

The laboratory exists for three purposes: First, to obtain and preserve standards for the measurement of electrical quantities; second, for giving the standard measurements of those quantities; and third, to enable the Electrical Adviser of the Board of Trade to make such tests of instruments and material as may be necessary in the performance of his duties. No scientific work outside these purposes can be undertaken.

Six rooms are in occupation in the basement of No. 8, Richmond-terrace, Whitehall, London. One of these is occupied by the gas engine and dynamos used solely for charging accumulators; another by the accumulators, from which all the power used in making electrical measurements is obtained. These two rooms are outside the main building. The rooms within the main building are, first, the room containing the standards for current and pressure and the necessary adjuncts; second, the room containing the transforming machinery, where the power derived from the accumulators is transformed as required; third, the room occupied by the standards of resistance and the Clark cells used as sub-standards, this room being used exclusively for measurements connected with resistance and the comparison of electromotive forces; and, fourth, the verification room, where commercial instruments sent for the purpose are verified. Beyond the verification room is fitted a small chemical laboratory.

The general arrangement of the laboratory is shown in the plan in Fig. 78, and consists of the various rooms already enumerated. The accumulators in use consist of a battery of four large 61-plate Crompton-Howell cells, which are generally used two in parallel for the direct production of large currents and 108 11-L. E.P.S. cells. The large cells can supply currents of any amount up to 2,000 amperes for a short time, and are charged in series at the rate of 200 amperes. The E.P.S. cells are used for running the transforming machinery and for supplying continuous pressure up to 200 volts. The discharge rate is never allowed to exceed 22 amperes even for short periods. Current from the large cells is brought into the room containing the standards of current by mains consisting of flat copper strips, the lead and return being sandwiched together to avoid magnetic disturbance. A break is provided just outside this room for the insertion of regulating resistances which for the most part consist of carbon rods with suitable terminals. The use of carbon in this connection has this great advantage in addition to that of handiness, that as the resistance diminishes with increase of temperature the fall of current which would otherwise take place from the slight polarisation of the cells and the warming of the metallic circuit can be completely obviated by choosing suitable lengths and cross sections of carbon. Great steadiness of even the largest currents for ample time has thus been obtained. Fine adjustments of current are secured by the use of rheostats formed of carbon plates under variable pressure inserted either in the main circuit or as a shunt to the measuring instruments.
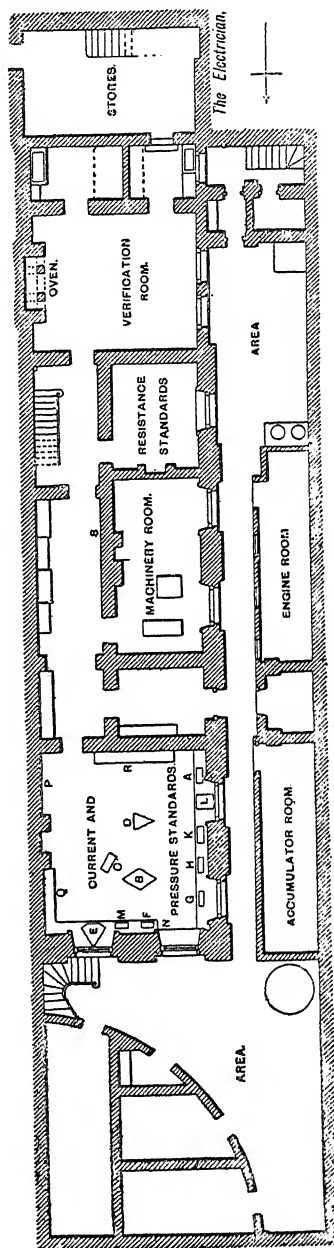
FIG. 78.—General Basement Plan of the Electrical Standards Department of the Board of Trade.

A large electric balance (A, Fig. 78), made under Lord Kelvin's supervision, is fixed immediately at the point of entry of the above-mentioned large mains. This instrument can read up to 10,000 amperes. Round two sides of the current and pressure standards room is fixed a concentric main of the same pattern as those used by the London Electric Supply Corporation for trunk mains from Deptford. The outer conductor is cut across at intervals, and large terminals clamped on at each side of these breaks, to which the current-measuring instruments are attached, while copper bars are provided for short-circuiting every instrument. The inner conductor is only exposed at the two ends, where it can be similarly connected to the outer conductor either through an instrument or by means of a short-circuiting strap. A complete anti-induction circuit is thus provided for current. A shelf of enamelled slate is carried on corbels let into the external wall of the building underneath the concentric main, as a support for the current-measuring instruments. The ampere standard B is placed upon a stone pedestal near the centre of the room, and the auxiliary ampere balance C upon a similar adjacent support. The volt standard D, which really measures 100 volts, or one hekto-volt, occupies a third stone pedestal in the centre of the room. Another instrument of the same pattern is placed at E. On the shelves round the wall of the room are five sub-standard Kelvin balances of the following ranges : 1 to 5 amperes (F, Fig. 78), 5 to 30 (G), 30 to 120 (H), 100 to 600 (K), and 500 to 2,500 (L). The first two of these have aluminium beams. A composite watt-balance is placed at M. One of the remaining walls R is occupied by various pressure-measuring instruments for low pressures, and the fourth wall partly by pressure-measuring instruments for high pressures Q and partly by a switchboard P for regulating the transforming machinery. A highly insulated platform is provided for standing on when manipulating the high-pressure instruments. On this board are a pair of Cardew voltmeters, supplemented by twelve resistance tubes. Each of these is practically a complete Cardew voltmeter, but without hand movement and dial. Each contains the usual wires and pulleys, under exactly the same "live" condition as in a working voltmeter. By means of these resistances, which can, of course, be independently checked and mutually compared, differences of potential up to 2,150 volts can be measured. A "chain" of voltmeters — that is to say, a set of instruments, the range of each one overlapping two others, as with the sub-standard ampere balances — are placed in this room.

The chief work which has been done in this room is the comparison of the three standards of current, electromotive force, and resistance. The standard ampere, as obtained by many repeated determinations by the silver volta-meter, was passed through the ampere standard, the auxiliary balance, and through a resistance of 100 ohms, and gave a difference of potential of 100 volts, which was observed on the volt standard. This 100-ohm resistance is made of manganin, and is maintained at a constant temperature by oil circulated by a small electric motor, and by the circulation of water in an outer jacket. This 100-ohm resistance was compared with the ohm standard, the value of which was accepted from the British Association determinations. The volt was thus deduced from the correlation of the ampere and the ohm.

The scale of the volt standard is long enough to include the pressures due to 69, 70, and 71 Clark cells, and by direct comparison the value of the cell was thus determined. The work of carrying out this triple comparison occupied many months.

A current of about 0·8 ampere is kept continually passing through the ampere standard to keep it warm and dry. The period of a complete swing of the standard of current is about half a minute, and hence the need for an auxiliary balance, to act as the "finder" of a telescope.

The machinery for generating the required currents consists of a continuous-current motor and an alternator with shafts in line and both keyed to a common pulley, while a continuous-current high-pressure dynamo can be run from this pulley by means of belting. By varying the exciting currents of these different machines a very delicate adjustment of the transformed current or pressure is obtained. There are, in addition, various alternating-current transformers for giving a range of current up to 500 amperes and of pressure up to 10,000 volts.

The room containing the standard of resistance and Clark cells is a small room arranged to be kept at a uniform temperature of 16°C. In this room is a Carey Foster bridge, and a Kelvin reflecting galvanometer with a transparent scale.

The verification room is fitted with a concentric main running around three sides, slate shelves for the support of instruments ; and a large fireclay oven 2ft. 3in. wide, 2ft. 6in. deep, and 3ft. high, with plate glass front perforated with holes for leads, is heated by gas, for testing instruments at various temperatures. It has been run to a very high temperature, but it is not proposed to use it for more than 30° C. This room is fitted up with secondary standards for the measurement of current and pressure up to the limits of instruments in common use. In this room calibration and tests of meters have been carried out. This work has nothing to do with the calibration and sealing of actual house meters, which is carried out by the London County Council, but relates to the approval of the patterns. In this room the deci-ampere, deka-ampere, and hekto-ampere Kelvin balances are fixed.

In the verification of instruments in this laboratory definite methods are employed for defining the inaccuracy. For instance, if an instrument reading to 100 volts is sent for verification at certain points, say 90, 95, and 100, the readings of those instruments when pressures of those amounts are applied are given, and not the true values of the scale indications 90, 95, and 100 on the instrument. It is obvious that where a large number of instruments have to be checked, this method results in economy of time and power.

CHAPTER II.

THE MEASUREMENT OF ELECTRICAL RESISTANCE.

1. The Comparison of Electrical Resistances.—The electrical resistance of a conductor may be defined as that physical quality of it in virtue of which energy is dissipated in it when an electric current flows through it. This amounts to saying that a fall of electrical potential accompanies the flow of a steady unvarying current along a conductor. The ratio between the numerical values of the fall or drop in potential (P.D.) down a conductor and the current (C) in it is a measure of the electrical resistance of that conductor, if the current is an unvarying or continuous current.

The energy dissipated in the conductor, measured by the product of the values of the potential fall and the current, takes the form of heat, and raises the temperature of the conductor. This temperature change affects the physical state of the conductor and alters the ratio of potential fall to current, and hence changes the numerical value of the resistance. If, however, the heat is so rapidly removed that change in temperature is not allowed to occur in the conductor, or if a correction is made equivalent to making allowance for this change in temperature, then it has been found that the ratio between the fall in electric potential down the conductor and the electric current in it remains the same for the same conductor whether the current is large or small. This interdependence, or rather exact proportionality of potential fall to current strength, in the case of conductors traversed by steady currents is called Ohm's Law.

Ohm's law is not a mere truism : it is the expression of a physical fact, and our confidence in the general truth of the statement is an induction from the results of experiment in particular instances. Ohm's law states the exact proportionality of the current when unvarying to the unvarying electromotive force producing it. This is not a necessary truth. The electromotive force E required to produce a current C in a conductor might, for instance, have been a function of the current C of the form

$$E = aC + bC^3 + cC^5 +, \text{ &c.}$$

Only odd powers of C could have occurred in the expression, because, as the current reverses its direction when the electromotive force is reversed, the right hand side of the expansion representing E in terms of C must change sign with C.

Suppose we consider as important only the first two terms, and write the expression for E in the form

$$E = RC (1 - hC^2).$$

It has been experimentally shown by Chrystal and Saunders (*see* British Association *Report*, Glasgow, 1876) that in the case of copper h is a quantity less than 10^{-12}, assuming all proper corrections for temperature made. It has been also shown by FitzGerald and Trouton* that in the case of a solution of sulphate of copper h in the above formula is less than 3×10^{-6}, the maximum current used being 10 amperes per square centimetre. Similar verifications have been made by Beetz† for zinc sulphate solution, by F. Kohlrausch‡ for dilute sulphuric acid employing electromotive forces between 0·1 and 1 volt, and by E. Cohn§ for solutions of sulphuric acid and sulphate of copper, using alternating currents of low (100 ∿) and high (25,000 ∿) frequency.

Hence there is a certain mass of experimental proof that the electromotive force is proportional simply to the first power of the current when it reaches a steady value.

Although no *à priori* reasoning would suffice to establish this law as a general truth, yet, as remarked by W. N. Shaw (B. A. *Report* on "Electrolysis," 1890), Ohm's law evidently belongs to that class of physical law which, though in the first instance discovered empirically, expresses in numeral relations necessary consequences of the nature of the physical quantities involved. J Hopkinson has suggested (*Phil. Trans.* R.S., 1877, p. 614) that the law asserts the superposition of the effects of electromotive force in bodies in which conduction is not complicated by any residual effects, and may therefore be regarded as a special instance of the general law of superposition.

Although, therefore, demonstrated experimentally only in the case of a few metallic and electrolytic conductors, no facts have been found which are inconsistent with a conviction of its universal truth as regards metals. It is,

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* B. A. *Report*, 1888. p. 341 ; 1886, p. 312 ; 1887, p. 345.
† *Pogg. Ann.*, 125, 1865, p. 126 ; 117, 1867, p. 15.
‡ *Pogg. Ann.*, 138, 1869, pp. 280, 370.
§ *Wied. Ann.* 21, 1884, p. 646.

however, not true in the case of every conductor. According to Braun,[*] it is not true for psilomelane, iron pyrites, and copper pyrites. Quincke[†] states that some liquids of high resistance, such as ether, carbon bisulphide, turpentine oil, and benzene are disobedient for electromotive forces of 30,000 volts and upwards. He also refers to a similar departure from Ohm's law in the case of thin layers of gutta-percha, sulphur, paraffin, and shellac for small electromotive forces.

In the case of liquids, when a departure from Ohm's law shows itself evidence of chemical decomposition also appears. Actual or initial chemical decomposition may, in all cases, be at the root of the deviations from exact obedience to Ohm's law so far found. It has been asserted that the conductivity of a plumbago line or pencil mark drawn on ground glass does not follow Ohm's law, and it is well known that the conduction current through gases is not proportional to the total electromotive force acting.[‡]

Assuming the *Volt* (equal to 10^8 absolute C.G.S. electromagnetic units) as the practical unit of potential difference, and the *Ampere* (equal to 10^{-1} absolute C.G.S. electromagnetic units) the practical unit of current, the practical unit of resistance is the *Ohm* (equal to 10^9 absolute C.G.S. units). For the denomination of very large or very small resistances the terms *Megohm* or *Microhm* are used, denoting respectively one million ohms and one-millionth part of an ohm. A resistance measured or appropriately reckoned in *megohms* is called a *high resistance ;* one conveniently expressed in *microhms* is called a *low resistance.* An extra high resistance can be measured in *mega-megohms* or billions of ohms. Hence methods of resistance measurement are correspondingly described as methods for the measurement of ordinary or moderate, of low, and of high resistances.

We shall, in the following pages, deal separately with the methods for the measurement of electrical resistance in the three cases: (i) when the resulting value is most conveniently expressed in ohms; (ii) when the result is best expressed in microhms; and (iii) when the result is most suitably expressed in megohms.

Each range of resistance measurement has its own most appropriate methods. Processes in which the numerical

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[*] *Pogg. Ann.;* 153, 1874, p. 556.
[†] *Wied. Ann.,* 28, 1886, p. 542.
[‡] Maxwell, "Electricity and Magnetism," 2nd ed Vol. ..p. 463, § 370.

value of a resistance is determined by comparing it with a known standard resistance are called *comparison methods*. Measurements in which the value of a resistance is determined by immediate reference to the fundamental measurements of length, mass, and time are called *absolute methods*.

2. Networks of Conductors.

2. Networks of Conductors.—In the generality of cases the conductor whose electrical resistance has to be determined takes the form of a wire, strip, or rod of metal of uniform cross-section, or a column of a liquid conductor of similar form. A number of wires or conductors, so joined together that their ends meet in certain common points, is called a *network of conductors*. A network of conductors has a common or resultant electrical resistance, which can be expressed as a function of the separate or individual resistances of the members forming the network.

As the majority of methods for the measurement of the resistances of wires involve arrangements of networks, it will be useful to indicate the best method of calculating the resultant resistance of a network from its constituent resistances, and also the method of calculating the current flowing through any constituent branch by the application of a known electromotive force to the network.

The following method of calculating the resultant resistance of a network of conductors was given by the Author in a Paper read before the Physical Society of London, June 27, 1885 (see *Phil. Mag.*, Sept., 1885, or *Proc.* Phys. Soc., London, Vol. VII., 1885) :—

If at any two points in the network connection is made with a source of electromotive force by conductors called respectively the anode and cathode conductor, then, after a short period, depending on the self and mutual induction coefficients of the various conductors, the total quantity of electricity arriving by the anode will distribute itself throughout the network and settle down into a steady flow. When this is the case, there is a certain definite difference of potential between the anode or source-point and the cathode or sink-point, and there is also a certain definite and constant strength of current in the anode conductor and in every mesh or branch of the network. Call a and γ the potentials of these source and sink-points, and x the strength of the current in the anode lead (that is, the whole quantity of electricity flowing per second through the network), then $(\gamma - a)/x$ measures the resistance of the network.

We can imagine the network replaced by a single linear conductor or wire of such sort that if the anode and cathode conductors are applied to its ends the difference of potentials at the ends of this simple conductor and the strength of the current flowing through it have the same numerical values, γ, a, and x. The resistance of this single conductor is then the same as that of the complex network.

The resistance of the network is obviously some function of the resistances of the separate conductors or wires which compose it, and is capable of being calculated from them. Experimentally, the resistance of a complicated network would best be determined by the measurement of the current-strength in the anode lead and the difference of potential between the source and the sink. Theoretically, it is interesting to examine the law of distribution of currents in a network, and to reduce to a function of the separate resistances the total resistance of the whole network between any two points.

In his treatise on "Electricity and Magnetism" Clerk Maxwell has treated the general case to determine the differences of potentials and the currents in a linear system of n points connected together in pairs by $\frac{1}{2}n(n-1)$ linear conductors,* and has shown how to form the linear equations, the solution of which gives the condition of the network when given electromotive forces acting along some or all of the branches have established steady currents in them.

The usual method of obtaining a solution for the distribution of currents is the application of Ohm's law round the several circuits of the network, controlled by the condition of continuity that there is no creation nor destruction of electricity at the junctions.

Since the publication of the first edition of his treatise, Maxwell reduced these two sets of equations to one set by the simple device of regarding the real currents in the meshes of the network as the *differences* of imaginary currents round each cycle or mesh of the network, all directed in the same direction, and thus obtained by the application of Ohm's law a single set of linear equations, the solution of which gives the required currents in each branch. Maxwell's method is as follows† : If we have p points in space and join them together by lines, the least number of lines which will connect all the points together is $p-1$. If we add one line more we make a closed circuit somewhere in the system—that is to say, a portion of space is enclosed, and forms a cell, cycle, or mesh. Every fresh line added then makes a fresh mesh, and hence, if there are l lines altogether joining p points, the number of cycles or cells will be $k = l - (p-1)$. Let such a system of points and lines represent conducting wires joining fixed points and forming a conducting network. Let a symbol be affixed to each point which represents the electrical potential at that point, and also a symbol affixed to each line representing the

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* Maxwell's "Electricity and Magnetism," 2nd edition, Vol. I., p. 374, §§ 280 and 282b.

† This method was first given by Clerk Maxwell in his last course of University lectures. It is alluded to in the second edition of his larger treatise and in the Appendix of his smaller treatise by their respective editors. Sir W. D. Niven and Dr. W. Garnett, to whom it was communicated by the present Author.

electrical resistance of the conductor represented by it. In such a diagram of conductors the form is a matter of indifference so long as the connections are not disturbed and lines are not made to cross unless the conductors they represent are in contact at that point.

Consider a network (Fig. 1) formed by joining nine points by thirteen conductors. Then there will be $13 - (9 - 1) = 5$ cycles or cells. Let an electromotive force E act in one branch B, and give rise to a distribution of currents in the network. Take $\alpha, \beta, \gamma, \delta$, &c., to represent the potentials at the points, and A, B, C, D, &c., the electrical resistances of the conductors joining these points, and consider that round each cycle or circuit an *imaginary current* flows, all such currents flowing in the same direction of rotation.

A circuit is considered to be circumnavigated positively if we move round it so as to keep the boundary on the right hand. Hence, going round an area A in the direction of the arrow is positive as regards the inside if we walk inside
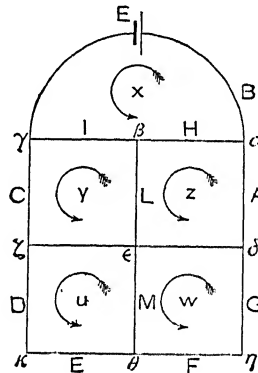

Fig. 1.

the boundary-line, and negative as regards external space B' if we walk in the same direction round the outside. We shall consider a current as positive when it flows round a cycle in the opposite direction to the hands of a watch. Returning then to the network, we consider that round each cycle an imaginary current flows in the positive direction. The real currents in the conductors are *the differences* of these in adjacent cycles or meshes, and the imaginary currents will necessarily fulfil the condition of continuity, because any point is merely a place *through* which imaginary currents flow, and at which therefore there can be no accumulation nor disappearance of electricity.

Let x, y, z, &c., denote these imaginary like-directed currents. Then $x - y$ denotes the real current in the branch I, and similarly $x - z$ that in branch H. Then x, y, z, &c., may be called the cyclic symbols of these areas. The cyclic symbol of external space is taken as zero; hence the real current in branch B is simply x.

Let an electromotive force act in the branch B. Let the internal resistance of the source of electromotive force be included in the quantity B, representing the resistance of the branch A. Then apply Ohm's law to the cycle x formed by the conductors B, I, H, and we have

$$E - Bx = \gamma - a.$$

In this case x is the actual current flowing in the resistance B, and the potential at the ends of B is equal to the effective electromotive force acting in it, *less* the product of the resistance of the conductor multiplied by the current flowing in it. For the conductor I we have similarly

$$\gamma - \beta = (x - y) I.$$

Hence $x - y$ represents the actual current in I: it is the difference of the imaginary currents flowing round the x and y cycles in the positive direction. And for the conductor H we have also

$$\beta - a = (x - z) H.$$

Add together these three equations,

$$E = \gamma - a + Bx,$$
$$0 = \beta - \gamma + (x - y) I,$$
$$0 = a - \beta + (x - z) H,$$

and we have, as the result of going round the cycle x, formed of conductors B, I, and H,

$$E = x(B + I + H) - yI - zH, \quad \ldots \ldots \ldots \quad (1)$$

a, β, γ having disappeared in virtue of these opposite signs.

The above equation (1) is called the equation of the x cycle, and we see that it is formed by writing as coefficient of the cyclic symbol x the sum of all the resistances which bound that cycle and subtracting the cyclic symbol of each neighbouring cycle multiplied respectively by the common bounding resistance as coefficient, and equating this result to the effective electromotive force acting in the cycle, written as positive or negative according as it acts with or against the imaginary current in the cycle. This statement may be called Maxwell's rule for the formation of the cycle or mesh equations.

Since there are k cycles or meshes, we can in this way form k independent equations, and by the solution of these determine the k independent variables, x, y, z, &c. The value of the current in any branch is then obtained by simply taking the difference of these variables belonging to the adjacent meshes, of which the conductor or branch considered is the common boundary.

The rule for forming the cycle equations, as given above, is merely a modification of the two statements concerning current networks generally known as Kirchhoff's Laws or Corollaries. These laws are usually stated as follows :—

(i.) In any network of conductors conveying steady electric currents the algebraic sum of all the currents meeting at a common point is zero, or

$$\Sigma(C) = 0.$$

In applying the above rule the current magnitude or value must be considered as algebraically *positive* if it flows *to* the common point, as *negative* if it flows *from* it.

(ii.) In any mesh of a network of conductors conveying steady currents, the sum of the products of the resistance of each conductor bounding the mesh and the current conveyed by it, is equal to the sum of all the electromotive forces acting round the mesh, or

$$\Sigma(CR) = \Sigma(E).$$

If we assume the existence of imaginary cycle electric currents, all circulating round the several meshes of the network in the same direction, it is obvious that this in itself implies the truth of (i.) ; and the second law (ii.) follows at once from an application of Ohm's law and the additive property of electromotive forces.

Let us now consider the most general case possible, in which we have a network composed of linear conductors sufficiently far apart to have no sensible mutual induction, and let there be electromotive forces acting in each branch or conductor. Let the system be considered to have arrived at
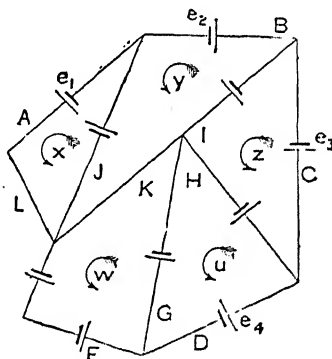

FIG. 2.

the steady condition. Let x, y, z, &c., be the cyclic symbols or measure of the imaginary current circulating counter-clockwise round each mesh. Let A, B, C, &c. (Fig. 2) be the resistances, and e_1, e_2, e_3, &c., the electromotive forces acting in each branch. These are reckoned positive when they tend to force a current round the mesh counter-clockwise, and negative when they act in the opposite direction. Then the equation to the x cycle will be

$$x(A+J+L) - yJ + 0z + 0u + 0w = e_1.$$

The symbols of all the cycles are written down, putting in those of z, u, and w with zero coefficients, as they are not adjacent cycles to that of x. We shall have four equations similar to the above for the other cycles, y, z, w and u. These equations involving only first powers of the variables x, y, z, &c., are called *linear equations*, and the student of physics is constantly met by the necessity for quickly obtaining the solution of such simultaneous linear equations, so as to evaluate the variables in terms of the constants.

In order that it may be possible to obtain a solution for the n variables in n simultaneous linear equations, certain conditions must be satisfied, and these conditions are *determined by* the relation between the constants.

The solution of linear equations is best effected by means of the method of *determinants*, and a small knowledge of this department of higher algebra is exceedingly useful to the student of electrical physics. The following elementary principles may be here explained :—

Consider the two linear equations

$$a_1x + b_1y = c_1, \quad \dots \dots \dots \quad \text{(i.)}$$
$$a_2x + b_2y = c_2. \quad \dots \dots \dots \quad \text{(ii.)}$$

Multiply (i.) by b_2 and (ii.) by b_1, and subtract the results. We obtain

$$(a_1b_2 - a_2b_1)\, x = c_1b_2 - c_2b_1.$$

This result is symbolically written in the form

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}.$$

The solution of the equations expresses the value of x as the quotient of two expressions called *determinants*.

A determinant may therefore be defined as an algebraical expression which consists of the sum (algebraic) of a number of *terms* denoted by letters or other signs, each of which is a product of certain *elements*. Into each product or term every element enters only once, and the elements can be arranged in a square or rectangular form consisting of *columns* and *rows* such that every product is formed by taking one element progressively from every row and every column once and multiplying together.

Thus $\begin{vmatrix} a & b \\ x & y \end{vmatrix}$ stands for the expression $ay - bx$, the minus sign occurring before the product bx because the order of operation or sign of the term must always be *positive* when the selection of elements proceeds forward—*i.e.*, from first row to second row, &c., or from first column to second column, &c.; and *negative* when the order of selection is reversed—that is, from the second row to the first row, or second column to the first column.

It can be similarly shown that if there are n linear equations of the type

$$a_1x_1 + a_2x_2 + \quad \dots \dots \quad a_nx_n = p_1,$$
$$b_1x_1 + b_2x_2 + \quad \dots \dots \quad b_nx_n = p_2,$$
$$\vdots \qquad \vdots \qquad\qquad\qquad \vdots$$
$$k_1x_1 + k_2x_2 + \quad \dots \dots \quad k_nx_n = p_n,$$

the solution for any variable x_1 is capable of being expressed as the quotient of two determinants, thus :—

$$x_1 = \cfrac{\begin{vmatrix} p_1 & a_2 & \cdot & \cdot & \cdot & \cdot & a_n \\ p_2 & b_2 & \cdot & \cdot & \cdot & \cdot & b_n \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ p_n & k_2 & & & & & k_n \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 & \cdot & \cdot & \cdot & \cdot & a_n \\ b_1 & b_2 & \cdot & \cdot & \cdot & \cdot & b_n \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ k_1 & k_2 & & & & & k_n \end{vmatrix}},$$

The difference between the determinants which form the numerator and denominator in the solution for x_n results from writing as numerator the determinant of the n equations having the column $p_1, p_2 \ldots p_n$ substituted for its nth column, and then writing down as denominator the determinant of the n equations simply. For example, the solution of the three linear equations

$$ax + by + cz = d,$$
$$a_1 x + b_1 y + c_1 z = d_1,$$
$$a_2 x + b_2 y + c_2 z = d_2,$$

is
$$x = \cfrac{\begin{vmatrix} d & b & c \\ d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a & b & c \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}\,,$$

with similar expressions for y and z, differing only in having as numerators respectively the determinants

$$\begin{vmatrix} a & d & c \\ a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} a & b & d \\ a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \end{vmatrix},$$

the denominator being in each case the same.

In each case the evaluation of these determinants is easy : a simple symmetrical process of taking products, according to the rule

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (ecg + bdi + afh),$$

suffices to give the equivalent of the three-row determinant in an ordinary algebraic form.

The properties of determinants enable us also very easily to evaluate a numerical determinant of any order. The process consists in a gradual reduction in the order of the determinant by such transformations as will render all the elements of the first row or column zero except the first. The determinant is then reduced to the product of its leading elements and the corresponding *minor*. The minor of any determinant is one formed from it by omitting all the elements in one row and one column and closing up all the rest of the elements into one determinant. A repetition of this process lowers the determinant one degree at each stage ; and finally, when it is resolved into a numerical two-row determinant, a simple cross multiplication gives its value.

The process of evaluation of a numerical determinant is dependent on four principles :—

(1) That the value of a determinant is not altered if rows are changed into columns, or columns into rows.

(2) The interchange of two rows or two columns reverses the sign of the determinant.

(3) If every constituent in any row or column be multiplied by the same factor, then the determinant is multiplied by that factor.

(4) A determinant is not altered if we add to each constituent of any row or column the corresponding constituents of any other row or column multiplied respectively by an identical factor, positive or negative.

For example, suppose that the solution of a series of network equations with numerical coefficients of resistance yields the determinant

$$\begin{vmatrix} 5 & 3 & 1 & 6 \\ 7 & 8 & 9 & 2 \\ 2 & 1 & 4 & 3 \\ 10 & 7 & 5 & 7 \end{vmatrix},$$

we proceed to operate on this as follows : Subtract the second column from the first and write the remainder as a new first column, we get

$$\begin{vmatrix} 2 & 3 & 1 & 6 \\ -1 & 8 & 9 & 2 \\ 1 & 1 & 4 & 3 \\ 3 & 7 & 5 & 7 \end{vmatrix}.$$

Subtract the third row from the first and put the remainder as a new first row, also add the third row to the second for a new second row, and we get

$$\begin{vmatrix} 1 & 2 & -3 & 3 \\ 0 & 9 & 13 & 5 \\ 1 & 1 & 4 & 3 \\ 3 & 7 & 5 & 7 \end{vmatrix}.$$

Again, subtract the first row from the third for a new third, and subtract three times the first row from the fourth row for a new fourth row, and we have

$$\begin{vmatrix} 1 & 2 & -3 & 3 \\ 0 & 9 & 13 & 5 \\ 0 & -1 & 7 & 0 \\ 0 & 1 & 14 & -2 \end{vmatrix}$$

which is equivalent to the third order determinant

$$\begin{vmatrix} 9 & 13 & 5 \\ -1 & 7 & 0 \\ 1 & 14 & -2 \end{vmatrix}.$$

And a similar series of operations reduces this to

$$\begin{vmatrix} 76 & 5 \\ 21 & -2 \end{vmatrix},$$

which is equal to

$$-76 \times 2 - 5 \times 21 = -257.$$

Accordingly, a series of simple subtractions and multiplications will effect the evaluation of any numerical determinant, and enable us to solve a series of linear network equations for the currents in all the branches when the numerical values of the resistances of the conductors are given.

The linear current equations as written above give as solutions the values of the cyclic symbols or imaginary currents round each mesh. To obtain the actual current in any branch we should have to obtain the values of the cyclic symbols or imaginary currents, for the adjacent meshes of which the given branch is a common boundary, and then take their difference. Maxwell ingeniously saves labour in this operation by taking as the symbol for one
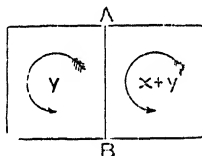

Fig. 3.

mesh say $x + y$, and for an adjacent mesh y (Fig. 3), and then the real current in the branch AB is

$$x + y - y = x.$$

And the simple rearrangement and solution of the network equation gives at once as value for x the current in the resistance AB, which is the common partition of the two meshes.

Returning to the case when there is only one impressed electromotive force in one branch, we see that in forming the cycle equations only one will be equated to an electromotive force—viz., the equation for the mesh containing the impressed electromotive force in one of its branches. All the other equations will be equated to zero ; and accordingly the equation for the current in any conductor will be of the form

$$x = \frac{E \Delta_{n-1}}{\Delta_n} ;$$

where Δ_n is a determinant of the nth order, and Δ_{n-1} is a first minor of this. Referring to Fig. 1, we see that, by writing down the five equations of the cycles x, y, z, u, w, we obtain equations by which to calculate the currents in

any of the thirteen branches, and the current in branch B will be

$$x = \frac{E\Delta_{n-1}}{\Delta_n};$$

where Δ_n is the determinant formed of the coefficients of the five equations, and Δ_{n-1} is the first minor corresponding to the coefficient of x in the equation of the x-cycle.

We also saw that if γ and α are the potentials at the ends of the branch B,

$$E - Bx = \gamma - \alpha.$$

Consider that part of the network which remains if the conductor B is removed, and let us imagine that a current x continues to be forced into it at γ and drained out at α. The total resistance of that part of the network, not counting B, is

$$\frac{\gamma - \alpha}{x};$$

but this is equal to

$$\frac{E}{x} - B.$$

Since the resistance of B may be anything, let it be zero ; then the total resistance of the network between γ and α will be

$$R = \frac{E}{x};$$

but

$$x = \left[\frac{E\Delta_{n-1}}{\Delta_n} \right]_{B=0},$$

where the suffix and bracket denote that after the determinants are formed from the cycle equations, according to Maxwell's rule, then in them B is put equal to zero.

If we denote the determinant of all the n-cycle equations under the condition of $B = 0$ by d_n, and by d_{n-1} the first minor of this latter, or the minor of its leading element corresponding to the coefficient of x with the resistance of the circuit containing the effective electromotive force put equal to zero, we have for the total resistance R of the network between the points at which the current enters and leaves the expression

$$R = \frac{d_n}{d_{n-1}}.$$

Since, then, as we have seen, the linear equations for the cycles can always be solved by evaluating the determinants, it follows that in all cases, no matter how complicated, the resistance of any network can be calculated by simple arithmetic processes from the given resistances of the branches or conductors which compose it. We have therefore here an interesting extension of Maxwell's method of calculating the currents in a network and the potentials at the junctions to a method of calculating the combined resistance of a number of conductors forming a network ; which method consists, as seen above, in forming a certain determinant whose elements are formed of the separate resistances of the branches, and dividing this determinant by another of an order next below—viz., the first minor of its leading elements ; and we find that the resistance between any two points of any network of conductors, however complicated, is expressible as the quotient of a certain determinant by another formed from it.

We shall proceed to illustrate this method by a few examples.

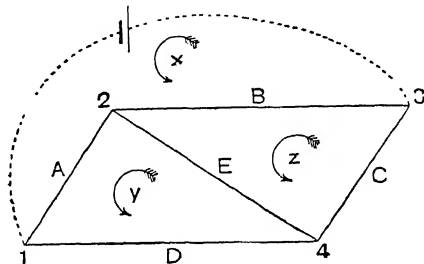1. Find the resistance between the points 1 and 3 (Fig. 4) of a network

Fig. **4.**

consisting of five conductors, whose resistances are A, B, C, D, E, joining four points, 1, 2, 3, and 4.

Connect 1 and 3 by an imaginary conductor of zero resistance, and having an electromotive force, e, supposed to act in it. Let x, y, z denote the cycles or imaginary like-directed currents in the three meshes so formed, and write down the current equations, according to Maxwell, for these three cycles

$$
\begin{aligned}
(A+B)x \quad &-Ay \quad &-Bz \quad &=e, \\
-Ax \quad &+(A+E+D)y \quad &-Ez \quad &=0, \\
-Bx \quad &-Ey \quad &+(B+C+E)z &=0.
\end{aligned}
$$

Then, by what has been shown above, the resistance R between the points 1 and 3 of the network is given by the expression

$$
R = \frac{\begin{vmatrix} (A+B), & -A, & -B \\ -A, & (A+E+D), & -E \\ -B, & -E, & (B+C+E) \end{vmatrix}}{\begin{vmatrix} (A+E+D), & -E \\ -E, & (B+C+E) \end{vmatrix}}.
$$

In dealing with numerical cases we need no longer introduce any notice of imaginary electromotive forces, but proceed according to the following rule :—

TO DETERMINE THE RESISTANCE OF A NETWORK OF CONDUCTORS BETWEEN ANY TWO POINTS ON THE NETWORK.—*Join these two points by a line whose resistance is supposed zero, and give symbols to the meshes of the network so formed, calling the additional mesh produced by this added zero conductor the added mesh. Then write down a determinant whose dexter diagonal has for elements the sum of the resistances which bound each mesh, beginning with the added mesh; and for the other elements of each row the resistances, having the minus sign prefixed, which separate this mesh respectively from adjacent meshes, zeros being placed for elements corresponding to non-adjacent meshes.*

More explicitly, if we denote by x, y, z, &c., the meshes, x being the added mesh, and by ΣR_x, ΣR_y, ΣR_z, &c., the sum of the resistances which bound each cycle, then these will be the elements along the dexter diagonal of the determinant. And if x and y are adjacent meshes, and $_xR_y$ represents the resistance of the common boundary, then $-_xR_y$ will be the element in the xth row and yth column, and also in the yth row and xth column; but if x and z are non-adjacent meshes, then 0 will be the element in the xth row and zth column, and also in the zth row and xth column. Having formed this determinant, which we call the network determinant, we divide it by the first minor of its leading element; and the quotient is the resistance of the network between the two points, joined by the zero-conductor forming the added mesh. It is seen that, owing to the mode of formation of the network equations, the network determinant is a symmetrical determinant—that is, one half of the determinant is the reflection, as it were, of the other half in the diagonal considered as a mirror.

As a means of comparing the results of this method with other known results, let us take the exceedingly simple case of three conductors joining two points in so-called multiple arc

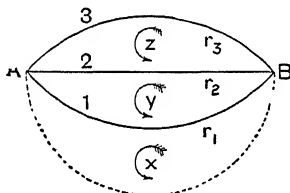Let 1, 2, and 3 (Fig. 5) be th three conductors joining two points A and B;

Fig. 5.

heir respective resistances be r_1, r_2, r_3; then join A, B by a dotted line so as to make one added mesh, and let the resistance of this added circuit be zero. Then, without writing down the equations to the cycles, we see that the network determinant is

$$d_n = \begin{vmatrix} r_1 & -r_1 & 0 \\ -r_1 & r_1+r_2 & -r_2 \\ 0 & -r_2 & r_2+r_3 \end{vmatrix}.$$

The elements r_1, $r_1 + r_2$, $r_2 + r_3$ of the dexter diagonal are the sums of the resistances which bound each mesh, x, y, and z, taking the added mesh x first.

The other elements of the first row are the resistances, with minus sign prefixed, which separate the mesh x from mesh y and mesh z; or are common to x and y and x and z—viz., r_1 and zero, because x and z are non-adjacent. And, similarly, if m and n are any two meshes, then the element in the nth row and mth column is the resistance separating or common to the two meshes; and the element in the nth row and mth column is identical with that in the mth row and nth column; zero being placed as an element if these meshes, m and n, have no common boundary or circuit.

The above determinant is easily evaluated. By adding the first row to the second for a new second row, and this second row to the third for a new third row, we transform the determinant easily into

$$\begin{vmatrix} r_1 & -r & 0 \\ 0 & r_2 & -r_2 \\ 0 & 0 & r_3 \end{vmatrix}$$

which is equal to $r_1 r_2 r_3$.

The first minor of the leading term of the network determinant

$$\begin{vmatrix} r_1 + r_2 & -r_2 \\ -r_2 & r_2 + r_3 \end{vmatrix} = d_{n-1},$$

which is equal to $r_1 r_2 + r_2 r_3 + r_3 r_1$;

and hence the resistance of the network between A and B is

$$\frac{d_n}{d_{n-1}} = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1},$$

which is a known result. In these simple cases the above general rule is of course a less easy method of finding the combined resistance than the direct application of Kirchhoff's corollaries of Ohm's law ; but whereas the general
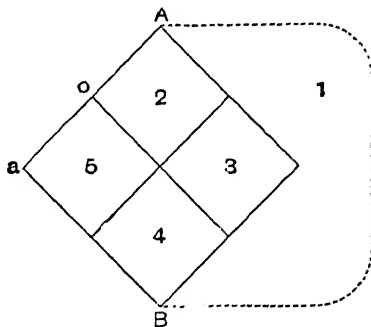

Fig. 6.

method is alike applicable to the most complicated as well as to the most simple cases, the simple direct method requires twice as many equations, and does not determine the direction as well as magnitude of the current in each branch.*

As a simple numerical example we may take the case of a crossed square of wires. Let twelve conductors join nine points (Fig. 6) so as to form a square

* The joint resistance of two conductors in parallel may be obtained by the following single geometrical construction : Draw two straight lines perpendicular to a common base line and at any distance apart. Set off on each lengths proportional to the two resistances respectively. Join the top of each line so set off with the base of the other line by a straight line, and from the point where these diagonals cross drop a perpendicular on the common base line. The length of the last perpendicular will represent to scale the joint resistance of the two conductors. We leave the proof as an exercise to the student. (See *The Electrician*, Vol. XXVIII., p. 167.)

divided into four squares, or a four-mesh network of conductors. Let the
resistance of each branch, as *ab*, be unity. It is required to find the combined
resistance between A and B. Number the meshes 2, 3, 4, 5 ; 1 being the
added mesh formed by joining A B by a dotted line, making an additional
fifth mesh, the resistance of this additional ideal conductor being zero. Then
the network determinant is

$$\begin{vmatrix} 4 & -1 & -2 & -1 & 0 \\ -1 & 4 & -1 & 0 & -1 \\ -2 & -1 & 4 & -1 & 0 \\ -1 & 0 & -1 & 4 & -1 \\ 0 & -1 & 0 & -1 & 4 \end{vmatrix} = d_n.$$

The dexter diagonal has for each element 4—viz., the sum of the four
resistances, each to unity, which form each mesh or cell. And all the other
figures, say in the *n*th row, are the resistances (with minus sign prefixed)
separating the [*n*th mesh from all other meshes, zero being placed in the
column corresponding to any mesh which has no common conductor or branch
with this *n*th mesh. The order in which the columns stand and also the rows
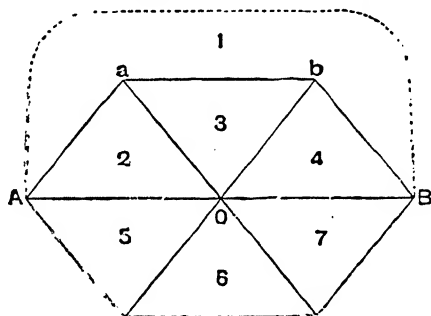correspond to the order in which the meshes are numbered in Fig. 6.

Fig. 7.

The numerical value of this determinant is easily found to be $288 = 3 \times 96 = d_n$.
Now if we take the first minor of its leading element, we get a determinant
formed of the elements included in the dotted rectangle ; and taking this as a
separate determinant and evaluating it, we have its value

$$d_{n-1} = 192 = 2 \times 96 ;$$

hence the resistance of the network between the points A and B is

$$\frac{d_n}{d_{n-1}} = \frac{288}{192} = 1\tfrac{1}{2} \text{ units.}$$

One more simple numerical case may be taken and compared with the
results of known methods.

Let a hexagon of conductors be taken (Fig. 7) having crossed diagonals all
meeting in the centre. Let the resistance of each side, as *ab*, be unity, and

also let the resistance of each semidiagonal, as $0a$, be unity. Then required the combined resistance of this network of 12 conductors between the points A and B diametrically opposite. Join the points A and B by a dotted line of zero resistance, making an added mesh 1. Mark the other meshes 2, 3, 4, 5, 6, 7. Then by forming the network equations it is easily seen that the network determinant d^n is

$$\begin{vmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 3 & 0 & 0 & -1 \\ 0 & -0 & 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 0 & -1 & 3 \end{vmatrix} = d_n.$$

The value of this determinant is 256.

The first minor of the leading element of d_n is d_{n-1}

$$= \begin{vmatrix} 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 3 \end{vmatrix}.$$

The value of this last is 320.

Hence the resistance of the network between the points A and B is

$$R = \frac{d_n}{d_{n-1}} = \frac{256}{320} = \frac{4}{5}.$$
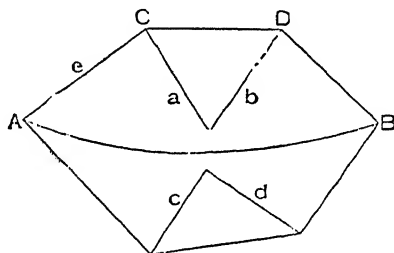

FIG. 8.

We can easily verify this result in the above symmetrical case, for the hexagonal framework in Fig. 7 is traversed symmetrically by the current flowing through it ; and hence no disturbance of the distribution of currents will take place by separating it, as in Fig. 8. We break the connection between the semi-diagonal conductors a, b and the mean diagonal A B, whilst

keeping them in contact with each other, the resistance of each branch still remaining unity. It is then easily seen that the hexagon so arranged must offer exactly the same resistance between the points A and B as in its original form.

The combined resistance of a, b, and f, each equal to unity, between the points C D is $\frac{2}{3}$, and the combined resistance of this with e and g in series is $2\frac{2}{3}$; and hence the total resistance of the whole network between A and B is equal to that of three conductors in multiple arc whose resistances are respectively $2\frac{2}{3}$. 2, and $2\frac{2}{3}$, which is equal to

$$\frac{1}{\frac{1}{2\frac{2}{3}}+\frac{1}{2}+\frac{1}{2\frac{2}{3}}}=\frac{4}{5},$$

the same result as obtained above.

These numerical examples show conclusively that, in cases in which the resistance of a network can be obtained by simple direct methods, the results coincide, as should be the case, with those obtained by the employment of the general method ; but at the same time the general method is capable of conducting easily to a solution in the most unsymmetrical cases. The general rule will, for instance, just as easily give the determinants when the selected points between which the resistance is required are not symmetrically placed, but are say adjacent angles of the hexagon, in which case no such simple direct method as employed above can be used.

A little practice will enable the student to apply the above rules to very complicated cases of networks of conductors, and to calculate the resultant resistance between any two points of a network.

§ 3. The Wheatstone-Kirchhoff Bridge. Slide Wire Form.

The arrangement or network of conductors called Wheatstone's bridge has already been briefly explained in principle (*see* § 13, Chap. I.). It consists essentially of six conductors joining four points (*see* Fig. 59, Chap. I.). In one of these circuits is placed a source of electromotive force, and in the other a galvanometer. In the laboratory the actual apparatus takes many different forms. In its most simple modification it is known usually in England as the slide wire bridge, or, as Continental writers call it, the Wheatstone-Kirchhoff bridge. In this form its most important portion is a uniform straight metallic wire of platinum-iridium alloy, platinum-silver alloy, manganin (which must be surface-gilt), or platinoid. This wire is stretched over a scale between

terminal blocks of copper, and may conveniently be one or two metres in length. The scale in contiguity to it may be either of boxwood, ebonite, or metal, and it should be divided into centimetres and millimetres. The slide wire must be very carefully selected, and its uniformity in electrical resistance per centimetre of length tested as described in § 5 of this chapter.

In the construction of a high-class instrument it is desirable to employ a wire drawn of an alloy formed of 90 per cent. platinum and 10 per cent. iridium. This alloy is very hard and non-oxidisable by exposure to the air. An alloy of platinum-silver may also be adopted having a composition of 66·6 per cent. silver and 33·3 per cent. platinum.

In cheaper instruments a German-silver or platinoid wire is generally used, but both these last alloys tarnish slightly when exposed to air. Manganin can be preserved from atmospheric action only by being gilt on the surface. The gilding must necessarily not be very thick, and in consequence is somewhat easily rubbed off the manganin. The extremities of the wire are screwed or clamped to massive terminal blocks of high-conductivity copper.

If the wire is soldered to the terminals, care must be taken that the solder does not affect the composition of the alloy at the extreme ends of the wire. Thus, for instance, manganin should not be soldered with any solder containing zinc, and platinum alloys should not be soldered with any solder containing lead.

The instrument must be provided with a travelling block or slider, such that an electrical contact may be made with the wire at any point by pressing a contact key. This contact must be made by a knife-edge, and it is desirable that the pressure of the finger should not apply the knife-edge directly to the slide wire, but merely release a spring. Thus, in Fig. 9, ww' is a portion of the slide wire, and EE' is a part of the sliding block, R a knife-edge carried on the extremity of a spring x, and P a finger-piece or trigger. A pressure applied to P will release the knife-edge and permit it to make

contact with the wire. It is easily seen that the pressure of the knife-edge on the wire is determined by the elasticity spring x carrying it, but not by the pressure of the finger on the finger-piece. The object of employing the above device is to prevent any undue pressure on the knife-edge causing "nicks" or deformations in the slide wire if contact is repeatedly made at one place on the wire. The knife-edge should consist of a piece of the same alloy used for the slide wire itself, carefully shaped to a wedge shape and soldered to a metallic carrier spring. The block holding the contact knife-edge should be capable of being traversed easily to and fro along the base-board in such a way that the knife-edge can make contact with the slide wire at any point in its length, and the exact position is observed and recorded by an indicating point, or vernier, moving over the divided scale.
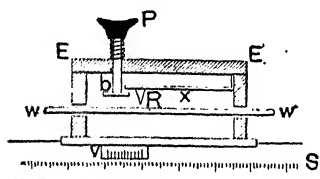

Fig. 9.

The "bridge" arrangement is then completed, as shown in Fig. 10, in which the third and fourth arms of the bridge consist respectively of a variable standard resistance S and the resistance R to be measured. A battery, B, consisting of two or three dry cells, is connected to the ends of the slide wire, and a movable coil galvanometer, G, between the knife-edge on the slider block and the junction between the third and fourth arms. Keys K_1 and K_2 are inserted in the battery and galvanometer circuits respectively. The comparison resistance may best be a dial or series plug pattern resistance box. The resistance unplugged or inserted into the third arm of the bridge should be as nearly as possible equal to the

resistance to be measured. Under these conditions, if the slider C is moved along the wire and trial contacts made with the knife-edge at various places, a point, C, will be found at which the galvanometer G indicates no current when the knife-edge contact is made. The ratio of the resistance being measured to that of the standard is then the same as that of the resistances or lengths of the two sections into which the knife-edge divides the slide wire. Hence, if S is the value of the standard in ohms and R that of the resistance being measured, and if P and Q are the resistances or the
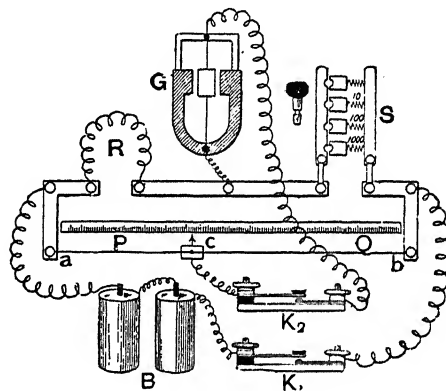
Fig. 10.

lengths of the uniform slide wire sections on the respectively adjacent sides of the bridge arrangement, we have, when the galvanometer current is zero,

$$P : Q = R : S,$$

or
$$R = S\frac{P}{Q}.$$

If the electrical resistances of various lengths of the slide wire are exactly proportional to those lengths, if the total length of the slide wire is 1,000 millimetres, and the slide wire reading at the knife-edge contact when the bridge is

balanced is at x millimetres from one end of the wire, then we have

$$P : Q = x : 1,000 - x.$$

Hence
$$R = S\left(\frac{x}{1,000-x}\right).$$

Accordingly, we can determine the value of the unknown resistance R in terms of the value of a known standard, S, and the ratio of the lengths of the two sections of the slide wire when the balance is obtained.

§ 4. The Plug Pattern Resistance Bridge.—In practice it is generally much more convenient to employ a *plug pattern Wheatstone bridge* for the measurement of resistances. In this case the resistances which form the three arms of the
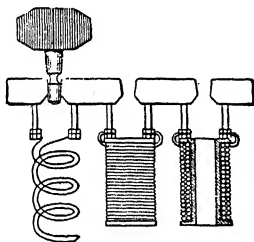

Fig. 11.—Sectional View of Blocks, Bobbins and Coil in Series Pattern Plug Bridge.

bridge are formed by adding in series the resistances of fixed coils of wire contained in a box. This addition is effected by the insertion of coned metallic plugs into holes bored out partly in one block of metal and partly in another adjacent one.

The resistance coils may be arranged either in *series* or in *parallel*, and the plugs may be arranged so as to *short-circuit coils* or to interconnect the junction between any coil of the series and a common or omnibus bar alongside the coils. In Fig. 11 is shown the *series system* of short-circuiting arrangement of plugs and blocks interconnected by resistance coils. In this case we have a series of bobbins of insulated wire of

various resistances, and one or more coils can be short-circuited and cut out of series by the insertion of plugs between the metal interconnecting blocks. In the second arrangement (Fig. 12) the coils are also arranged in series, but a junction can be effected between any interconnecting block and a parallel common or omnibus bar, so that the resistance between the bar and one end of the coil series can be varied by steps. In this latter case it is customary to employ a series of nine or ten coils of exactly equal value—say ten units, ten tens, or ten hundreds. The plug may be stepped along from hole to hole, and thus insert a resistance of one,
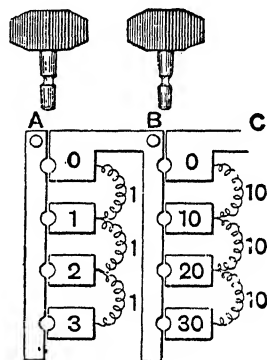
Fig. 12.

two, three, four, &c., units, tens, or hundreds of ohms between the end or terminal block and the common bar or block alongside. In some cases the blocks are arranged for this second pattern in a circle or dial (*see* Fig. 13), and in other cases in a row or rows (*see* Fig. 14) so connected that the resistances are joined in parallel and not in series, when one or more plugs are inserted.

A "bridge" is constructed of a series of coils arranged so as to form the *two ratio arms* of the bridge by having a terminal placed in some position in the series, and another series of resistances forming the *measuring arm*. These

resistances may be arranged as follows : In the so-called
" Post Office " pattern of Wheatstone bridge the ratio arms
are each formed of a series of coils of 1,000, 100, 10, and 1 ohms
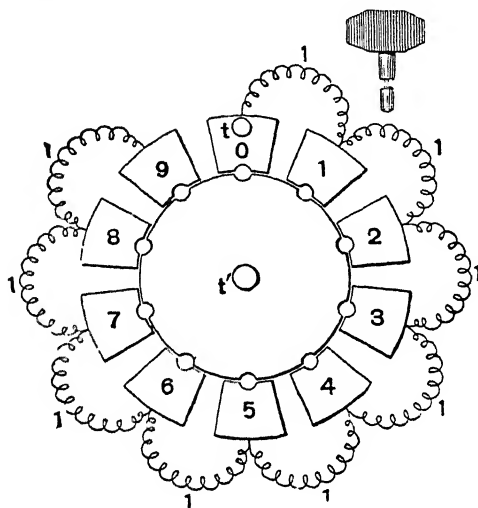
FIG. 13.—Arrangement of Blocks and Coils in Dial Pattern Resistance Box.

resistance arranged in the series fashion.   The measuring arm
is then made of a series of coils (generally of 1, 2, 3, 4, 10, 20,
30, 40, 100, 200, 300, 400, 1,000, 2,000, 3,000, 4,000 ohms in

FIG. 14.—Arrangement of Resistances in a Conductivity Box.

series) ; and terminals for the battery, galvanometer, and
resistance to be measured, together with a contact key in the
battery and one in the galvanometer circuit, are also added.

The block pieces are of brass, generally mounted on an ebonite slab, which forms the top of the box. The arrangement of circuits in a series coil bridge is shown in Fig. 16, and the external appearance in Fig. 15. The particular design illustrated is that of Messrs. Nalder Bros. and Co.*

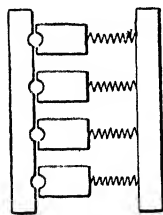In using this bridge the operator connects the resistance to be measured to the terminals marked L and L, the galvanometer to the terminals G G, and a battery of three or four dry cells to the terminals B and B. He then removes one of the plugs from the one ratio arm
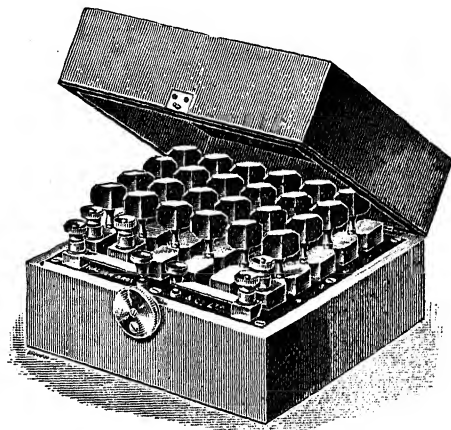
Fig. 15.—Series Plug Pattern Wheatstone Bridge.

series and one from the other ratio arm series. In so doing it is desirable to have some notion of the value of the resistance to be measured, and in general it is well, unless the unknown resistance is suspected to be very large or very small, to begin by a ratio of equality—that is, the two ratio arms are made 1 and 1, or 10 and 10, or 1,000 and 1,000 ohms by removing the plugs from the holes marked 1, 10, or 1,000 on each side. This done, the observer removes plugs from the other or measuring arm series until, on pressing first

* See *The Electrician,* Vol. XLI., p. 596.

the battery key and afterwards the galvanometer key, the galvanometer indicates no current. The bridge is then said to be *balanced*. It will generally happen that with integer value coils of the three arms it is not possible to secure a perfect or absolute balance. For instance, if the ratio arms are 10 and 10 ohms, then with 230 ohms resistance unplugged on the measuring arm, the galvanometer may show a small current in one direction through it, whereas with 231 ohms unplugged the galvanometer may show a small deflection in the opposite direction.
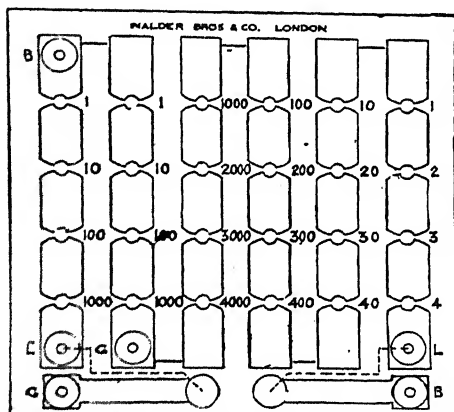
Fig. 16.—Arrangement of Circuits in Series Plug Pattern Wheatstone Bridge.

A further approximation to the true value of the unknown resistance may be obtained as follows: If the galvanometer is one reading by a mirror and scale, note the small steady scale deflection of the galvanometer when the galvanometer and battery key are both held down and the lower of the two measuring arm resistances is unplugged. Thus, for instance, suppose that, with 230 ohms out in the measuring arm, the steady scale deflection of the galvanometer is 10 millimetres to the left. This small deflection of the spot of light or indicating needle is proportional to the galvanometer or bridge current under those

conditions. Suppose, then, that the measuring arm is made 231 ohms and the galvanometer deflection is changed to 15 scale divisions to the right. We may deduce the true value of the measuring arm resistance, which would exactly balance this unknown resistance being measured, by a simple calculation. Since a change of one unit in the measuring arm resistance changes the galvanometer deflection from $+15$ to -10, it is clear that, since the whole scale deflection change is 25 divisions for the whole change from 230 to 231 ohms in the measuring arm resistance, a value of the measuring arm of $230 + \dfrac{10}{25}$ ohms, or of 230·4 ohms, would exactly balance the unknown resistance being measured. Hence, if the ratio arms are 10 and 10, the resistance being measured is 230·4 ohms. The above-described method, called taking right and left galvanometer deflections, enables us to make an initial approximation to the value of the unknown resistance.

When this is done a different ratio may be selected for the ratio arms. The ratio may, for instance, be made 10 to 1 by removing the plugs corresponding to 1,000 and 100 ohms in the two ratio arms. Balancing again, suppose we find that a value of 2,304 ohms unplugged out of the measuring arm makes a scale deflection of one division to the left, and with 2,305 ohms unplugged in the measuring arm we obtain a scale deflection of two divisions to the right; then similar reasoning shows that the exact balancing resistance would be 2,304·33 ohms. Hence the resistance being measured is 230·433 ohms.

In using a series pattern plug bridge the following precautions must not be neglected:—

(*i.*) The plugs when not in use should never be laid on the table, or be held in a hot hand, but should be placed in the lid of the bridge box, which is generally lined with velvet or cloth.

(*ii.*) The greatest care should be taken to prevent the plugs becoming permanently soiled, oxidised, or amalgamated with mercury. If necessary to clean a plug, only a very careful application of the finest glass-paper should be made.

(*iii.*) In putting the plug into a hole, give it a slight twist to make it seat itself accurately in the coned hole, and, if the box is a series plug pattern box, after each rearrangement of plugs go over the other plugs in the same manner again to see that there are no loose plugs. The plugs should, however, not be put into the holes so violently that there is risk of twisting off the heads in getting them out again.

(*iv.*) The ebonite slab carrying the brass blocks should be kept carefully dusted, and the bridge when not in use should be put away in its box with all the plugs loosely placed in their holes.

The advantage of the *dial* pattern of bridge (*see* Fig. 13) is that there is only one plug to each decimal series of coils. A resistance of 0 to 9 ohms is introduced into the measuring arm by merely moving the plug to the holes marked 0 to 9 in the dial or group or row of blocks marked *units*. A resistance of 0 to 90 ohms or 0 to 900 ohms is introduced by placing a plug in one or other of the holes in the block group marked *tens* or *hundreds*, and so on.

In using a dial pattern bridge to measure an unknown resistance, set off first a ratio of 1 to 1 in the ratio arms, and then systematically increase the measuring arm resistance by changing the unit plug; then move the ten ohm plug, next the hundred ohm plug, from hole to hole progressively until a resistance is found such that the galvanometer is deflected one way for the lower and the opposite way for the higher. Thus, suppose that with a ratio arm of 100 and 100 ohms we find a resistance of 500 ohms in the balancing arm makes the galvanometer needle fly to the left and 600 ohms makes it fly to the right, we may try next increasing regularly from 500 to 600 by tens. If then we find left and right deflections for 550 and 560 ohms, we proceed to increase by units; and when we find small left and right deflections for 553 and 554 ohms we adopt the deflectional method and unequal ratio arms already explained to determine the next decimal place or places.

In measuring any inductive resistance it is essential to close the battery circuit of the bridge before the galvanometer

circuit, in order to permit the currents in the various branches to become steady. This is achieved without thought by the employment of a double key (*see* Fig. 17), which closes first the battery circuit B B, and afterwards the galvanometer circuit G G.

In making careful measurements it is necessary to apply a correction to the bridge reading for the temperature variation of its own coils, and also for any error there may be in the actual coil resistances.

The possessor of a plug pattern bridge should not take for granted that it is correct, but should proceed to check its readings as follows:—A standard 1-ohm coil should be connected to the bridge, using very thick copper rods or strips of copper and mercury cups to make the connection.
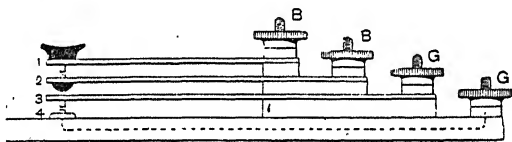
FIG. 17.—Wheatstone Bridge Combined Battery and Galvanometer Key.

The bridge reading should then be taken, using various ratio arms—say 1 to 1 ohm, 10 to 1 ohm, 1,000 to 100 ohms. If the bridge is properly adjusted each of these readings, when reduced, should give a value of unity for the known standard 1-ohm resistance. The maker of the bridge generally marks on it the temperature at which it is correct in its readings, and he will always furnish the purchaser with a statement of the nature of the wire used for the coils and its temperature coefficient. Thus, if the bridge coils are made of platinum-silver wire, the temperature coefficient of which at about 15°C. is 0·026 per cent. per degree, and if the bridge is correct at 15°C., then it follows that, if the bridge coils at the moment of reading are at a temperature t°C., the true value of one unit of the bridge reading is $1 + 0·00026(t - 15)$ ohms.

The real difficulty is the uncertainty as to the actual temperature of the wire of the bridge coils at the moment of measurement. A thermometer placed in the box containing the coils merely gives the temperature of the air round the coils, and if these have become internally heated by currents passing through them, the actual temperature of the wire may be, and probably is, very different from that of the air outside them. To avoid this uncertainty, it is necessary that bridges intended for very accurate work should be made up with manganin wire properly *aged,* and that the coils of wire should be wound on brass or copper tubes merely having one layer of silk tape laid over it. The box containing all the coils should be filled with an insulating oil which can be kept stirred. In this manner it is possible to maintain the coils at a temperature near to that at which the manganin has a zero temperature coefficient, and at this temperature the coils should be adjusted to be correct.

In the case of the measurements made in an electro-technical laboratory, it will not often happen that a degree of refinement is necessary in the resistance measurements which will make it worth while to apply the above correction for the temperature of the bridge coils. For very accurate work, however, it has been proposed to employ coils of bare wire wound on mica frames.* The coils *after* winding are annealed by passing a strong current through them. When in use they are kept immersed in a fluid insulating oil which is well stirred.

We may conclude this section with a description of a standard form of Wheatstone bridge for very accurate resistance measurements which has been designed by Prof. H. L. Callendar and Mr. E. H. Griffiths.†

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* *See* Mr. F. W. Burstall, " On the Use of Bare Wire for Resistance Coils." *Proc.* Phys. Soc., Vol. XIV., p. 286.

† The following paragraphs describing the Callendar and Griffiths bridge are taken by permission *verbatim* from a series of articles in *The Electrician* for 1898 by Mr. G. M. Clark, entitled "On the Measurement of Temperature : an Application of the Measurement of Resistance." (*The Electrician,* Vol. XXXVIII., p. 747.)

A general view in its case is given in Fig. 18, and a view of the top of the box in Fig. 19. It will be seen that, besides a
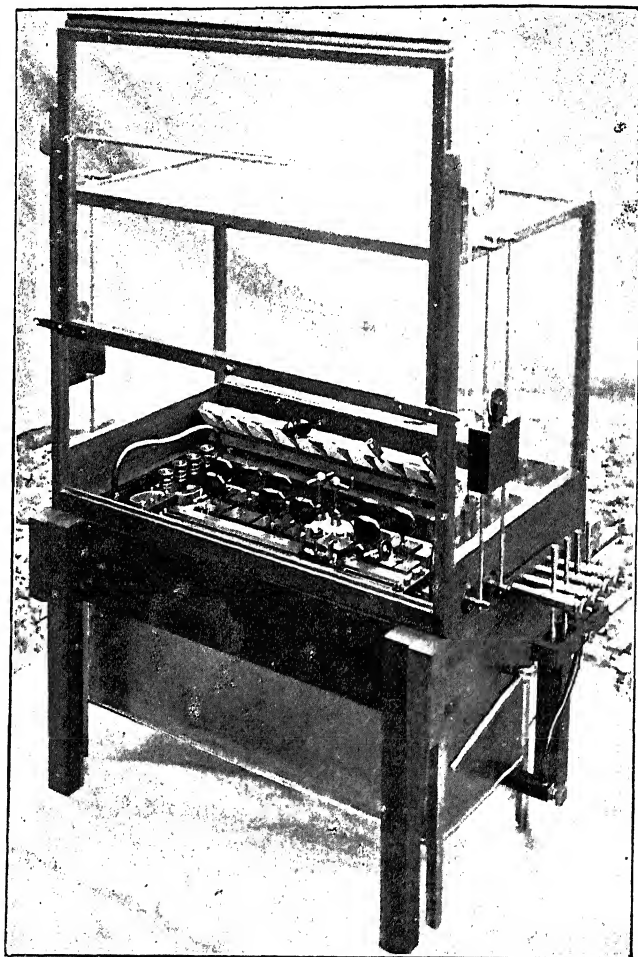

FIG 18.—Callendar and Griffith's Resistance Bridge.

set of coils, the box has a bridge wire in addition. One arm of the bridge is formed by the resistance to be measured and
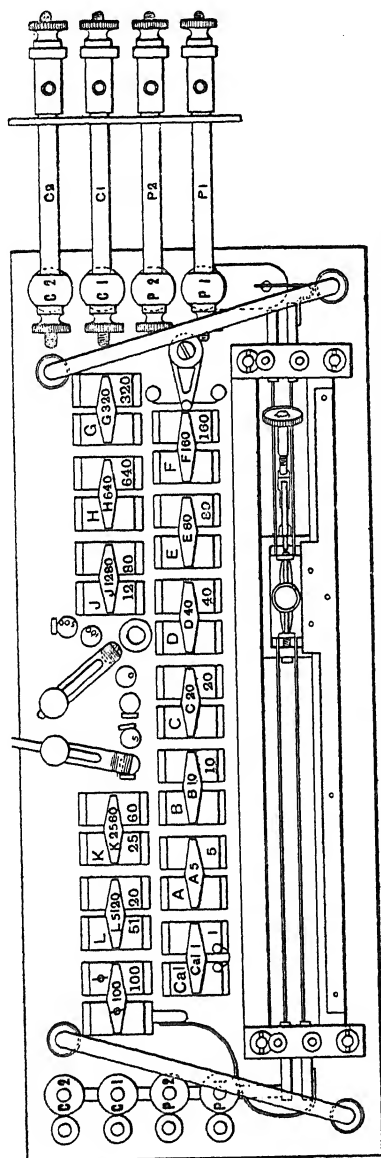
Fig. 19. — Arrangement of Circuits in Callendar and Griffith's Resistance Bridge.

part of the bridge wire, the rest of the bridge wire and the necessary coils forming the other arm. As the back coils of the bridge are equal, a pair of compensating leads (equal in resistance to those leading to the resistance to be tested) may also be introduced into this arm. It is therefore not necessary to make separate measurements for the leads. When the resistance to be tested is short-circuited, the bridge should be in balance with the contact at the centre of the bridge wire. Thus, by short-circuiting the bridge by the four connectors shown at the right of Fig. 18, the zero of the bridge can be readily tested at any time.

The coils of the bridge are thirteen in number. Eleven of these form the ordinary working coils of the bridge. These are arranged on the binary scale. The smallest coil, marked A, has a resistance of 5 box units. The unit of the box is 0·01 ohm. The other coils are respectively 10, 20, &c., of such units. The largest coil, L, is 5,120 units. Thus the whole resistance of the box is a little over 100 ohms. Since the back coils are equal, this is also the greatest resistance that can be measured with the box. The other two coils are marked respectively Cal. and ϕ. The coil Cal. has a resistance of approximately the box unit, and is used in the calibration of the bridge wire. The coil ϕ has a resistance of 100 box units, and is useful for the adjustment of platinum thermometers. For if these are constructed so that their fundamental interval is 100 box units, then the calculation of the platinum temperature is greatly simplified.*

A movement of two centimetres on the bridge wire corresponds to the box unit. As it is easy to read this movement by means of a vernier to $\frac{1}{50}$mm., the least count of the bridge is $\frac{1}{1000}$th of the box unit, or 0·00001 ohm. The contact maker seen on the bridge wire is of a special design to permit of this fine adjustment. It also, by its construction, renders injury to the bridge wire impossible.

In calibrating the bridge wire all that is required is a rough adjustable resistance, whose value need not be known. Having adjusted this until the bridge is in balance with the slider at one end of the scale, the plug Cal. is removed and the slider moved until a fresh balance is obtained. The plug is then replaced and the external resistance adjusted until the balance

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* For $pt = 100(R - R_0)/R_1 - R_0$: and $F.I. = R_1 - R_0 = 100$, then $pt = R - R_0$. If the F. I. is not exactly 100, but differs only by a small quantity, then pt can be found by the usual methods of making small corrections.

is again attained at, or sufficiently near to, the same place in the bridge wire, and the process is repeated until the whole bridge wire has been traversed. We have thus a series of steps on the bridge wire, each equal in resistance to one box unit. The length of these steps has been read off on the scale, so that if there are any errors in the scale, these have been introduced into the calibration, and are therefore of no consequence.

In the calibration of the coils a somewhat similar process is followed. As each coil only differs from the sum of all that precede it by five box units, a balance obtained with any one coil can also be obtained with the preceding coils, together with a length of bridge wire. Thus every coil in the box can be readily determined in terms of the bridge wire. By using different combinations of coils to measure any resistance, there is a perpetual check on the calibration of the whole box. Any change which takes place in the value of any of the coils can be readily detected. It is, however, very improbable that much will take place. These coils are of naked platinum silver, and are wound on mica frames. By lifting the box out of its case all the coils can be seen, and are easily accessible. Coils so formed can be very thoroughly annealed by passing a strong current through them, sufficient to raise them to a red heat, after the coil has been wound. They are thus entirely free from all strain. The bare wire coils are immersed in a bath of non-volatile hydrocarbon oil of high insulating power. This oil can be stirred, and there is no doubt that the temperature indicated by a thermometer in the oil is also the temperature of the coils themselves. The temperature of the box can be rapidly raised or lowered if necessary; and as the coils follow these changes, the temperature coefficient of the coils can very readily be found.

The back coils of the bridge also consist of two naked platinum silver coils wound together on the same mica frame. There is thus no chance of the two being at different temperatures. A short length of wire on the top of the box joins the two ends of these coils, and on this length is the galvanometer contact. If the back coils are not equal the galvanometer contact can be adjusted till equality in the back arms is obtained. The galvanometer contact is a potential one only, so that resistance through the contact is of no importance.

The top of the box is formed of white marble. This has high insulating properties, and is not open to the same

objections as ebonite. Separate brasses are used for each plug, so that the movement of one plug in no way affects the rest. Each block and plug is also marked with its distinguishing letter and value, so that in case of wear either can be refitted. As no part of the plug is wider than the top of the plug hole, no shoulder can be worn on the taper, and the wear is reduced to a minimum. The contact resistance of these plugs is found to be very constant. When not in use, the plugs are placed in a spring rack lined with wash-leather, so that they are always kept thoroughly cleaned.

The box case is constructed of a double copper box with asbestos lining between. Thus the inner surface of this must be at a very uniform temperature throughout. The top of the box is protected by a glass case similar to a balance case. The
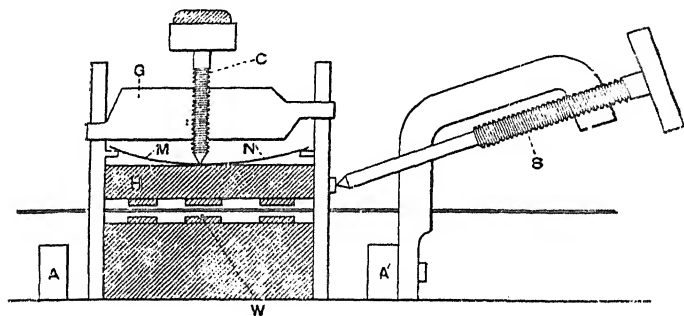
Fig. 20.

front of this is made to slide up or down. This serves the purpose not only of keeping the instrument free from dirt and dust, but also adds very considerably to the uniform distribution of temperature throughout the box. The terminals are brought through the side of the case.

The bridge wire is connected in such a manner that its tension is not affected by changes in temperature. The coefficient of expansion of platinum-silver is intermediate between that of steel and brass. The rectangular framework carrying the bridge wire has its two longer sides one of steel and the other of brass. By placing the bridge wire in the correct position it will suffer no further strain, no matter what temperature changes it may be subjected to. These bars further tend

to protect the bridge wire, so that there is but little danger of differences in temperature existing in the bridge wire. Alongside the bridge wire proper is stretched a second wire, on which the galvanometer contact is made.

This contact is made through the contact maker, which is shown in section in Fig. 20 and in plan in Fig. 21. ABA′ is a brass framework, which slides between the steel and brass bars. A second block, FEE′H, is within the brass framework. Springs at A and A′ press the brass frame against the steel bar, and springs at E and E′ press the inner block against the brass bar. Thus if the screw S is turned the inner block is alone moved, for the pressure of the inner block against the brass bar is that of the springs at E and E′ only, whereas the
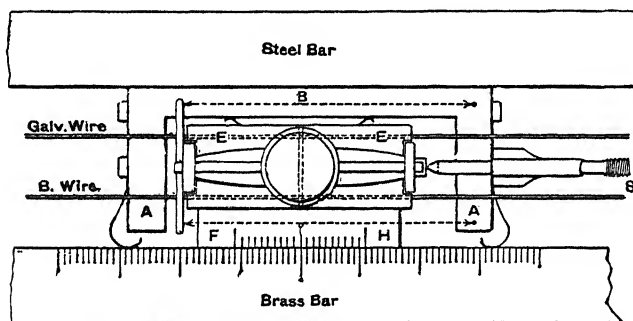
FIG. 21.

pressure of the framework against the steel bar is that of the springs at A and A′ in addition. If the screw S is receded instead of advanced, then the inner block is made to follow it by long springs indicated by the dotted lines in Fig. 21. The screw C clamps the bridge wire and galvanometer wire together on to a short length of wire W, as shown in section in Fig. 20. If it is desired to make a temporary connection only, then the block G must be pressed down. The spring MN releases the bridge wire from the contact wire. If by mischance the screw S is turned whilst the bridge wire is clamped by the screw C, then the outer framework ABA′ alone moves, and it is thus impossible with this form of contact maker to scrape or in any way damage the bridge wire. At the same time a very sharply defined contact is obtained.

Q 2

§5. **Portable Forms of Wheatstone Bridge and Testing Sets. Trotter Bridge.**—Instrument makers have devised numerous forms of portable slide-wire and plug bridges suitable for laboratory and outdoor work. For the details of these the trade circulars of the principal electrical instrument makers may be consulted. In most of these arrangements the galvanometer is one with a movable nearly astatic double-magnetic needle carried on a jewelled centre. The vibrations of the

FIG. 22.—Portable Form of Wheatstone Bridge.

needle are arrested by a stop which raises the needle off its steel centre-suspension point when the box lid enclosing the instrument is shut.

For rapid work, where great accuracy is not required, or for obtaining a preliminary measurement of the resistance of a wire afterwards to be more accurately measured, some of these portable bridges are very useful. The resistances are thrown into circuit, not by removing plugs, which may be

lost, but by a revolving contact arm making contact against fixed brass studs. The stud surfaces must be kept clean. These instruments are arranged either for measuring ordinary and low resistance or for high resistances such as insulation. (*See* Figs. 22 and 23.) In the latter case the battery is replaced by a small continuous-current magneto machine, giving a voltage of from 80 to 100 volts when the handle is quickly turned.
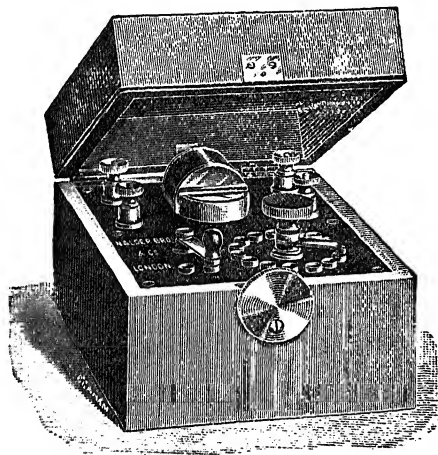
Fig. 23.

A form of portable slide-wire bridge has been developed by Messrs. A. P. Trotter and J. Swinburne out of a simpler form due to Major P. Cardew.* Two wires of nickel steel are stretched over a scale divided into equal parts. These wires are connected together at one end by a bar of negligible resistance. These wires are represented by the lines b and a and c, d, e in Fig. 24. A length b is set off one end of one wire, and a length c is set off the end of the

* See *The Electrician*, Vol. XXXVII., p. 691.

other wire, so that the resistance of b is equal to that of c. The scale extends over d and e, and the scale zero is at d. The resistance to be measured is represented by the coil x. A battery is joined in between the cross piece and the junction a to x and a galvanometer between junctions b to a and a slide which moves over the wire c, d, e and makes contact at any place.
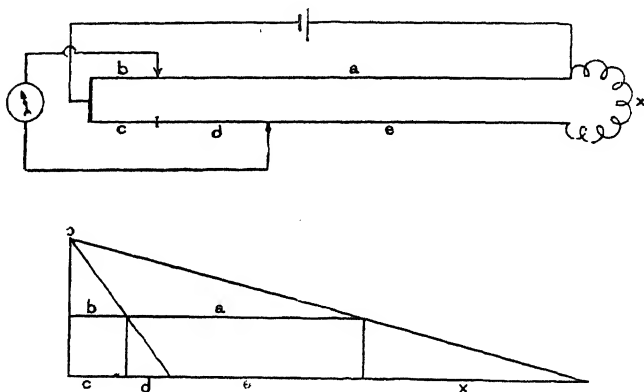
FIG. 24.

Then it is obvious that, if the bridge is balanced so that the galvanometer shows no current, we have

$$\frac{a+b}{b} = \frac{x+e+d+c}{c+d},$$

where the letters stand for the resistances of the segments of the two wires. If $b=c$ and $a=e+d$, then the above ratio is reduced to $\dfrac{a+b}{b} = \dfrac{x}{d}$,

or
$$x = d\,\frac{a+b}{b}.$$

Hence the length d on the slide wire is proportional to the resistance x if $a+b/b$ is a constant ratio. Accordingly, by

shifting the galvanometer contact on the wire $a\,b$, so as to give the ratio a/b any desired value, we can make the ratio $a+b/b$ what we please.

When once this ratio is fixed, the length of d, when the galvanometer balance is obtained, is proportional to x. Points may therefore be marked off on the wire $a\,b$, so that the segments a and b have such values that $a+b/b=1$ or 10 or 100. If, then, the resistance per unit of length of the wire $d\,e$ is known, we have at once the value of x given.

The Trotter bridge is made up in a portable form, as shown in Fig. 25.

A useful implement to possess in an electrical laboratory is a pair of equal uniform platinum-silver wires stretched parallel to each other over a pair of divided scales, both wires having contact sliders moving over them and terminal screws at each end of each wire. The platinum-silver wires may be replaced by platinoid or manganin. They should be about the size called No. 20 S.W.G. A thick strip of copper should be provided for connecting across corresponding ends of the two wires.
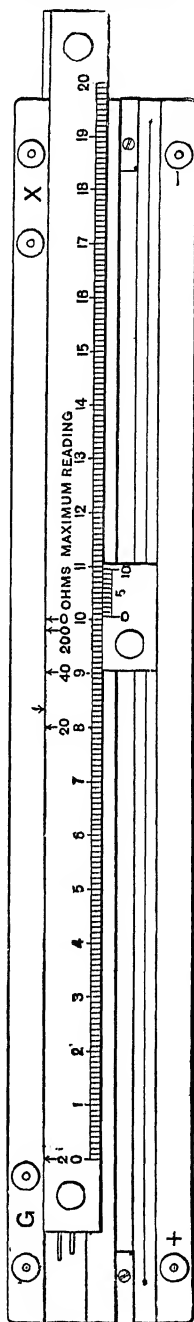

FIG. 25.—Trotter's Resistance Bridge.

§ 6. Theory of the Wheatstone Bridge.—In connection with the use of the Wheatstone bridge for resistance measurement, we have to consider the best arrangement of the circuits so that with a given galvanometer and battery the *sensitiveness* of the arrangement may be a maximum. The six conductors joining the four points consist of two *ratio arms* or resistances, the standard of comparison or *balancing arm*, the resistance to be determined, and the galvanometer and battery circuits. These can be shown arranged symmetrically as in Fig. 26, instead of in lozenge form as in Fig. 27.

Let the three meshes of the above network be supposed to be traversed by imaginary cycle currents $x + y$, y, and z, then the real current through the galvanometer circuit is x, and that through the battery circuit is z. Forming, by Maxwell's rule, the cycle equations (see page 197), we have

$$(R + G + P)\,\overline{(x+y)} - Gy \qquad - P\,z = 0,$$
$$(G + Q + S)\ y \qquad - G\,\overline{(x+y)} - Q\,z = 0,$$
$$(P + Q + B)\ z \qquad - P\,\overline{(x+y)} - Q\,y = E,$$

where G and B are taken as the respective resistances of the battery and galvanometer branch, and E is the electromotive force of the battery. Rearranging these equations, we have

$$-P\,x - (P + Q)\,y + (P + Q + B)\,z = E \quad . \quad . \quad . \quad \text{(i.)}$$
$$(P + R + G)\,x + (P + R)\,y \qquad - P\,z = 0 \quad . \quad . \quad . \quad \text{(ii.)}$$
$$-G\,x + (Q + S)\,y \qquad - Q\,z = 0 \quad . \quad . \quad . \quad \text{(iii.)}$$

Hence, solving these equations for x, we obtain

$$x = \cfrac{E\begin{vmatrix} (P + R), & -P \\ (Q + S), & -Q \end{vmatrix}}{\begin{vmatrix} -P & , & -(P + Q), & (P + Q + B) \\ (P + R + G), & (P + R), & -P \\ -G & , & (Q + S), & -Q \end{vmatrix}} = \cfrac{E\,(P\,S - R\,Q)}{\Delta},$$

where Δ stands for the determinant, in the denominator with the fu algebraic expression of which we are not for the moment concerned.

Accordingly, if $PS - RQ = 0$, that is, if the relation $\dfrac{P}{Q} = \dfrac{R}{S}$ holds good, then the current through the galvanometer is zero; the bridge is then said to be *balanced*, and $x = 0$. The determinant Δ can be written out symmetrically into the expression

$$\Delta = BG(P + Q + R + S) + B(Q + S)\,(P + R) + G(R + S)\,(P + Q)$$
$$+ RS(P + Q) + PQ(R + S). \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv.)}$$

The full expression for the current x through the galvanometer circuit of the

bridge, in terms of the electromotive force (E) of the battery and the resistances (P, Q, R, S, B, G) of the six branches, is therefore

$$x = \frac{\text{E(PS - RQ)}}{\text{BG(P + Q + R + S) + B(Q + S)(P + R) + G(R + S)(P + Q) + RS(P + Q) + PQ(R + S)}}$$

The question then arises whether for given values of P, Q, R and S the current x through the galvanometer will be greater if the battery circuit is connected in between the junctions where (P and R) and (Q and S) meet and the galvanometer in between the junctions where (P and Q) and (R and S) meet, or *vice versa*.

If we consider the position of the galvanometer and battery exchanged— that is, if we write B instead of G and G instead of B in the above expression, (iv.), and call the result Δ_1, we have for the value of $\Delta_1 - \Delta$ the equations

$$\Delta_1 - \Delta = (G - B) \left\{ (Q + S)(P + R) - (R + S)(P + Q) \right\}$$

or

$$\Delta_1 - \Delta = (G - B)(S - P)(R - Q).$$
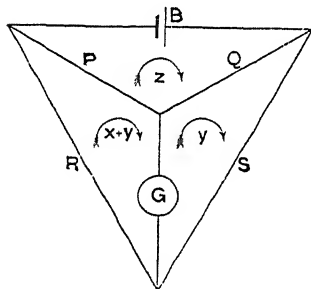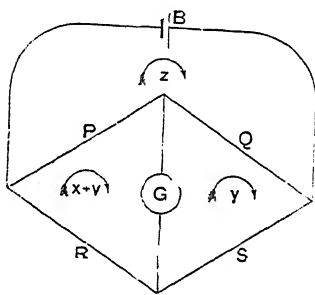
FIG. 26.　　　　FIG. 27.

The best arrangement of the battery and galvanometer therefore is that which will make x a maximum, other arrangements remaining the same ; for if the balance is nearly obtained, the galvanometer should show the greatest possible deflection for a small defection from the fulfilment of the relation $PS = RQ$. It will generally happen that the value of G, the galvanometer resistance, is greater than that of the battery B.

Let us suppose the four resistances arranged in consecutive order of magnitude are denoted by S, R, Q, P. That is, assume

$$S > R \quad R > Q \quad Q > P,$$

or else

$$S < R \quad R < Q \quad Q < P ;$$

then $(S - P)$ and $(R - Q)$ are both positive quantities or both negative ones, so that their product is always positive. Therefore, in the expression given above $\Delta_1 - \Delta = (G - B)(S - P)(R - Q)$, $\Delta_1 - \Delta$ must have the same sign as $(G - B)$. Accordingly, if G is greater than B, Δ_1 will be greater than Δ ; and hence x will be less in value when the galvanometer and battery have the positions shown in Fig. 28 than when they are exchanged.

We have therefore the following rule:—Assuming the galvanometer to have a greater resistance than the battery, the current through the galvanometer G, when it and the four resistances S, R, Q, P in order of magnitude are arranged so that the galvanometer is connected in between the junctions of the two greatest resistances S, R and the two least resistances Q, P, is greater than when the battery occupies that position ; or the rule may be formulated in another manner, as follows :

Of the two resistances, that of the battery and that of the galvanometer, select that which is greater, and connect it to join the junction of the two greatest to that of the two least of the four resistances forming the arms of the bridge.   Let the remaining appliance occupy the conjugate position.

Thus, for instance, if we have to measure a resistance of the order of say 50 ohms, and we decide to adopt a ratio of 1,000 to 10 for the ratio arms of the bridge, then, when the balance is obtained, the resistance in the measuring arm will be about 5 000 ohms.   Suppose that in the quadrilateral of resistances we have two large and two small, as shown in Fig. 28.
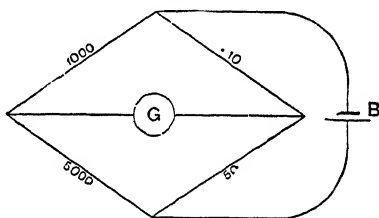
FIG. 28.

In this case the rule above given shows that if the battery circuit B has a lower resistance than the galvanometer circuit G, then the galvanometer and battery should be connected as shown in Fig. 28.   If, however, the battery has a higher resistance than the galvanometer, then their positions should be exchanged.

If the equations (i.), (ii.) and (iii.) for x, y and z are solved for z instead of x, we obtain the value of the current through the battery circuit, and we find by the above methods the equation

$$z = \frac{E \begin{vmatrix} (P+R+G), & (P+R) \\ -G, & (Q+S) \end{vmatrix}}{\Delta},$$

where Δ has the same value as in (iv.).   Hence the ratio of x to z, or current through galvanometer to current through battery, is

$$\frac{x}{z} = \frac{\begin{vmatrix} (P+R), & -P \\ (Q+S), & -Q \end{vmatrix}}{\begin{vmatrix} (P+R+G), & (P+R) \\ -G & (Q+S) \end{vmatrix}}$$

$$= \frac{PS - QR}{G(P+Q+R+S) + (P+R)(Q+S)}.$$

It has been shown independently by Mr. Oliver Heaviside (*Phil. Mag.*, Vol. XLV., 1873, p. 114) and by Mr. T. Gray (*Phil. Mag.*, Vol. XII., 1881, p. 283), that if R is the resistance to be measured, and B and G, the battery and galvanometer resistances, are fixed, the maximum sensitiveness is secured if the other resistances P, Q and S forming the bridge arms are selected so that

$$Q^2 = BG, \ S^2 = RB\frac{R+G}{R+B}, \text{ and } P^2 = RG\frac{R+B}{R+G}.$$

(For the proof of the above formulæ the reader is referred to "Absolute Measurements in Electricity and Magnetism," by Prof. A. Gray, Vol. I., p 332.)

§ **7. The Matthiessen and Hockin Bridge.**—We have already explained that the slide-wire bridge can be used so as to enable us to measure the *difference* in resistance between two coils and not their *ratio*. In this arrangement four coils are provided and connected with the bridge wire as shown in Fig. 29.
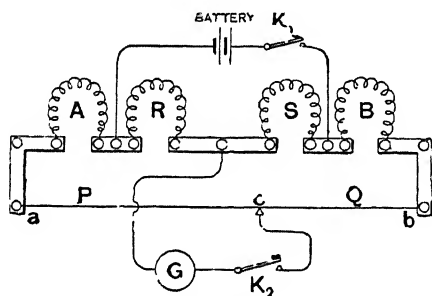
Fig. 29.

We have, also, already proved (*see* page 149) that if the coils A and B are exchanged in position, the resistance of that length of the slide wire over which the slider contact has to be moved to find a second balancing position after the

coils are exchanged is equal to the difference in electrical resistance of the two coils.　This method of resistance comparison is due to Prof. G. Carey Foster,* and is invariably used when comparisons have to be made between coils of nearly equal value, one being a standard and the other a coil of which the resistance is required with reference to the standard.

We have already also described some of the mechanical devices for effecting quickly the interchange of the coils A and B.　The simplest device for this purpose is to form a group
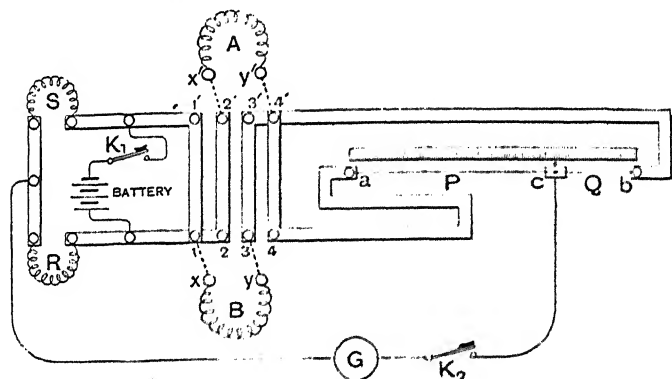
Fig. 30.

of 12 mercury cups, x, y, x', y', 1, 2, 3, 4, 1', 2', 3', 4', which may be made by holes bored out in a thick mahogany slab.　These mercury cups are connected, as shown in Fig. 30, by copper strips about half an inch wide and one-eighth of an inch thick. These strips are prevented from touching one another where they cross by slips of mica inserted between them　The coils to be compared are represented by A and B, and these coils have their legs or terminal rods placed in mercury cups x, y, x' y', which can be connected by thick copper $\cap$-shaped rods with the other cups, so that x is connected to 1, y to 3,

* See *Journal* of the Society of Telegraph Engineers, May 8, 1872.

x' to $2'$, y' to $4'$, or else x to 2, y to 4, x' to $1'$, y' to $3'$. The ends of the copper strips are connected up to a bridge wire and to two ratio arm coils R and S as shown.

To obtain very accurate results, all the coils A, B, R and S must be placed in melting ice or in paraffin oil surrounded by melting ice, to keep them at a known and constant temperature. In the bottom of each mercury cup must be placed an amalgamated copper dish, against which the ends of the copper strips, ∩-connectors, or coil terminals are placed. In the Author's form of circular bridge (*see* page 152) the mercury cups are formed by slipping pieces of indiarubber tube on to the ends of short cylinders of copper which are soldered to
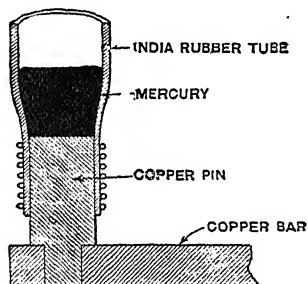
Fig. 31.

the copper bars or strips (*see* Fig. 31). The ends of these cylinders being well amalgamated, the coil legs or connectors are then well pressed down on them by weights or springs so that a good copper-to-copper contact is made between well amalgamated true copper surfaces immersed in mercury. A joint such as this offers an exceedingly small and constant resistance.

The process of taking a reading is then as follows :—Let A be a standard coil of known value at 0°C. After the coils have been kept immersed in ice for a sufficient time to ensure that in each the whole wire is all at the temperature of

melting ice, a bridge reading is taken by finding the balancing point of the slider on the wire. The position of the coils A and B is then exchanged and a second reading taken. The resistance of that part of the slide wire between the two positions of balance is equal to the difference between the resistances of the known resistance A and the unknown resistance B at 0°C. For if R and S are the values of the resistances of the ratio arm coils, and A and B the resistance of the coils the difference of which is desired, and if ρ is the resistance of the slide wire and connections at the side of A up to the point of contact of the slider, whilst W is the whole resistance of the slide wire and connections between A and B, we have the following relation between these resistances when the bridge is balanced:—

$$S : R = A + \rho : B + W - \rho.$$

Supposing, then, the coils A and B interchanged in position and that the reading ρ then becomes ρ', for a new position of balance; then :—

$$S : R = B + \rho' : A + W - \rho'.$$

Hence
$$\frac{A+\rho}{B+W-\rho} = \frac{B+\rho'}{A+W-\rho'}.$$

Therefore
$$\frac{A+B+W}{B+W-\rho} = \frac{A+B+W}{A+W-\rho'}.$$

Hence
$$A - B = \rho' - \rho.$$

Accordingly, the difference in the resistances of A and B becomes known when the slide wire is calibrated so that its resistance per centimetre or per scale division is discovered, and then, if A is a known resistance, the value of B is accurately determined.

In the accurate comparison of standard coils or of unknown resistances with a standard resistance, the great difficulty is that of discovering the actual temperature of the wire

corresponding to a certain evaluation or measurement. The passage of a current, however small, through a wire heats it and changes its resistance; hence the very accurate comparison of resistances is not easy to make unless the coils can be kept immersed in crushed and melting ice for long periods of time, or else can be placed in tanks containing a large quantity of water or oil in a state of motion and at the temperature of the air around them.

There are several methods by which this constancy of temperature can be secured. If the supply of water to the laboratory is from a constant town supply, it will generally be found that, if a current of this town water is kept flowing through a tank, after a time the temperature will be maintained at a nearly constant value, or at least within narrow limits. The coils to be compared can be both immersed in such a tank. Another way is to place both coils in a vessel filled with an insulating oil, which is kept in motion by a paddle driven by an electromotor, or by blowing air through it with a bellows, or by hand stirring. In any case the object must be to rapidly and thoroughly renew the layer of liquid in contact with the metallic case of the resistance coil or the wire itself, and it is only under these conditions that we can assume that a thermometer placed in the liquid will give even an approximation to the true temperature of the wire. Then, after discovering the points of balance on the bridge, the coils should be left for some time, and when quite at the temperature of the bath should be once more compared.

For the very accurate comparison of standard resistances a special room should be set apart, and this room should preferably be below ground, so as to be preserved nearly at a constant temperature all the year round. No person should be allowed in the room except the observer at the time the measurements are being made. If it is necessary to place the galvanometer scale at a distance, it can be read by means of a telescope suitably fixed near the observer.

§ **8. The Calibration of a Slide Wire.**—In the use of a standard wire bridge a preliminary operation is that of the calibration of the wire to determine the true resistance per centimetre length of the wire. The following method is the one which is most convenient :—Two resistances are provided of nearly equal value at the same temperature. These may be manganin wires, each having a resistance of 1 ohm and soldered to suitable terminal rods. These wires should be of sufficient diameter not to heat sensibly with the bridge currents. If they are not of exactly equal resistance, then one of them will be greater. Let two plug resistance boxes be provided, and let the terminals of these be connected to the ends of the 1-ohm wires, so that when a high resistance R is unplugged out of either box the resistance of the 1-ohm wire and this high resistance in parallel can be slightly varied. Thus, suppose the 1-ohm wire to have a value of exactly 1 ohm, and that we place in parallel with it a resistance of 2,000 ohms. The joint resistance of the two in parallel is

$$\frac{1}{\frac{1}{1}+\frac{1}{2,000}} = \frac{2,000}{2,001} = 1 - \frac{1}{2,001},$$

or is diminished by nearly 0·0005 of an ohm. Hence, by a series of trials we can adjust the parallel low and high resistances so that the combined resistances consisting of the two 1-ohm coils so shunted are of exactly equal resistance. The high resistance shunts must be adjusted until the two combined resistances are brought into a condition of equality as shown by the fact that when placed as coils A and B and exchanged on the bridge the balancing point on the slide wire is not changed.

This being done, let the two equal combined resistances be furthermore made different by a known small amount by shunting one of them with a resistance say of 1,000 ohms. We have then two resistances which differ by a known small resistance. For if each of the equal combined

resistances is x ohms, the oue shunted with 1,000 ohms is $\dfrac{1{,}000\,x}{1{,}000 + x}$ ohms, and hence their difference is then $\left(x - \dfrac{1{,}000\,x}{1{,}000 + x}\right)$ ohms. Let these two groups of coils be connected to the Matthiessen and Hockin bridge as the coils A and B, and interchanged in position, the balancing point on the slide wire $a\,b$ (*see* Fig. 32) being found for each position. Let the distance between the two balancing positions be y centimetres of wire. We then know that the resistance of the length y of the slide wire lying between the two points of balance is equal to the known difference of resistance of the two coils--viz., to $\left(x - \dfrac{1{,}000\,x}{1{,}000 + n}\right)$ ohms. The coils P and Q in the Matthiessen
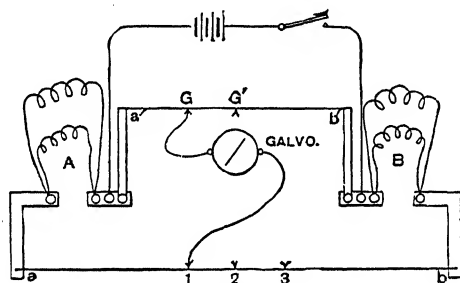
Fig. 32.

and Hockin bridge are then to be replaced by a wire of platinoid $a'b'$ about equal in resistance to the resistance of each of the coils A and B taken together. This wire need not be specially selected for uniformity. It is preferably in the form of a slide wire stretched over a board with a divided scale beneath it, and having a sliding contact piece to make contact with any point in its length.

The connections are then made as in Fig. 32, the ends of the galvanometer circuit being connected to the sliders on the wires. The observer begins by moving the contact G on the top slide wire until a place of balance, 1, is found on the bridge

slide wire $a\,b$ as near as possible to one end of the bridge wire. Then he interchanges the coils A and B, and finds a new balancing position at 2. The resistance of the lengths between points 1 and 2 on the bridge slide wire is equal to the resistance $A - B$. Then he changes the coils A and B back to their original position, and moves the contact G to G′ until a place of balance is found at the same point 2 on the bridge slide wire. Again he interchanges A and B, and finds a new place of balance at 3. The resistance of the length between 2 and 3 on the bridge slide wire is also equal to the resistance $A - B$. In this way, by changing alternately the position of the coils and the point of contact G, we are able to mark off the whole length of the bridge slide wire into little intervals of length, 1—2, 2—3, 3—4, &c., each of equal electrical resistance.

We can then from these values construct a table, by using proportional parts, which will give us very approximately the true resistance of each centimetre in length of the slide wire, and thus give us the mean resistance per unit of length or centimetre of the slide wire.

This calculation should be checked by increasing the difference between the coils A and B by a known shunting resistance, so that the difference $A - B$ is just a little less than the resistance of the whole slide wire. The constant application of corrections for inequality in the slide wire is, however, so troublesome, that in building a standard Matthiessen and Hockin slide wire bridge it is worth while to take some pains to secure a wire of such uniformity in resistance per unit of length that the correction to be applied for inequality in resistance per centimetre is practically negligible. When the resistance of the wire has been thus determined, we can at once employ the bridge to measure the difference between a standard resistance and a coil not differing from it by more than the whole resistance of the bridge wire, and this *method of differences* is more accurate for evaluating standard coils than is the method of ratios.

In this manner, given a standard 1-ohm coil, a standard or known value 10-ohm coil, &c., the experimentalist can fix the exact value at a known temperature of other coils intended to represent 1 ohm, 10 ohms, &c.

§ 9. To Determine the Temperature Coefficient of a Standard Resistance Coil.

—If the observer possesses a resistance coil the value of which, in terms of some standard of reference, is known at a stated temperature, it is essential to determine the *temperature coefficient* (T.C.), so that the true resistance of the coil may be known at any other temperature. In the case of the high-resistance alloys—platinum-silver, manganin, platinoid, &c., used in the construction of standard coils for ranges of temperature from about 0°C. to 25°C., it may be assumed that for all practical purposes the resistance (R_t) of the coil or wire at any temperature t°C. is related to its resistance (R_o) at 0°C., in the manner expressed by the formula

$$R_t = R_o(1 + \alpha t).$$

In this case α is called the temperature coefficient. It is generally expressed as a percentage per degree Centigrade of the resistance at 0°C. or at 15°C.

In the case of pure metals or alloys taken over wider ranges of temperature, the relation between the resistances at different temperatures is less simple, and can generally only be expressed in a graphical form by a curve of resistance in terms of temperature. If R is the value of the resistance of the conductor at any temperature t°C., then $\frac{dR}{dt}$ is the rate of change with temperature, and $\frac{1}{R}\frac{dR}{dt}$ is the rate of change per unit of resistance at that temperature. The value of the expression $\frac{1}{R}\frac{dR}{dt}$ may be taken as the temperature coefficient *corresponding to that temperature t°C.* It is usual to state the mean temperature coefficient between certain extreme temperatures. Thus, the mean temperature coefficient (mean

T.C.) between 0°C. and 25°C. must be known for every standard coil possessed by the laboratory. Also its true resistance at 0°C. Its resistance at t°C. is then at once calculated by the expression

$$R_t = R_o(1 + at).$$

Generally speaking, instrument makers mark on a coil the temperature at which it has its nominal value. Thus, a 1-ohm standard may be marked as "correct at 15·4°C." If, then, the temperature coefficient is known or given, the true resistance at any other temperature can be found provided it lies within the limits of practical constancy of the temperature coefficient.

The temperature coefficient of a standard coil is most easily determined as follows:—Select two coils of nearly equal value, say A and B. Place both coils in crushed melting ice contained in any convenient vessel, and let the coils remain in this ice until it is practically certain all parts of the wire of the coil are at 0°C. Place these coils on a Matthiessen and Hockin bridge (which may conveniently be the Fleming or Nalder form of differential bridge), and take the difference between the resistance of these coils in terms of the resistance of the bridge wire unit. Thus, suppose at 0°C. we have

$$A_o - B_o = x \text{ divisions of bridge wire.}$$

Next keep the coil B at 0°C., but immerse the coil A in tap water say at t°C. or the service water temperature, and keep it there until it is practically certain all parts of the wire are at the same temperature. Then again take the difference in resistance of A and B on the bridge, and we have

$$A_t - B_o = y \text{ divisions of bridge wire.}$$

Hence, by subtraction, $A_t - A_o = y - x$.

Accordingly, $\dfrac{y-x}{t}$ is the increment in bridge wire divisions of the resistance of coil A for one degree Cent. between 0° and t°. If ρ is the resistance of the bridge wire per unit of length, we

have the value of the temperature coefficient (a) of the coil
A between 0°C. and 15°C. given by the expression

$$a = \frac{100}{A} \frac{y-x}{t°}\rho.$$

§ 10. **To Determine the Mean Temperature Coefficient of
a Metallic Alloy in the Form of a Curve.**—The sample of
the alloy should be drawn into a wire of uniform diameter
and as far as possible of uniform resistance per unit of
length. It is then required to determine the mean tempera-
ture coefficient of the material. This is effected in the
following manner:—A boxwood cylinder of about 2in. (or
5cm.) in diameter and 3in. in length has a deep and coarse
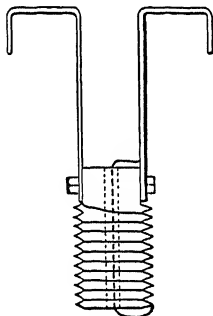
FIG. 33.

screw thread cut on it in the lathe. This screw thread ma
have a pitch of six or eight turns per inch. The opposite
sides of the cylinder should have deep grooves cut in them
and copper rods about $\frac{1}{16}$in. (or 4mm.) in diameter attached
to the cylinder, as shown in Fig. 33. These rods may be
slightly flattened where they lie against the wood cylinder,
and be screwed to it. The rods are bent over, as shown in
the diagram, to form electrodes. The alloy must be drawn
into the form of a wire of diameter between No. 22 and
No. 30 S.W.G., and must be wound loosely on the boxwood
cylinder in the grooves. The ends of the wire must be

soldered to the copper terminal rods. The cylinder is then
to be immersed in a copper vessel containing paraffin oil
enclosed in another vessel which contains water. The
resistance coil is connected with the resistance bridge by
means of thick stranded copper connecting leads. In the
first instance the copper vessel of paraffin oil may be
immersed in crushed ice and kept there until the paraffin
and the resistance wire immersed in it has a temperature of
0°C. as taken by a correct mercury thermometer. The
resistance of the wire is then observed on the bridge by
taking either its difference from, or ratio to, a known
resistance. The resistance coil is conveniently connected to
the bridge circuit by having its copper legs placed in
mercury cups, which are in connection with the bridge by
thick flexible leads of stranded copper. In order to eliminate
the resistance of the leads a copper loop must be provided, the
total length of which is equal to the total length of the two
copper terminal rods of the coil, and it must be made of a
sample of the same copper wire. This *blank*, as it is called, is
placed in the mercury cups and measured, and the difference
in measurements taken when the resistance coil is in the
cups and when the blank is in the cups is the resistance of
the wire of the resistance coil.

The measurements having been made at 0°C., the next step
is to make them at a temperature as near as possible to
100°C. For this purpose the outer jacket of the copper vessel
is filled with water, which is made to boil. The paraffin oil
in the inner vessel must be kept well stirred and the tempera-
ture taken by a correct mercury thermometer. The double
readings are then obtained as before. Other readings may
then be taken at intermediate temperatures. The chief diffi-
culty in obtaining good results consists in ascertaining the
true mean temperature of the wire at the moment when the
resistance measurement is made.

A very extensive series of measurements of the above kind
were made by the Author in conjunction with Sir James Dewar

in 1893, using metals of known purity and alloys of ascertained composition. The measurements were made between $-200°$C. and $+200°$C., using boiling liquid oxygen to provide a temperature of $-182·5°$C., a mixture of solid carbonic acid in ether to create a temperature of $-78·2°$C., melting ice to give a temperature of $0°$C., and boiling water under a pressure of 760mm. to fix a temperature of $100°$C. By this means a series of observations of resistances of the same metallic wire were taken at known temperatures, and the results set out in a series of curves. These curves show that the resistance of a wire of a pure metal steadily diminishes as the temperature falls in such a manner as to indicate that at or near the absolute zero of temperature ($-273°$C.) the resistivity of the metal would in all probability be zero. In other words, it would become a perfect conductor. Values of the ordinates of these curves showing the volume resistivity of different metals at fixed temperatures between $-150°$C. and $+150°$C. are given in Table V. at the end of this chapter.

The curves of temperature resistance (temperatures being the abscissæ) are in the case of some metals concave upwards and in other cases concave downwards. The curves of the magnetic metals iron and nickel are at first concave upwards and rise very rapidly. At a temperature very near to that at which the metals lose their marked magnetic qualities (the magnetic critical temperature) the temperature resistance curve has a point of inflexion and becomes concave downwards. The temperature coefficient thereafter becomes greatly diminished. In the case of iron this change temperature is near $780°$C., and in the case of nickel near $340°$C. The variation of resistance with temperature in the case of iron is therefore delineated by a curve not unlike its magnetisation (*see* Fig. 34).

The following table gives the values of the absolute volume resistivity of a certain specimen of annealed iron wire as observed by Dr. D. K. Morris.* The absolute volume

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* "On the Magnetic Properties and Electrical Resistance of Iron as Dependent upon Temperature." D. K. Morris, Ph.D. *Phil. Mag.*, Sept. 1897, p. 213.

resistivities (ρ) in C.G.S. measure have been calculated for exact centennial temperatures by interpolation from the observed values as given by Dr. Morris for numerous intermediate temperatures. The change in resistivity per degree $\left(\dfrac{d\rho}{dt}\right)$ in the neighbourhood of each century is also stated.
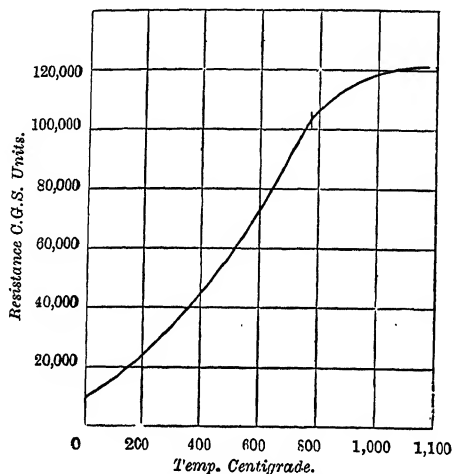
FIG. 34.—Temperature Resistance Curve of Iron.

The temperature coefficient *at* any temperature ($t°$) is obtained by dividing $\dfrac{d\rho}{dt}$ at that temperature by the absolute resistivity (ρ_t) corresponding to that temperature.

Volume electrical resistivity of iron annealed at 1,150°.

Magnetic critical temperature=778°C.

Centigrade temperature $t°$.	Volume resistivity in C.G.S. units.	Change in resistivity per degree Centigrade.
0°	10,050	61
100	16.527	71
200	24,308	82
300	34.537	115
400	45,024	116
500	57,416	124
600	71,764	152
765	100,025	195
780	103,200	—
800	.106,600	133
900	115,342	52
1,000	118,781	22
1,100	120,656	12

The temperature coefficient at 0°C. is 0·0057, and it rises to a maximum value of 0·0204 at 765°C. and falls again to a value 0·00244 at 1,000°C.

Observations on the specific heat of iron seem to show a remarkable similarity in variation to that of the resistance temperature coefficient. According to M. Pionchon, the specific heat of iron can be calculated for any temperature between 0°C. and 660°C. by the formula

$$\gamma_t = 0·1012 + 0·000050^1 66t + 0·0\,00000164t^2.$$

Hence we have the specific heat of iron at various temperatures as follows :—

Temperature.				Specific heat.
0°C. ...	...	...	...	0·1012
100°C. ...	...	...	...	0·1079
200°C. ...	...	...	...	0·1176
300°C. ...	...	...	...	0·1306
400°C. ...	...	...	...	0·1468
500°C. ...	...	...	...	0·1665
600°C. ...	...	...	...	0·1892

Also it has been shown that the average specific heat of iron

between 750°C. and 1,000°C. = 0·213,

between 954°C. „ 1,006°C. = 0·218, and

between 1,050°C. „ 1,200°C. = 0·1988.

If the above values are set out in a curve, we find that the specific heat of iron rises to a maximum and falls again in a very similar manner to the temperature resistivity coefficient. These remarkable changes in the temperature coefficient and in the form of the resistivity curve are doubtless connected with the changes in the internal energy which go on in iron at certain critical temperatures.

Many attempts have been made to express the relation between temperature and electrical resistivity by an algebraic formula. Over moderate ranges of temperature for most metals an expression of the form

$$\rho_t = \rho_o(1 + at \pm \beta t^2),$$

where ρ_t is the resistivity at $t°$C., ρ_o is the resistivity at $0°$C., and a and β are constants, will suffice to approximately express the facts.

It is, however, clear, from the form of the resistance temperature curve of iron given above, that no such simple formula will suffice over wide ranges of temperature or when including critical temperatures.

In some other cases, such as that of the alloy manganin there is a well-marked *maximum value* of the resistivity corresponding to a certain temperature, and in certain specimens of bismuth a *minimum value* of the resistivity has been found for a particular temperature. This is the case also with graphitic carbon. Hence all simple algebraic expressions, expressing the value of resistivity in terms of temperature, can only be made conformable with the facts of observation over a certain range of temperature, and such expressions cannot safely be extrapolated to yield results lying beyond the limits of temperature for which the formula was originally constructed.

When a pure metal is fused there are always rapid and generally large changes in the electrical resistivity in passing from the solid to the liquid condition. Thus, when solid mercury is heated up from $-200°$C. to above $+100°$C. it melts at $-40°$C., and just beyond that temperature its resistivity is increased nearly $4·1$ times,* as shown in Fig. 35. The temperature coefficient whilst solid is not very different from that of other pure metals.

In the case of pure platinum Mr. J. Hamilton Dickson, after discussing other formulæ, showed† that the resistivity of pure platinum could be very well represented in terms of the centigrade temperature by an empirical equation of the form

$$(R + a)^2 = p(t + b),$$

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* *See* Dewar and Fleming, "On the Electrical Resistivity of Mercury at the Temperature of Liquid Air." *Proc.* Roy. Soc., June, 1896, Vol. LX., p. 76. *See also* Cailletet and Bouty, *Comptes Rendus*, 1885, Vol. C., p. 1188, who found the ratio to be $4·08$ times.

† "On Platinum Temperatures," by J. D. H. Dickson, M.A. *Phil. Mag.*, Dec., 1897, p. 445.

where R is the resistance at the temperature $t°$C. and a, p, and b are suitably selected constants. He has found, by comparison with actual observations made on a pure annealed platinum wire by Profs. Fleming and Dewar over a range of temperature of $-200°$C. to $+200°$C., that a formula of the above kind can be made to express with considerable accuracy the variation in resistivity of platinum between the above
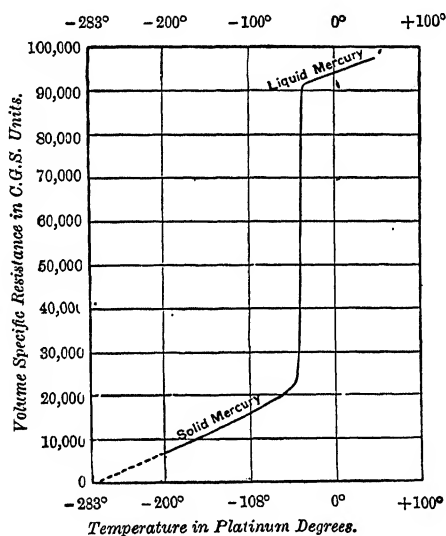
Fig. 35.—Temperature Resistance Curve of Mercury.

limits of temperature. The formula will be seen to be equivalent to the statement that the square of the resistance measured from an artificial zero is simply proportional to the temperature also measured from an artificial zero. In the case of the magnetic metals—iron, nickel, and cobalt, there are remarkable changes in resistivity at the temperatures at which the magnetic qualities are suddenly altered which prevent any empirical formula from adequately representing

the temperature change in resistance over ranges of temperature which include these critical points.*

§ 11. To Determine the Specific Resistance or the Resistivity of a Metal or Alloy.

—The *volume-resistivity* of a material is defined to be the resistance of a cube of the material having a side of unit length, taken between opposed faces of the cube at a defined temperature. The usual mode of stating the resistivity of metals and alloys is in C.G.S. units or in *microhms per centimetre-cube at* 0°C. In the case of electrolytes or conducting liquids it is expressed in *ohms per centimetre-cube at* 18°C. In the case of materials of very high resistivity, commonly called insulators, it is usual to state the resistivity taken at 75°F. in megohms or in *mega-megohms* —that is, in millions of megohms per centimetre-cube.

To determine the volume-resistivity of a sample of a metal or alloy, it is desirable to possess it in the form of a carefully drawn wire of uniform circular cross-section. Owing to the difficulty of determining the diameter of very fine wires, it is found more convenient to determine and define the resistivity of metals and alloys by the *resistance in ohms per metre-gramme at* 0°C.—that is to say, by stating the ohmic resistance at 0°C. of a wire of circular cross-section having a length of one metre and weighing one gramme.

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* For a discussion of the chief formulæ which have been proposed to represent the variation in the electrical resistivity of metals, and especially platinum, with temperature, the reader may consult with advantage the following papers :—

"On Platinum Temperatures," by J. D. Hamilton Dickson, *Phil. Mag.*, Dec., 1897. Also

"Notes on Platinum Thermometry," by H. L. Callendar, *Phil. Mag.*, Feb., 1899.

With regard to the temperature variation in resistance of magnetic metals, consult

W. Kohlrausch, *Wied. Ann.*, Vol. XXXIII., p. 42,

J. Hopkinson, *Proc. Roy. Soc.*, Vol. XLV., p. 457,

Le Chatelier, *Comptes Rendus*, Vol. CX., p. 283, and

D. K. Morris, *Phil. Mag.*, Sept., 1897.

For a record of very careful work on the variation of the resistance of metals with temperature, and the representation of results by a parabolic formula of the type $R_t/R_0 = 1 + a \pm bt^2$, see Benoit, *Comptes Rendus*, 1873, p. 342.

The relation between the resistance per centimetre-cube, called the *volume-resistivity*, and the resistance per metre-gramme, called the *mass-resistivity*, is as follows :—

Consider a wire of uniform circular cross-section of s square centimetres and of length l centimetres. Let the density of the material be d, its electrical volume-resistivity in C.G.S. units be ρ, and the total resistance of the wire be R ohms. Then we have

$$10^9 R = \frac{\rho l}{s};$$

also $$lsd = M,$$

where M is the mass of the wire in grammes.

Hence, if $M = 1$, we have

$$lsd = 1, \quad \text{or} \quad s = \frac{1}{ld}.$$

Hence $$10^9 R = \rho l^2 d.$$

If the wire has a length of one metre or 100 cms. we have

$$10^9 R = \rho d 10^4, \quad \text{or} \quad R = \rho \frac{d}{10^5}.$$

Accordingly, if ρ' is the mass-resistivity, or resistance expressed in ohms, of a wire of circular section one metre long and weighing one gramme, we have

$$\rho' = \rho \frac{d}{10^5}, \quad \text{or} \quad \rho = \frac{10^5 \rho'}{10d}.$$

If the mass-resistivity is expressed in *microhms per metre-gramme* $(=10^6 \rho')$, we have the rule : *Microhms per metre-gramme, divided by ten times density, is equal to resistivity per centimetre-cube in C.G.S. units.* Also, if the volume-resistivity of the material, reckoned in *microhms per centimetre-cube*, is denoted by ρ'' we have

$$1,000 \rho'' = \rho.$$

Hence $$100 \rho' = \rho'' d, \quad \text{or} \quad \frac{\rho'}{\rho''} = \frac{d}{100}.$$

Accordingly, *the resistivity of a uniform wire in ohms per metre-gramme is to the resistivity of the material in microhms per centimetre-cube as the density is to* 100. Or, again,

since we have $\rho'' = \rho' \dfrac{100}{d}$, we see that the *volume-resistivity* can always be deduced from the *mass-resistivity* when we know the density of the material.

The mass-resistivity per metre-gramme can always be obtained by measuring the resistance and the mass of any uniform sectioned wire of which the length is known. If, as above, ρ' stands for the mass-resistivity in ohms per metre-gramme and ρ for the volume-resistivity per centimetre-cube, and if R is the resistance in ohms of any uniform sectioned wire of length l centimetres and mass M grammes, we then have

$$10^9 \text{R} = \rho \frac{l}{s} = \rho \frac{l^2 d}{\text{M}} = \rho d \frac{l^2}{\text{M}}.$$

Also
$$10^9 \rho' = \rho \frac{10^4 d}{1}, \quad \text{or } 10^5 \rho' = \rho d.$$

$$\therefore\ 10^9 \text{R} = 10^5 \rho' \frac{l^2}{\text{M}}, \quad \text{or } 10^4 \text{R} = \rho' \frac{l^2}{\text{M}}.$$

$$\therefore\qquad \rho' = \frac{10^4 \text{M}}{l^2} \text{R}.$$

Hence a determination of the length (l), mass (M), density (d), and resistance (R) of any wire enables us to find the mass-resistivity in ohms per metre-gramme (ρ') and the volume-resistivity in C.G.S. units per centimetre-cube (ρ) by the equations

$$\rho' = \frac{10^4 \text{MR}}{l^2}, \quad \cdots \cdots \quad \text{(i)}$$

and
$$\rho = \frac{\rho' 10^5}{d}. \quad \cdots \cdots \cdots \quad \text{(ii)}$$

Hence we have the following practical rules :—

Given the resistance in ohms (R) of a uniform sectioned wire of length (l) in centimetres and mass (M) in grammes, calculate the mass-resistivity (ρ') in ohms per metre-gramme.

Answer.—Multiply together 10,000 times the mass in grammes and the resistance in ohms, and divide the product by the square of the length in centimetres ; the quotient is the ohmic mass-resistivity per metre-gramme.

Example.—The resistance of a column of pure mercury 106·3 centimetres in length, and weighing 14·4521 grammes, is one ohm at 0°C. ; find the mass-resistivity of mercury.

Answer.—12·789 ohms per metre-gramme at 0°C.

Again, given the mass-resistivity in ohms per metre-gramme (ρ'); find the volume-resistivity (ρ) in C.G.S. units per centimetre-cube.

Answer.—Multiply the ohmic mass-resistivity by 100,000, and divide the product by the density (d).

Example.—Given that the density of mercury at 0°C. is 13·595; find the volume-resistivity in C.G.S. units, knowing the mass-resistivity to be 12·789 ohms per metre-gramme.

Answer.—94,070 C.G.S. units per centimetre-cube at 0°C.

To determine the resistivity of a metallic material in the form of a wire of uniform section, we must, therefore, find the total resistance, length, mass, and density of the wire. The density may be taken with appropriate samples on the whole wire. It is convenient to proceed in the following manner :—

If the length and diameter of the wire furnished is such that the total resistance is not much less than one ohm, the wire may be laid in a flat circular coil the turns of which are prevented from touching each other by winding the wire on a suitable bobbin or frame. The ends of the wire must be soldered to thick copper terminal rods or leads by which it can be connected to the bridge. The wire must be immersed in a bath of paraffin oil kept continually stirred. The resistance of the wire is then carefully taken at several temperatures. It is best to take it at the temperature of the room and also at or near 0°C. by cooling the paraffin oil by ice. In this manner the temperature-coefficient becomes known.

Having satisfactorily ascertained the resistance of the wire at known temperatures, it is cut off close to the thick terminal rods, and these are soldered together and the resistance of the leads determined in order that a correction may be applied for lead resistance. The length of the wire used has then to be determined. This must be done without in the least degree stretching the wire, and is best achieved by pressing the wire gently into a shallow groove made with a saw in the surface of a long board. This groove holds the

wire straight and enables its length to be measured with an accurate metre scale. The length and resistance having been found, the wire may be cut up into sections and the density or specific gravity found in the usual way. If the wire is a fine wire it may be wound up into a sort of loose ball, and the specific gravity of the whole mass determined. The great difficulty which here occurs is that of the removal of all the air which is entangled in, or adherent to, the wire. The best way to proceed is as follows:—The hank of wire is suspended by a fine horse-hair from the beam of a delicate chemical balance and the weight taken. Let this weight be W grammes. The wire is then placed in distilled water and well boiled, and allowed to remain in the water whilst the water cools. The wire is then weighed again in the water without removing it from the water. Let the weight then be W′ grammes. The density d is equal to $\dfrac{W}{W-W'}$. If the wire is of such a material that it is chemically affected by being boiled in water at 100°C. the adherent air must be removed by gently heating the water and then placing it under the receiver of an air pump and exhausting the air from around it. The water will then boil under a reduced pressure at a much lower temperature. The presence of any air-bubble in contact with the wire when weighed in water will render the apparent density too small. In both weighings a correction must be applied by weighing the horse-hair suspension alone and deducting this from the weight of the mass. It is more difficult to get correct results the smaller the mass of the wire. Hence the mass of the material weighed should be as great as possible. The density d being obtained and the mass M of the wire in grammes used for the resistance measurement, as also the length l in centimetres, we have the means of determining the mean cross-section s of the wire for

$$lsd = M.$$

Hence

$$s = \frac{M}{l\,d}.$$

In the case of very fine circular-sectioned wires a close approximation can be made to the mean diameter of the wire by measuring with a microscope-micrometer the mean diameter in a large number of different places and positions: and taking the mean of all these measurements.

Having determined in one way or other the mean diameter of the wire, we have the volume-resistivity ρ in C.G.S. units given by the equation,

$$10^9 R = \frac{\rho l}{s} = \rho \frac{l^2 d}{M},$$

or

$$\rho = \frac{M R 10^9}{l^2 d}. \quad \cdots \quad \text{(iii.)}$$

Tables of the volume-resistivities of various metals and alloys are given at the end of the present chapter. (*See* Tables I., II., III., IV., V. and VII.)

§ 12. Determinations of Volume and Mass-Resistivity of Metals and Alloys.

—Great labour has been expended on the determination of the mass and volume-resistivity of metals and certain standard alloys. Considerable differences are, however, found in the values assigned by various authorities to the volume-resistivities at 0°C. of the various metals. The reason for this is that exceedingly minute proportions of impurity or other metals have an immense effect upon the electrical resistivity. Some metals, such as iron, have probably never yet been obtained in an absolutely pure annealed condition.

The work on this subject to which reference is most often made is that of Dr. A. Matthiessen, who published between 1860 and 1864 the results of numerous researches on electrical conductivity recorded in the *Philosophical Magazine*, in the *Philosophical Transactions* of the Royal Society, and the *Reports* of the British Association for those years.

For most electrical engineering purposes the constants of greatest importance are those of the mass and volume-resistivity of pure copper in its hard-drawn and annealed

conditions.   Matthiessen's value for the mass-resistivity of pure hard-drawn copper is called *Matthiessen's Standard.*

Matthiessen's Standard is defined in the following statement* :—

The resistance of a wire of pure hard-drawn copper, one metre long, weighing one gramme, is 0·1469 *British Association units at* 32°F.

Since the numerical value of a resistance measured in standard or international ohms is, by authoritative definition, equal to 0·9866 of the value of the same resistance stated in B.A. units, the above may be modernised into the following fundamental definition :—

The resistance of a wire of pure hard-drawn copper, one metre long, and weighing one gramme, is 0·14493 *standard ohms at* 0°C.

Since resistances are not conveniently determined at 0°C., but better at 60°F. or 15·55°C., the definition has again been re-cast in form by the Committee on Copper Conductors appointed to consider this question of Copper Conductivity Standards† as follows :—

Matthiessen's standard for hard-drawn high-conductivity commercial copper shall be considered to be a wire of pure hard-drawn copper one metre long and weighing one gramme, whose resistance at 60°F. *is* 0·153858 *standard ohms.*

Hard-drawn copper is defined as that which will not elongate more than 1 per cent. without fracture.

The conversion from B.A. units at 32°F. to standard ohms at 60°F. is made by employing a formula given by

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* British Association *Report*, 1864, or *Phil. Mag.*, Vol. XXIX., May, 1865, p. 362.

† The Committee on Copper Conductors was organised in 1899 by writing to the General Post Office and the Institution of Electrical Engineers and inviting them to send delegates to meet the representatives of eight of the principal manufacturers of insulated copper cables to consider and come to an arrangement as to the standard to be adopted for Copper Conductivity. The sittings of the Committee were held in London.   The Secretary was Mr. A. H. Howard.   The recommendations of the Committee have been adopted by the General Post Office and the chief cable manufacturing companies.   The Report of the Committee is published in the *Journal* of the Institution of Electrical Engineers, January, 1900, p. 169.

Matthiessen for the temperature coefficient of copper, viz. :—

$$R_t = \frac{R_{32}}{1 - 0.00215006\ (t-32) + 0.00000278\ (t-32)^2},$$

and the rule :—

Resistance in B.A.U. $\times$ 0.9866 = resistance in standard ohms. Hence, we have for *hard-drawn copper wire* the values below for the metre-gramme resistivity (ρ') :—

ρ' (at 32°F.) = 0.1469 B.A.U.

ρ' (at 0°C.) = 0.1449 standard ohms.

ρ' (at 60°F.) = 0.1539 standard ohms.

Matthiessen also measured the mass-resistivity of annealed copper in the form of wire. Annealed copper may exist in various states of annealing, and in this condition its resistivity is less by slightly variable amounts than in the hard-drawn condition. The resistivity of annealed copper was measured by Matthiessen at an earlier date (*Phil. Trans.* Roy. Soc., 1860, p. 86), and he found that it showed a conductivity greater than that of hard-drawn copper by about 2.26 per cent. to 2.5 per cent. Although this is undoubtedly correct for carefully annealed copper, it is a result which cannot be obtained in practice with ordinary commercial copper wire, as this latter is unannealed, or hardened somewhat by bending and winding in the process of manufacturing it into covered wire. In practice it is found that the resistance of commercial annealed copper wire is about 0.9875, or $1\frac{1}{4}$ per cent. less than that of hard-drawn wire of the same length and section.

Matthiessen, however, gave a later value for annealed copper as follows :—

The resistance of a wire of annealed copper one metre long and weighing one gramme is 0.1440 *British Association units at* 32°F.

In modern standard units the definition reads thus :—

The resistance of a wire of annealed copper one metre long and weighing one gramme is 0.1421 *standard ohms at* 0°C.

The ratio of the numbers 0·1421 to 0·1449 is 0·98025.

This value has accordingly been accepted by the Committee on Copper Conductors, and the ratio of the resistivity of soft or annealed to hard-drawn copper is taken at the above value, so that

$$\text{The resistivity of pure annealed copper} = 0.98025 \times \text{the resistivity of pure hard-drawn copper.}$$

Hence the Committee have formulated the definition for the standard for soft or annealed copper as follows :—

Matthiessen's standard for annealed high-conductivity commercial copper shall be considered to be a wire of pure annealed copper one metre long and weighing one gramme, whose resistance at 60°F. is 0·150822 *standard ohms.*

Employing the same temperature coefficient as for hard-drawn copper, we have the following values for the metre-gramme resistivity of annealed copper :—

$$\rho' \text{ (at } 32°F.\text{)} = 0.1440 \text{ B.A.U.}$$
$$\rho' \text{ (at } 0°C.\text{)} = 0.1421 \text{ standard ohms.}$$
$$\rho' \text{ (at } 60°F.\text{)} = 0.1508 \text{ standard ohms.}$$

It is to be noted that this standard for annealed copper is based upon an assumption as to the relative conductivities of hard and annealed copper. The figures given by Matthiessen for this ratio vary considerably in different Papers. See *Phil. Trans.*, 1860, p. 86; *Phil. Trans.*, 1864, p. 197; and *Phil. Mag.*, May, 1865, p. 363; also *The Electrician*, Vol. XLV., p. 59.

The next question with which we are concerned is the equivalents of the above numbers in volume-resistivity. The specific gravity of copper varies from 8·89 to 8·95, and the standard value which is now accepted for high-conductivity commercial copper is 8·912, corresponding to a weight of 555lb. per cubic foot at 60°F. Hence, multiplying the values for the metre-gramme resistivity by 100 and then dividing by 8·912, we have the corresponding volume-resistivities in

microhms per centimetre-cube for pure commercial hard-drawn and pure annealed copper as follows :—

The volume-resistivity of pure hard-drawn copper at $0°C.$ = $1·626$ *microhms per centimetre-cube, or* $1,626$ *C.G.S. units.*

The volume-resistivity of pure annealed copper at $0°C.$ = $1·594$ *microhms per centimetre-cube, or* $1,594$ *C.G.S. units.*

A table giving the resistances of annealed copper wire calculated on the basis of the above value is given at the end of this chapter. (*See* Table XIII.)

The calculation of the resistivity at any other temperature (t) requires a knowledge of the *temperature coefficient* (T.C.) for copper. Matthiessen's formula for the reduction of the resistivity at $t°\text{F.}(\rho_t)$ to the resistivity (ρ_{32}) at 32°F. was :—

$$\rho_{32F} = \rho_t \ (1 - 0·00215006 \ (t-32) + 0·00000278 \ (t-32)^2),$$

or approximately $\rho_t = \rho_0 \ (1 + 0·00387 \ t')$, if $t' = $ Centigrade temp.

It is now known that this correcting factor is rather too small, owing to the improvement in the quality of copper made since Matthiessen's time. Hence the Committee on Copper Conductors have recommended that the *average temperature coefficient* between the temperatures 30°F. and 100°F. shall be that value obtained by Messrs. Clark, Forde and Taylor, as given in a pamphlet published by them on February 20th, 1899, viz., 0·00238 per degree Fahr., or 0·00428 per degree Centigrade*, so that for Centigrade temperatures we shall have

$$\rho_t = \rho_0 \ (1 + 0·00428t),$$

or

$$\rho_{60F.} = \rho_{32F.} \times 1·06665.$$

Since the date when Matthiessen's work was carried out the most careful research carried out on the conductivity of copper is that by Mr. T. C. Fitzpatrick, described in a Paper read before the British Association at Leeds† in 1890. Mr. Fitzpatrick's experiments on the conductivity of hard-

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* This is precisely the same mean value for the temperature coefficient of annealed copper as was independently obtained by Profs. Fleming and Dewar. See *Phil. Mag.*, Sept., 1893, p. 299. Messrs. Clark, Forde and Taylor's pamphlet is published by "The Electrician" Printing and Publishing Co.

† See *Report* B.A., Leeds, 1890 ; Appendix III. to the *Reports* of the Committee on Electrical Standards for 1890 ; also *The Electrician*, Vol. 25, p. 608 1890.

drawn copper led him to confirm exactly Matthiess·
for the resistivity (per metre-gramme) of a wire of h²
copper wire taken at 18°C., viz., 0·1571 ohms p·
gramme. Also, he found substantially the same p·
difference as Matthiessen did between the metr·
resistance of a hard-drawn copper wire and that of
wire when annealed and soft.

Mr. Fitzpatrick found, as others have done, great ·
in the specific gravity of copper, so that wires having
mass-resistivity per metre-gramme do not give   t
volume-resistivity per centimetre-cube at the same ter·
Matthiessen therefore expressed all his results in m
tivity, believing it to be more accurate and affording
definition of the real conductivity.

In addition to the determination of the conduc
copper, Matthiessen carried out very elaborate resea·
the electrical conductivity of different metals and
researches which are justly regarded as classical. A·
times numerous other observers have obtained value
volume-resistivities of different metals and alloys.

A table of resistances of platinoid and manganin
various sizes is given at the end of this chapte·
Tables XII. and XIII.)

As regards the metals, with the exception of silver
and mercury, and one or two others, such as tin an
obtained without much difficulty in a state of chemical
it is found that the greatest differences exist betw·
electric conductivities as found by different observers
is due partly to the influence of minute proport
impurity, and the all but impossibility of obtaining
metals, such as iron, in a state of absolute chemical
It is also due to differences in physical condition.
in the case of platinum and nickel, which are cap
being prepared almost, if not quite, perfectly pu
greatest differences are found in the values assign
different observers to the electric conductivity. In the

at the end of this chapter are given Matthiessen's results for the principal metals reduced to express them in terms of the standard ohm; also values obtained from a very extensive series of observations made in 1892 and 1893 by Fleming and Dewar.* (*See* Table V.)

In these last-mentioned researches great care was taken to obtain the metals in the highest state of chemical purity. These materials were drawn into uniform wires and the diameters of these wires measured by a micrometer method as described in the above-mentioned papers. Hence the measurement made was a volume-resistivity determination. The resistances of these various wires were measured over a range of temperature lying been $+200°C$. and $-200°C$., this latter temperature being obtained by the use of liquid air. The temperatures were measured by means of a platinum thermometer. A table of densities (*see* Table I.) of the principal metals is added to enable mass-resistivity to be converted to volume-resistivity.

The differences which occur in the values assigned by different authors and experimentalists to the volume-resistivity of the various metals are in some cases very great.

It appears as if, in some metals, very considerable proportions of impurity or changes in physical state make but little difference in the electric conductivity, whereas in the case of other metals very great differences are created in the electric conductivity by the presence of mere traces of other metals. Matthiessen showed that metallic alloys may be broadly divided into two classes :—

(i.) Those in which an admixture or alloy composed of them has a very much higher resistivity than either or any of the constituents. To this class belong alloys of copper, silver, gold, aluminium, platinum, nickel and most other metals.

(ii.) On the other hand, alloys which consist only of two or more of the following metals, viz., lead, tin, zinc, and cadmium, have a conductivity which is nearly the mean of those of their constituents, and may be roughly calculated from the proportions in which the elements are mixed.†

In the case of alloys of these last four metals (ii.), the resultant or mean conductivity C can be calculated by the formula

$$C = \frac{c_1 v_1 + c_2 v_2 + \&c.}{v_1 + v_2 + \&c.},$$

* See *Phil. Mag.*, Oct. 1892 and Sept. 1893. † Matthiessen, B.A. *Report*, 1864.

while $c_1 c_2$, &c., are the specific conductivities of the constituents, and $v_1 v_2$, &c. are the volumes of the respective constituents.

For alloys containing only two or more of the above-named metals, viz. zinc, lead, tin and cadmium, the mean density D of the alloy is also obtainable from the densities $d_1 d_2$, &c., of the constituents, and their relative volumes $v_1 v_2$, &c., by a similar formula, viz. :

$$D = \frac{d_1 v_1 + d_2 v_2 + \&c.}{v_1 + v_2 + \&c.}.$$

The alloys formed with metals of class (i.) with one another have a much inferior conductivity to that of any of the components, but as the percentage of one component rises to 100 the conductivity of the alloy rises also very quickly to that of the pure metal.

Alloys of metals taken partly from class (i.) and partly from class (ii.) have a specific gravity and conductivity which is always less than that of the mean. If a metal of class (ii.) is alloyed with a considerable percentage of a metal from class (i.) the conductivity is not much altered, but if a metal from class (i.) is alloyed with a very little of a metal from class (ii.) the conductivity is very much reduced. Hence we find an immense effect produced by the presence of a little zinc in reducing the conductivity of pure copper, or of lead in reducing that of pure silver. Accordingly large variations may be expected in the determinations of resistivity in the case of chemically-prepared metals which are not easily prepared pure.

The reader may be referred to the following Papers and sources of information for additional knowledge on the subject of resistivity measurement :—

"The Specific Resistance of Pure Copper." By Messrs. J. W. Swan and Rhodin. *The Electrician*, Vol. XXXIII., p. 303.

As the mean of a number of determinations of the volume-resistivity made with hard-drawn and annealed copper the authors find the following values for the resistivity (ρ_0) at 0°C. in C.G.S. units and the temperature coefficient (α).

$$\text{Hard-drawn copper } \rho_0 = 1603 \quad \alpha = 0{\cdot}00408.$$
$$\text{Annealed copper } \quad \rho_0 = 1563 \quad \alpha = 0{\cdot}00416.$$

The density of the copper at 15°C. was 8·96.

"The Electrical Conductivity of Aluminium." By Messrs. J. W. Richards and J. A. Thomson. *The Electrician*, Vol. XXXVIII., p. 801.

The authors find the volume resistivity at 0 to be as follows :—

$$\text{Hard-drawn aluminium } \rho_0 = 2684$$
$$\text{Annealed aluminium } \quad \rho_0 = 2659$$

"The Electric Conductivity of Steels." By M. Campredon. *The Electrician*, Vol. XXVIII., p. 345.

The author gives a useful table of the electric conductivity of steels of given composition. The resistance of a wire 1mm. diameter and 1 kilometre

long at 15°C. would be 21 ohms if of copper and 125 ohms if of iron. The resistivity of steel is lower in proportion as the purity and softness is greater. Manganese is the element which has the greatest effect in raising the resistivity.

"The Electric Resistance of Copper at Low Temperatures." By Wroblewski. *The Electrician*, Vol. XXI., p. 432.

"Electric and Thermal Conductivity." By H. F. Weber. *The Electrician*, Vol. VII., p. 6.

A remarkable connection is established between the electrical conductivity (K), the thermal conductivity (H), and the specific heat per unit of volume (S). This relation is $\frac{H}{K} = a + bS$, where a and b are constants.

"Electric Conductivity and Atomic Volume." By W. P. Granville. *The Electrician*, Vol. XXI., p. 381.

An attempt to establish a relation between these quantities.

"The Electrical Resistivity of Silicon." *The Electrician*, Vol. XL., p. 580.

"The Conductivity of Cement and Concrete." By Dr. St. Lindeck. *The Electrician*, Vol. XXXVI., p. 788.

Useful data on the electrical conductivity of road-making materials.

§ 13. Determination of Low Resistances by Fall of Potential.

—For the determination of very low resistances many of the above-described bridge methods are not applicable. If it is desired to measure the resistance of a dynamo armature or transformer coil or short length of electric lighting cable, the uncertain resistance at the contacts of any conductors used in connecting the resistance to be measured to other circuits would perhaps be greater than the whole resistance to be measured.

In the case of such low resistances one method which may be adopted is that of measuring the resistance by the fall of potential down it when a known current is sent through it. The resistance to be measured is joined in series with a known low resistance standard, say one-tenth or one-hundredth of an ohm. A suitable adjustable resistance is then added in series with the two above-mentioned resistances, and a few cells of

a primary or secondary battery are employed to send a current through the circuit. The terminals of a high resistance galvanometer, or a galvanometer having a resistance of 1,000 or 2,000 ohms in series with it, are then connected, first to the terminals of the known low resistance and next to the terminals of the unknown low resistance. In each case the galvanometer deflection is noted. It is desirable to reduce the current in the circuit to such a value that neither of these galvanometer deflections is very large. The ratio of these scale deflections then gives us the ratio of the unknown to the known deflection, on the assumption, which must previously be justified, that the deflections of the galvanometer are proportional to the current flowing through it.

The methods of testing the correctness of this assumption in the case of the galvanometer used will be given in the chapter on CURRENT MEASUREMENT. Meanwhile, assuming it to be the case, let G denote the *galvanometer constant*, or the number by which the scale deflection of the galvanometer must be multiplied to give the current in amperes flowing through it. Let R_g be the galvanometer resistance, R_s the resistance of the known standard, and R the resistance to be determined. Also let V_s be the potential difference (P.D.) between the terminals of the standard resistance, and V that between the ends of the unknown resistance. If, then, D_s and D are the galvanometer scale deflections in the two cases, we have, by Ohm's law,

$$\frac{V}{R_g} = GD, \quad \text{also} \quad \frac{V_s}{R_g} = GD_s, \quad \text{and} \quad \frac{V}{R} = \frac{V_s}{R_s},$$

since the current in the main circuit is everywhere the same. Hence, from the above equations,

$$\frac{D}{D_s} = \frac{R}{R_s}, \quad \text{or} \quad R = R_s \frac{D}{D_s}.$$

Accordingly, the value of the resistance being measured is given as the product of the value of the standard resistance (R_s) and the ratio of the two scale deflections D and D_s.

In practically measuring, for instance, the resistance of a dynamo armature, we must first give a rough guess at the probable *order* of the resistance. Let it be ascertained to be something of the order of 0·01 of an ohm. We should then select as a standard resistance a resistance strip having a resistance of 0·01 ohm. This may be a suitable strip or strand of manganin or platinoid. The strip is then joined in series with the armature by connecting it to the brushes or terminals, and a few dry cells may be employed to send a current through the circuit. The ends of a pair of wires are then connected to the terminals of the standard resistance strip, and a pair to the terminals of the armature. These last may preferably be pressed into contact with opposite sections of the commutator by being placed under the brushes, if the armature being measured is a continuous-current armature. The other ends of these wires are conveniently brought to a set of six mercury cups mounted on a board, so that, by changing the position of a pair of copper bridge pieces, either of the potential wires can quickly be brought into connection with the galvanometer.

It is necessary to make both the measurements quickly after one another, and to ascertain that the main current has not altered in the meantime. It is best, therefore, to take several readings of the values of D and D_s alternately. If, say, D is taken first, and we find a value D_1, and then D_s is taken, and after an equal interval D is taken again, and we find a value D_2, the scale reading which must be taken for D is half the sum of D_1 and D_2, or its mean value, and it may be assumed that this is the proper value corresponding to D_s. In any case, a large number of such readings should be taken and the value ultimately accepted, for the ratio of $D : D_s$ should be the mean of a large number of observations.

An essential condition of success in this method is that the galvanometer circuit shall be so high that its connection as a shunt on one of the low resistances does not sensibly alter the potential difference of the terminals of the latter.

§ 14. Measurement of Low Resistances by the Matthiessen and Hockin Bridge.

—We may employ the slide wire bridge with a carefully calibrated or uniform slide wire to compare together two resistances which are small, one of them having a known value. Let the resistances be arranged in bridge form as in Fig. 36.

Let PR be the slide wire extended electrically by the resistance coils A and C, and let a battery of a few dry cells be joined up to points x and y. Let ab and cd be the low resistances to be compared joined in so as to complete the four arms of the bridge. Let G be a galvanometer of high or not very low resistance but great sensitiveness, and let
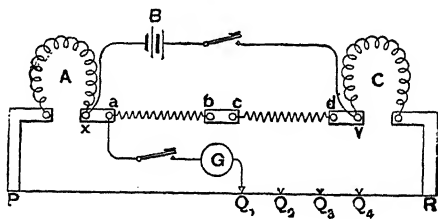

FIG. 36.

contact keys be inserted in the galvanometer and battery circuits. The following operations are then performed :—

Connect the galvanometer G in between the sliding contact on the bridge wire and the point a. Find a point Q_1 on the slide wire such that the galvanometer shows no current. Then we have

$$\text{(i.)} \qquad \frac{xa}{ay} = \frac{A + PQ_1}{C + Q_1R}, \quad \text{and hence} \quad \frac{xa}{xy} = \frac{A + PQ_1}{A + C + PR},$$

where xa stands for the resistance of the conductor between points x and a, and A stands for the resistance of the conductors between the points x and P, and so forth for the other points.

We then connect the galvanometer in between points b and the slider, and find a second position of the slider at which the

galvanometer indicates no current.   Similarly we have,

(ii.) $\dfrac{xb}{xy} = \dfrac{A + PQ_2}{W}$,   where $W = A + C + PR$.

Hence from (i.) and (ii.) we find

$$ab = xb - xa = \frac{xy}{W}(Q_1 Q_2).$$

Also in the same way we find

$$cd = \frac{xy}{W}(Q_3 Q_4)$$

where $Q_3 Q_4$ are the balancing positions on the slide wire when the galvanometer is connected to the points c and d respectively.

Accordingly, we find the ratio of ab to cd to be

$$\frac{ab}{cd} = \frac{Q_1 Q_2}{Q_3 Q_4}.$$

In other words, assuming the uniformity of the slide wire, we have

$$\frac{\text{The resistance between } ab}{\text{The resistance between } cd} =$$

$$\frac{\text{The length of the slide wire between } Q_1 \text{ and } Q_2}{\text{The length of the slide wire between } Q_3 \text{ and } Q_4}$$

It may so happen that the resistances A and C will have to be changed between the readings, so as to make all the balancing positions come on the slide wire.   If in the case of the four readings of the resistance A takes the values A_1, A_2, A_3 and A_4, and the same for the resistance C, then, provided $A_1 + C_1 = A_2 + C_2$, the four equations will take the form

$$\frac{xa}{ay} = \frac{A_1 + PQ_1}{C_1 + Q_1 R}, \text{ or } \frac{xa}{xy} = \frac{A_1 + PQ_1}{A_1 + C_1 + PR}.$$

Also $\dfrac{xb}{xy} = \dfrac{A_2 + PQ_2}{A_2 + C_2 + PR}$, and hence $ab = \dfrac{xy}{W}(A_2 - A_1 + Q_1 Q_2)$,

where W, as before, is the constant resistance $A_1 + C_1 + PR$.

In the same way we can find that

$$cd = \frac{xy}{R}(A_4 - A_3 + Q_3 Q_4).$$

Hence,

$$\frac{ab}{cd} = \frac{A_2 - A_1 + Q_1 Q_2}{A_4 - A_3 + Q_3 Q_4}.$$

If $A_1 = A_2 = A_3 = A_4$, the last equation reduces as before to

$$\frac{ab}{cd} = \frac{Q_1 Q_2}{Q_3 Q_4}.$$

If, then, cd is a known low resistance, we obtain the value of ab in terms of cd and the ratio of two lengths of a uniform slide wire.

The accuracy of the above method is not dependent upon the absence of any variation of the battery current. It can be carried out with the current supplied from public supply circuits if necessary.*

§ 15. **The Kelvin Double Bridge.**—The Kelvin double bridge is an arrangement of nine conductors joining six points and having a source of electromotive force in one branch and a galvanometer in another.

Let $P, Q, R, S, B, G, a, b, c$ (Fig. 37) be the nine conductors, B being the battery branch and G the galvanometer branch. Let the above letters stand for the respective resistances of these branches.   It will be seen that if the conductor c is cut at any point x the arrangement then becomes a simple Wheatstone's bridge. It is always possible to find some point x in the conductor c dividing c into two segments, a and β, such that the point x and the two points x_1 and x_2 are all at the same potential. When this is the case we must have the relation $\frac{a}{\beta} = \frac{a}{b}$. For,

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* The above method was reproduced in *The Electrician*, July 1, 1898, Vol. XLI., p 320, as a new method due to Messrs. Müller and Wallau. It was, however, described many years previously in Maxwell's Treatise on "Electricity and Magnetism," and is due to Messrs. Matthiessen and Hockin.

under the above conditions, the current along the conductor a must be the same as the current along b.   Also, for the same reason, we must have the relation

$$\frac{R}{S} = \frac{P + \dfrac{1}{\dfrac{1}{\alpha} + \dfrac{1}{a}}}{Q + \dfrac{1}{\dfrac{1}{\beta} + \dfrac{1}{b}}}.$$

Or, since $\alpha + \beta = c$, the above becomes

$$\frac{R}{S} = \frac{P + \dfrac{a}{1 + \dfrac{a+b}{c}}}{Q + \dfrac{b}{1 + \dfrac{a+b}{c}}}.$$

If, then, we make $a + b = c$, we have

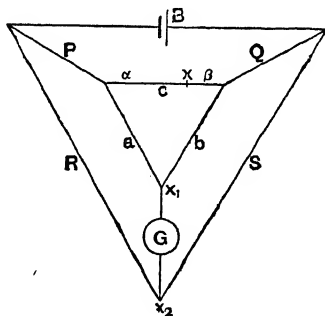$$\frac{R}{S} = \frac{P + \dfrac{a}{2}}{Q + \dfrac{b}{2}}.$$

Fig. 37.—Diagram of Kelvin Bridge.

Under this last condition the Kelvin bridge becomes modified into a form called the Thomson (Kelvin) and Varley

slide. This latter instrument may be described in principle as follows:—

Let two slide wires XY, X'Y' (Fig. 38) be stretched over scales parallel to one another. Let a battery B be attached to the terminals of one slide wire, XY, so as to create a fall of potential down it. Along the wire XY a slider is arranged to move having double contact edges, so as to make contact at two places, nn' on XY, separated by a constant resistance or length of slide wire, nn'. Let the resistance from X to n be P, that from Y to n' be Q, and let nn' be c. The second slide wire X'Y' must have a resistance equal to $a+b=c$, and
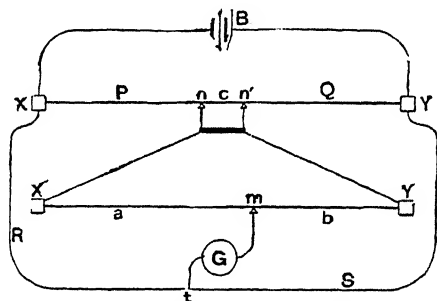

FIG. 38.—Diagram showing arrangement of Conductors in Kelvin and Varley Slide.

it has a slider m running on it and making contact at any point, and thus dividing it into two sections a and b. A pair of adjustable resistances R, and S, are connected to the points X and Y, and from their common junction, t, a galvanometer having a resistance G is connected in between the slider m and t. The double contact slider nn' has its contacts connected to the ends of the wire X'Y'.

In Fig. 38, showing the diagram of the above arrangement of circuits, the letters denoting resistances are the same as those used in Fig. 37, showing diagrammatically the form of the Kelvin bridge.

In the laboratory the practical form of this double bridge is known as the *Kelvin and Varley slide.* In this instrument (shown in Fig. 39) the slide wire XY consists of 101 coils of wire in series, each of 1,000 ohms. The common junctions of these coils are brought to terminal pins on the top board of the instrument, which is generally an ebonite slab. The wire X'Y' is represented by a similar series of 100 coils of 20 ohms each so that the whole resistance of X'Y'=2,000 ohms = that of two coils of XY. The double contact piece *nn'* is represented by a double branched revolving arm which as it moves round makes contact with
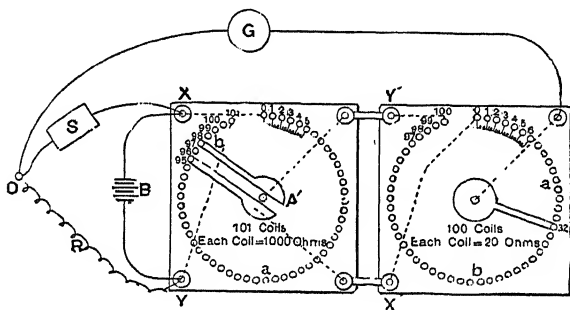

FIG 39—Kelvin and Varley Slide.

a pair of studs including between them two of the 1,000 ohm coils in series. This interval of 2,000 ohms corresponds with the resistance *c* in the diagrams.

The resistances to be compared are the R and S in the diagrams, and these are represented in practice by a plug resistance box and the unknown resistance to be determined. The revolving contact arms of the two resistance boxes, which represent the wires XY and X'Y' are then moved round to touch the various contact pins until positions are found where the galvanometer connected in circuit as indicated shows no current.

T

If R is the unknown resistance and S is the standard or resistance of the plug box, we have

$$\frac{R}{S} = \frac{P + \frac{a}{2}}{Q + \frac{b}{2}}$$

If the point n is indicated by a scale reading N_1 on the first box and the point m by a scale reading N_2 on the second box

we have $P = 1,000 \ N_1$ ohms,

and $a = 20 \ N_2$ ohms.

Also, we have $Q = 1,000 \ (99 - N_1)$,

and $b = 20 \ (100 - N_2)$.

Accordingly $P + \dfrac{a}{2} = 1,000 \ N_1 + 10 \ N_2$,

and $Q + \dfrac{b}{2} = 1,000 \ (99 - N_1) + 10 \ (100 - N_2)$

$= 100,000 - (1,000 \ N_1 + 10 \ N_2)$;

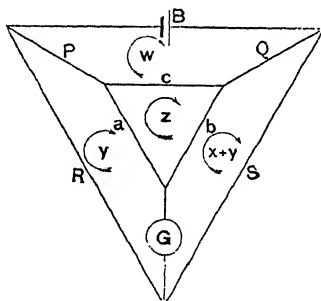or $\dfrac{R}{S} = \dfrac{1,000 \ N_1 + 10 \ N_2}{100,000 - (1,000 \ N_1 + 10 \ N_2)}$

FIG. 40.

The theory of the Kelvin bridge is as follows: Referring again to Fig. 37, it is easy to show that the absence of a current in the galvanometer circuit necessitates a certain relation between the values of the various resistances. Assign letters $\overline{x + y}$, y, z, and w to denote the imaginary cycle currents in the meshes of the bridge arrangement (*see* Fig. 40) and consider the network so formed of the nine conductors.

Let imaginary currents $x + y$ and y circulate clockwise round the circuits bounded by the resistances R, G, P, a and G, b, Q, S. Then the actual current through the galvanometer is x. Form, as before described, the cycle equations by the method of Maxwell, and we have

$$(B + P + Q + c)w - Q\overline{x+y} - Py - cz = E$$
$$(G + S + Q + b)\overline{x+y} - Gy - Qw - bz = 0$$
$$(G + R + P + a)y - G\overline{x+y} - Pw - az = 0$$
$$(a + b + c)z - b\overline{x+y} - ay - cw = 0.$$

Re-arranging terms, we have the four equations in x, y, z, w as follows :—

$$- Qx - (P + Q)y - cz + (P + Q + B + c)w = E$$
$$(G + S + Q + b)x + (S + Q + b)y - bz - Qw = 0$$
$$- Gx + (R + P + a)y - az - Pw = 0$$
$$- bx - (a + b)y + (a + b + c)z - cw = 0.$$

The solution for x (the galvanometer current) is then

$$x = \frac{E}{D} \begin{vmatrix} (S + Q + b), & -b, & -Q \\ (R + P + a), & -a, & -P \\ -(a + b), & (a + b + c), & -c \end{vmatrix},$$

where D in the denominator is a determinant whose value does not concern us. The above equation writes out into

$$x = \frac{E}{D}\{(a + b + c)(RQ - SP) + c(Rb - Sa)\}.$$

Hence the condition that the galvanometer shall show no current, or that x shall be zero, is that the relations $RQ = SP$ and $Rb = Sa$ must simultaneously hold good. If, however, c is a very small resistance, then the galvanometer current will be very nearly zero if $RQ = SP$, even though Rb is not quite equal to Sa.

Accordingly, in the Kelvin double bridge we have a double relation which must hold good in order that the bridge may be balanced. We must have $\frac{P}{Q} = \frac{R}{S}$ as a relation between the resistances P, Q, R and S, and also $\frac{R}{S} = \frac{a}{b}$ as a relation between the resistances a, b, R and S.

§16. Modifications of the Kelvin Double Bridge for Low Resistance Measurement. Practical Forms.—The Kelvin double bridge can be arranged so as to be a convenient instrument for measuring low resistances, such as lengths of electric arc light carbons or short lengths of copper cable. One form which it then takes is as follows :—

The resistance to be measured is clamped in between massive clamps, C_1 C_2, fixed on a board, and contact knife edges, e_1 e_2, arranged to press against it, intercepting any required length of the conductor. A slide wire, A_1 A_2, is fastened to the same board, and on it move two sliders, v_1 v_2,

one or both of which are movable, which can make contact with the wire at any position. In the centre of the board are two plug resistance boxes of series pattern, having resistances 10, 10, 100, 1,000 ohms each, and each provided with two plugs. The galvanometer G and battery B (which should be a couple of storage cells) are connected in as shown. The resistances in Fig 41 are lettered to correspond with those in Fig. 37.

In the first place, the plugs are inserted so as to make the resistances R and S in the same ratio as *a* to *b*, viz., either 10 : 10, 10 : 100, or 10 : 1,000. The sliders v_1 and v_2 are then moved until the galvanometer shows no current. Calling the
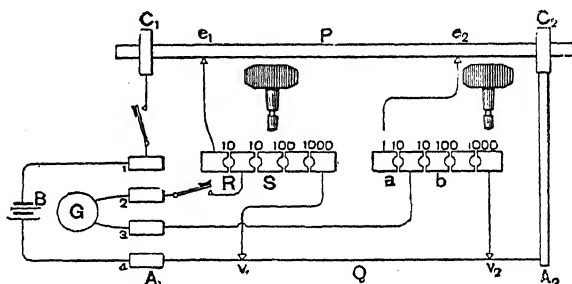

Fig. 41.

ratio of $R : S = a : b = n$, we then have $P = Qn$. If the slide wire has been calibrated so that its resistance per centimetre of length is known, we have at once the resistance of P in terms of a certain length of slide wire, $v_1 v_2$.

Another form of the Kelvin bridge adapted for the purpose of low resistance measurement has been devised by Mr. J. H. Reeves.* In this apparatus A and E (*see* Fig. 42) are two massive pieces of copper, which can be joined by a plug when desired. To one is connected the wire or bar EFGH of which the resistance is to be determined, and to the other a

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*See *Proc.* Phys. Soc. Lond., Vol. XIV., p. 166.

comparison wire ABCD of known resistance per unit of length. A pair of contact edges K and M make contact with the wire FG at two places L and N of known distance apart. It is convenient to make the distance LN equal to one metre. A pair of connecting wires BO, CT are soldered to the wire BC intercepting a known resistance, which may be conveniently 0.01 ohm. The same board or another carries a calibrated slide wire, *ab*, the terminals of which are connected to O and K.

A pair of adjustable resistances *x* and *y* are connected to M and T. These last may be ordinary resistance boxes, one say *y*, to 1,000 ohms, and the other is a series plug box
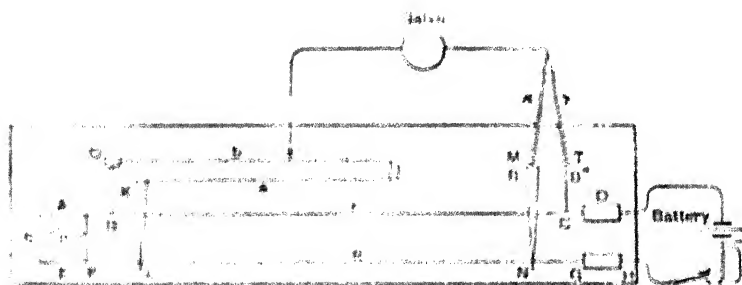

Fig. 42.

reading from 1 to 5,000 ohms. Between the junction of *x* and *y* and the slider on the slide wire is joined in a galvanometer, and a battery is connected to the terminals D and H. If the plug P is removed so that the blocks A and E are not connected, the arrangement is an ordinary Wheatstone's bridge. The slider *s* can be then adjusted so as to divide the slide wire into two segments *a* and *b* such that the resistance *a* is to that of *b* as *x* is to *y*. This being done, the plug P is inserted to connect A and E. The arrangement then becomes a Kelvin double bridge. If the resistance of the wire under test between L and N is called R, and if that of the standard wire between B and C is called *z*, we can then further adjust

x and y so that the galvanometer shows no current. When this is the case we have again

$$\frac{R}{r} = \frac{x}{y}.$$

If the wire under test is a copper wire, then we know from the tables of wire resistances approximately the resistance of one metre length of it. Calling this resistance R_1, we choose x_1 as the value of the resistance unplugged out of the box represented by x, so that $\frac{x_1}{y} = \frac{R_1}{r}$. If y is made 1,000 ohms, and if $r = 0\cdot01$ ohm, we have $\frac{x_1}{1,000} = \frac{R_1}{0\cdot01}$.

Removing the plug and making the bridge an ordinary one we find a position of the slider so that the slide wire is divided into sections a and b, and when the balance is obtained

$$\frac{a+R_1}{b+r} = \frac{x_1}{y} = \frac{R_1}{r},$$

or $$\frac{a}{b} = \frac{R_1}{r} = \frac{R}{r} \text{ approximately,}$$

where R is the true resistance of the wire between L and N.

Next, inserting the plug P and passing a stronger current from the battery, we find a new balancing position by altering x_1 to x_2 but keeping a and b the same. Then, since $\frac{a}{b} = \frac{R}{r}$ nearly, we have

$$\frac{R}{r} = \frac{x_2}{y}, \text{ or } R = \frac{x_2}{y} r \text{ very nearly.}$$

In any case the value of R so obtained is much nearer to the true value of the resistance of the metre length of the wire than R_1.

If the value of x_2 is not very different from x_1 the value $\frac{x_2}{y} r$ may be taken as the true value of R, but if x_2 differs considerably from x_1 then we must begin over again, and employing x_2 as the resistance in branch x, obtain a new position of the slider and new values for a and b.

In carrying out the test the battery employed should be such that a small current can be sent from it when the bridge is being employed as a simple bridge, but a much larger current when as a double bridge. This is best achieved by employing a couple of secondary cells in series with a resistance which can be more or less short circuited as required. This resistance may be a simple length of platinoid wire with terminals at each end so arranged that the resistance can be short-circuited by bringing the ends together.

The above method enables us to measure with great ease and accuracy the resistance of a metre length of ordinary copper wire of, say, No. 16 S.W.G., and deduce the volume or mass-resistivity from the observations. It is, therefore, of great utility when only a short sample of bare wire is obtainable. It may be employed also to measure the resistance of short lengths of strand copper cable or copper rod. The sensibility of the method is that an accuracy of 0·1 per cent. can be obtained, or, in other words, by this means the resistance of a yard or two of bare copper wire of No. 16 or No. 14 S.W.G. can be as accurately measured as a resistance of the order of an ohm or more can be measured on the ordinary plug form of Wheatstone's bridge.

§17. Modifications of the Ordinary Wheatstone's Bridge for Low Resistance Measurement.

—A simple yet effective bridge arrangement for measuring low resistances is that devised by Mr. R. H. Housman.[*] The principle of the method consists in making an ordinary bridge measurement in two stages in such a manner that the value of a low resistance standard of 0·001 or even 0·0001 of an ohm can be compared with a standard one-ohm resistance with an accuracy of about one-tenth per cent. This test can be carried out with an ordinary plug Wheatstone's bridge, using with it a movable coil mirror galvanometer of fairly high sensibility, say, giving a scale deflection of 1mm. at a

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[*] See *The Electrician*, Vol. XL., p. 300, December 24 1897

metre distance per microvolt at the galvanometer terminals. The method requires the possession of a one-ohm standard resistance, which will carry safely a current of one or two amperes for a few seconds without sensible heating. For this purpose a No. 18 bare manganin wire, wound on a frame but otherwise exposed to the air, may be employed. This wire resistance standard should terminate in thick copper connecting rods. The standard should be very carefully compared, by any of the methods already described in this chapter, with an ordinary small current one-ohm standard resistance of platinum-silver or manganin of known value.

Let us suppose, then, that a low-resistance standard in the form of a manganin strip has to be measured, and that its
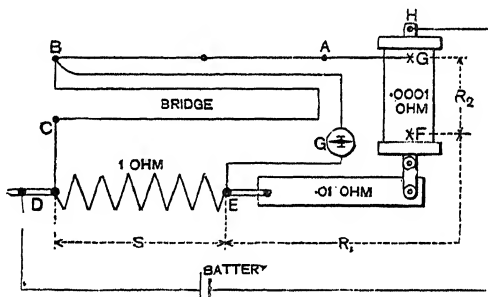

Fig. 43.—Diagram of Housman Bridge Arrangement.

resistance value is something of the order of 0·0001 of an ohm This strip will be provided with current terminals in the usual way and also with potential terminals. Let the resistance of the strip to be tested be taken between the potential terminals and called R_2. It is then to be joined in series with a second resistance or strip, having a resistance (denoted by R_1) of about 0·01 ohm, and with the one-ohm resistance standard, whose exact value will be denoted by S. These three resistances must be joined up in series by stout copper rods or strips, and connected to a plug resistance, which may be an ordinary resistance box, as shown in Fig. 43

The first stage in the process is to measure the ratio of the resistance of the 0·01 ohm strip (R_1) with contacts and connections plus the resistance of the strip under test (R_2) to the resistance of the one-ohm standard S. Let $\frac{R_1 + R_2}{S} = a$. The one-ohm standard, S, should be provided with double terminals, two for current entrance and exit, and two for potential terminals, between which its resistance is known. The Wheatstone bridge employed with the above resistances, R_1, R_2 and S is used merely as a potentiometer wire, the side AB, or the ratio arms, being used as one branch, and BC, or the measuring arm, being used as the other branch. When this bridge arrangement is connected with a galvanometer, as shown in Fig. 43, and with a single storage cell, the cell sends a current of about two amperes along the circuit $R_2 + R_1 + S$. Since the ratio of $R_2 + R_1$ to S is about 1 : 100, it can be determined with an accuracy of 1 to 10,000 when a suitably sensitive galvanometer G is employed, connected across between the terminals B and E, as shown in Fig. 43.

The second stage in the process is to shift one galvanometer connection from E to F and one battery connection from D to E. The storage cell used should be one which can for a short time send a current of 200 amperes or so without injury. The arrangement in the second case is again a simple bridge, but in this last form the circuit $R_1 + R_2$ is traversed by about 200 amperes. No contact which was included in the first stage is altered, but we can now measure the ratio of R_2 to R_1. This is again a ratio of about 1 to 100. Hence we can find with great accuracy the ratio $\frac{R_2}{R_1 + R_2} = b$. Accordingly, we have, by the first measurement,

$$\frac{R_1 + R_2}{S} = a,$$

and by the second one $\quad \dfrac{R_2}{R_1 + R_2} = b.$

Hence, $\qquad\qquad\qquad R_2 = abS.$

The contact resistances are all eliminated. If the galvanometer is one having a resistance of 100 ohms, and giving a scale deflection of 1 mm. at 1 metre distance with 1 microvolt, we can measure the ratio of R_2 to $(R_1 + R_2)$ with an accuracy of 1 in 10,000.

The connections of the bridge wire or bridge circuit ABC to the series of three resistance S_1, R_1, R_2 should be made as small as possible. If, for instance, $R_1 = 0.01$ ohm, R_2 0.0001 ohm, then in the measurement the arm AB may be 10 ohms and the arm BC may be 1,000 ohms. The fall in potential down each side will be 2 volts. Hence, if a current of 200 amperes is flowing through $R_1 + R_2 = 0.010$ ohm, it is clear that a change in resistance of $\frac{1}{1000}$th of the 0.0001 ohm resistance, or of 0.0000001 ohm, or of one-tenth of a microhm, will produce a potential change of nearly 20 microvolts on the galvanometer terminals, and hence produce 20mm. scale divisions deflection, which it is impossible to avoid detecting. This method is therefore one which can quite easily be put into practice in any laboratory possessing a good plug resistance bridge and a sensitive galvanometer.

When once a low resistance strip is obtained, the value of which is accurately known at any temperature, then it is easily copied and others reproduced from it.

Bridge methods, speaking generally, have a great advantage over methods for the comparison of resistances which depend upon the measurement of the relative value of the fall of potential down conductors to be compared when traversed by the same current, in that, in the case of bridge measurements, slight variations in the value of the currents passing through the network do not affect the accuracy of the result.*

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* See *The Electrician*, Vol. XLI., p. 354, 1898, for remarks by Prof. H. L. Callendar " On the Bridge Method of Comparing Low Resistances," in which the advantages of bridge over potentiometer methods of comparing low resistances are upheld. For a trenchant criticism by the same writer on the subject, see also *The Electrician*, Vol. XLI., 1898, p. 501 and p. 631.

§ 18. Measurement of High Resistances by Direct Galvanometer Deflection. — The measurement of high resistances, or resistances of the order of a megohm or upwards, cannot be well effected by any of the above described bridge methods. The method by which it is generally achieved is by observing the current which can be sent through a calibrated galvanometer of great deflectional sensibility, but good zero-keeping quality, by a battery of high electromotive force placed in series with the galvanometer and with the resistance to be measured. If the resistance to be measured is placed in series with the galvanometer and with a battery of small secondary cells, we have then a circuit of which the resistance is made up of the resistance to be measured (which may be reckoned as megohms and denoted by R), the internal resistance of the cells denoted by r_1, the resistance of the galvanometer (represented by r), and that of the connections by r_2, these last three being measured in ohms. If, then, E is the electromotive force of the battery in volts, and A the current in amperes which flows through the circuit, we have

$$A = \frac{E}{10^6 R + r + r_1 + r_2}.$$

If the battery is composed of secondary cells, and the connections are of copper wire and not long, then r_1 and r_2 may be neglected in comparison with $10^6 R$, and we have

$$A = \frac{E}{10^6 R + r}.$$

Under these circumstances the galvanometer will give a deflection δ such that $A = G\delta$, where G is the galvanometer constant for steady currents. The galvanometer must, for this purpose, be a highly sensitive mirror instrument, with either movable needle or movable coil, placed at a distance, say, 2 metres, from a scale divided into millimetres. On the scale the sharp image of a portion of an incandescent lamp filament is focussed so that any deflection of the coil or

needle can be carefully measured in terms of the scale deflections. In the case of such a mirror galvanometer the deflections will be proportional to the current flowing through the galvanometer. The first step, therefore, is to determine the galvanometer constant. This is best done by the employment of a *potentiometer*. A fall of potential is created down a long fine wire of high resistance laid over a divided scale, and this fall of potential is adjusted by means of a Clark cell, so that the fall of potential per scale division is known. For the details of this process the reader is referred to the description given of the potentiometer and its uses in Chapters I., III. and IV.

Suppose the potentiometer is so adjusted that we have a fall in potential of 0·001 volt per scale division of the scale over which the wire is laid. The terminals of the galvanometer are then connected through a resistance of several thousand ohms with two points on this slide curve and the distance between these points adjusted until the galvanometer gives a convenient deflection. Thus, suppose the galvanometer has a resistance of 6,000 ohms and we place in series with it 4,000 ohms and find that when the terminals are connected to two points on the slide wire separated by 20 scale divisions we have a galvanometer deflection of 40 divisions. Then we know that the potential difference on the terminals of the galvanometer is $20 \times 0\cdot001$ volt $= 0\cdot02$ volt and the resistance of the galvanometer circuit is 10,000 ohms. Hence the current through it is $(0\cdot02 \div 10,000)$ amperes.

If G is the galvanometer constant, then

$$40G = (0\cdot02 \div 10,000),$$

or
$$G = \frac{1}{20,000,000} = \frac{1}{20} \times \frac{1}{10^{6}}$$

or 0·05 of a microampere, per scale division. A determination of G should be made for a great many different values of δ (the galvanometer deflection) and the mean value taken.

In the next place we must determine, by means of the potentiometer, the value of the electromotive force (E) of the

battery. This can be done in the way described in Chapter IV., in which are given methods for the measurement of electromotive force. Having thus determined the value of G and E, and knowing the value of the galvanometer resistance r, we can calculate the value of R in the expression

$$G\delta = \frac{E}{10^6 R + r}$$

for
$$R = \frac{1}{10^6}\left(\frac{E}{G\delta} - r\right).$$

In this test the electromotive force E should be produced by a number, say, 50 to 100, of small secondary cells, so that it can be measured with a good voltmeter. It may even be the circuit pressure of an electric supply circuit if it is sufficiently constant.

If neither potentiometer nor voltmeter is at hand, but merely a battery of cells and resistance boxes of the ordinary type, we may proceed as follows :—

Let g be the galvanometer resistance, and s be the resistance of a shunt across its terminals. Then the total resistance, g_1, of the shunted galvanometer is

$$\frac{gs}{g+s} = g_1.$$

Employing one cell of the battery, send first a current from it through the shunted galvanometer and a resistance, r_1, in series with it. Let r be the internal resistance of the cell and c its electromotive force. If, then, the resulting galvanometer deflection is d_1, and if G is the galvanometer constant, or the number by which the galvanometer deflection must be multiplied to give the current in amperes producing that deflection, then, by Ohm's law, we have

$$\frac{c}{r + r_1 + g_1} = Gd_1,$$

or
$$\frac{c}{G} = rd_1 + r_1 d_1 + d_1 g_1 \quad . \quad . \quad . \quad . \quad \text{(i.)}$$

Next alter the resistance r_1 to r_2 and obtain a corresponding scale deflection, d_2, of the galvanometer; then

$$\frac{e}{r+r_2+g_1}=Gd_2,$$

or
$$\frac{e}{G}=rd_2+r_2d_2+d_2g_1. \quad . \quad . \quad . \quad . \quad \text{(ii.)}$$

Eliminating $\frac{e}{G}$ from (i.) and (ii.), we have

$$r=\frac{d_2r_2-d_1r_1}{d_1-d_2}-g_1. \quad . \quad . \quad . \quad \text{(iii.)}$$

If the resistance r_2 is so adjusted that the second deflection $d_2=\frac{1}{2}d_1$, then we may reduce the equation (iii.) to

$$r=(r_2-2r_1)-g_1.$$

Hence the internal resistance of the cell becomes known under the conditions that it is sending a very small current.

In the next place, let a battery of n cells be joined in series with the galvanometer and the large resistance to be measured of which the value in megohms may be denoted by R. Let the galvanometer deflection then be d_3. Hence we have

$$\frac{ne}{nr+10^6R+g}=Gd_3. \quad . \quad . \quad \text{(iv.)}$$

Change the number of cells to n', and obtain another galvanometer deflection d_4, then

$$\frac{n'e}{n'r+10^6R+g}=Gd_4. \quad . \quad . \quad . \quad \text{(v.)}$$

Eliminating $\frac{e}{G}$ from (iv.) and (v.), we have

$$R=\frac{(nd_4-n'd_3)g+nn'(d_4-d_3)r}{10^6(n'd_3-nd_4)}. \quad . \quad . \quad \text{(vi.)}$$

Substituting in (vi.) the value found above by equation (iii.) for r, we obtain the value of the resistance R in megohms in terms of the scale deflections and the resistances of the galvanometer and cells.

The assumption made that each cell has the same internal resistance must be tested by measuring the internal resistance

of various cells, and if they differ, as they probably will do, then the mean value of all must be taken.

The method, however, cannot be regarded as affording more than an approximation to the value of the high resistance unless each of the cells has absolutely the same internal resistance and electromotive force. This is never the case in any actual battery. If, however, we employ a battery, say of 50 moderately large secondary cells which have been about half discharged, and in which each cell is in good order, the voltage of each cell will be very nearly 2 volts and the internal resistance of the whole battery will be negligible in comparison with that of a galvanometer having a resistance of say 5,000 ohms. In this case the equations become reduced as follows:—

Let a single secondary cell be first applied to create a current through the shunted galvanometer in series with a resistance. Then, taking the same symbols as before, we have

$$\frac{e}{r_1+g_1}=Gd_1. \quad . \quad . \quad . \quad . \quad \text{(vii.)}$$

Next let the high resistance be placed in series with the battery of n cells and the galvanometer be unshunted, and we have

$$\frac{ne}{10^6R+g}=Gd_2. \quad . \quad . \quad . \quad \text{(viii.)}$$

Hence, from (vii.) and (viii.) we obtain

$$\frac{ne}{G}=nr_1d_1+rg_1d_1=10^6Rd_2+gd_2.$$

$$\therefore \quad R=\frac{nr_1d_1+ng_1d_1-gd_2}{10^6d_2}=\frac{nd_1(r_1+g_1)-gd_2}{10^6d_2}; \quad \text{(ix.)}$$

or, if the high resistance is measured in ohms, then

$$R=n(r_1+g_1)\frac{d_1}{d_2}-g. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(x.)}$$

The most satisfactory method is, however, to standardize the galvanometer by the potentiometer, and then to measure

by the same means the actual value of the high electromotive force employed to transmit the currents through the high resistance.

The apparatus therefore required for this measurement consists of three parts: First, the sensitive high-resistance galvanometer combining good zero-keeping quality and proportionality of deflection to current or constancy of deflectional constant over the useful range of the scale. These qualities must not be taken for granted, but be carefully proved to exist. The galvanometer should have various electromotive forces applied to its terminals by the potentiometer, and the ratio between the observed scale deflection and the potential difference of the terminals observed. This ratio should prove to be constant. In the next place, when the circuit is opened, the galvanometer needle or coil should return accurately to the same zero position. If it does not, the galvanometer is no use for the present purpose. In the next place a good potentiometer is required by means of which to make the above-described calibration and to determine the galvanometer constant at any moment. In the third place a battery of small secondary cells is required. This is most conveniently provided in the form of a series of trays containing rows of lithanode cells made up in large test tubes. For most work in measuring insulation from 6 to 12 trays will be required, each containing 25 cells. These trays should be exceedingly well insulated by being placed on ebonite legs or paraffin blocks. High resistance keys are also required. The insulation of the galvanometer itself is a matter of importance, and it should be supported on three ebonite blocks in which the levelling screws rest.

The above apparatus serves for the measurement of the insulation resistance of cables or of highly insulating materials such as indiarubber or gutta-percha.

§ 19. Measurement of Insulation Resistance.—Let it be
desired to measure the insulation resistance of the dielectric

of a cable or insulated wire.   The method of procedure is as follows :—

A large tank is provided of sufficient capacity to contain one or more coils of the wire or cable to be tested.   For laboratory purposes it is convenient to have a galvanised iron tank about 3ft. wide, 3ft. long, and 2ft. deep, and to rest it upon a wooden platform on castors for facility of movement. The coil of cable is placed in the tank, and the tank filled up with water.   The two ends of the cable are kept well above the surface of the water and must be prepared by stripping
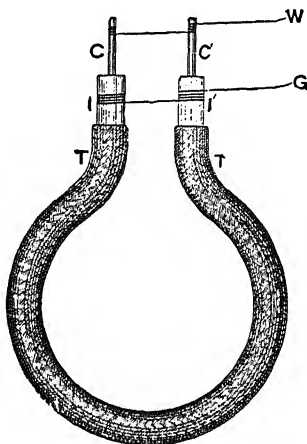
Fɪɢ   44.—Use of Price's Guard Wire.

back for some distance the plait or tape outer covering so as to expose the indiarubber (I.R.) or gutta-percha (G.P.) insulation.   The two ends of the copper conductor are also bared and twisted together.   Round the exposed length of I.R. or G.P., which should be a foot or more in length, is twisted a fine wire, G, called a *guard wire*.   This should be put on at about 6in. from the outer end of the insulation.

The cable then presents the appearance shown in Fig. 44. C, C' are the exposed copper ends connected by a wire W. I, I′ are the exposed indiarubber or gutta-percha surfaces, the

tape or protective covering being neatly stripped off and the ends connected by the guard wire G. T,T is the tape or twist-covered complete cable which remains under water in the tank.

The connections are then made as follows : One terminal of the battery is connected to the tank or to a metal plate placed in the water. The other terminal is connected to one terminal of the galvanometer, and the remaining terminal of the galvanometer G (*see* Fig. 45) is connected to the copper wire w of the cable. The guard wire g is wound round the exposed ends of the insulating covering I and connected to the junction point of the battery and galvanometer. It is best to insert a key in circuit with the battery and to place this next to the tank. The battery and galvanometer should be well insulated by placing them on ebonite
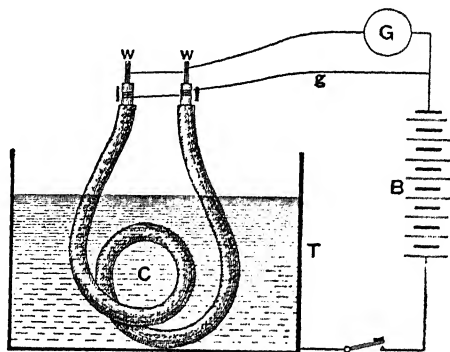

Fig. 45.—Testing Insulation of Cable.

slabs or blocks of paraffin. The use of the guard wire is as follows : If any leakage of current takes place over the cleaned surfaces of indiarubber, this current would, unless prevented, pass through the galvanometer and be added to the true conduction current or flow through the dielectric of the cable. The guard wire, however, arrests the leakage current and conducts it harmlessly past the galvanometer.*

* This ingenious yet simple means for annulling the effect of the leakage current over the exposed dielectric ends of a cable under insulation test is called Price's Guard wire, being due to Mr. W. A. Price (see *Electrical Review*, Vol. XXXVII., p. 702, 1895). It is described also in a Paper by Mr. R. Apple-yard on Dielectrics (see *Proc. Phys. Soc.,* Lond., Vol. XIV., p. 256 ; or *The Electrician,* Vol. XXXVII., p. 159). *See* also, Prof. W. E. Ayrton, "Some Developments in the Use of Price's Guard Wire in Insulation Tests," *Phil. Mag.,* April, 1900, Vol. XLIX., p. 343.

In making such an insulation test it will be found that if the material of which the dielectric of the cable consists is indiarubber, gutta-percha, okonite or any of the usual insulators, the deflection of the galvanometer does not remain constant when the same electromotive force is continuously applied. It will be seen to decrease with the time during which the battery is kept in circuit. If the cable is at all a long one, then at the moment of first making contact there will, in addition, be a rush of current into the cable. In the case of very large cables this rush is so violent that it is better to short-circuit the galvanometer for a moment whilst putting down the battery key, and then to withdraw the galvanometer short-circuit immediately.

If we operate in this manner we shall find that the galvanometer deflection decreases from moment to moment in a manner which indicates that the insulation resistance is increasing with time. It is the custom to define the insulation by stating its value in megohms after *one minute's electrification*, naming also the electromotive force of the battery and the temperature of the cable.

In the case of such materials as indiarubber and gutta-percha, and probably in the case of most dielectrics, the apparent insulation resistance is a function of the electromotive force and of the time during which it acts. The apparent resistance increases with the time of application of the electromotive force, and in some cases it decreases also slightly as the electromotive force is increased, even if taken after the same interval. The insulation resistance decreases markedly with rise of temperature.

Hence the voltage, temperature, and time of electrification must always be stated, or else the numerical value of the insulation resistance is a meaningless number. Thus a cable may be marked as having an insulation resistance of 300 megohms per mile tested with 600 volts after one minute's electrification on application of the electromotive force, the insulator being at a temperature of 75°Fahr. Such a

statement is precise, but the mere statement that the insulation resistance is 300 megohms per mile is a very imperfect description of the actual insulation quality of the cable.

The relation between the strength of the current flowing through a non-metallic conductor and the electromotive force producing it is in general much less simple than in the case of metals.

In addition to the metals and alloys formed of them, which are called the *metallic conductors*, we have bodies—such as carbon, selenium, tellurium, phosphorus, certain metallic oxides and sulphides, &c.—which are non-metals, and yet in certain states are electrical conductors. These are called *non-metallic conductors*. They are generally characterised by the property of existing in two or more allotropic conditions, in which their electrical conductivity differs widely. Carbon exists in three forms, as diamond, charcoal and graphite. In the first state it is a good insulator, in the second a poor or indifferent conductor, and in the third a fairly good conductor. Selenium, also, can exist in two allotropic forms, and is remarkable from the fact that light falling on it when it is in one of these forms causes a sudden and considerable diminution in resistance. For the most part, the electrical volume-resistivity of all these non-metallic conductors is much higher than that of any of the metallic conductors.

The third class of conductors comprises such substances as indiarubber, gutta-percha, mica, glass, ebonite, etc. These are all substances of very high electrical resistivity, and are generally called *insulators*. They have a volume-resistivity enormously greater than any metallic or non-metallic conductor. They are usually termed non-conductors, but may also be denominated as *dielectric conductors*. This class includes numerous solids and liquids.

The fourth class of conductors comprise the conducting liquids. These are called *electrolytes*. In their case the process of conduction is accompanied by an evident chemical decomposition of the material, and it is described as electrolytic conduction, the liquids themselves being spoken of as *electrolytic conductors*. It is a difficult matter to give a precise answer to the question, What is an electrolyte?

A fifth class may be said to comprise the *gaseous conductors ;* and although there is some evidence that in their case the process of conduction is electrolytic in nature, it may be well to retain them at present in a distinct sub-division. Gases at ordinary pressures are good insulators, but if subjected to an electromotive force exceeding a certain limit, they pass into the conductive condition.

As examples of these five classes of conductors, we may select copper, graphite, mica, melted silver chloride, or an aqueous solution of sodic chloride, and hydrochloric acid gas.

It may be that these divisions are not sharply marked. The conduction in the case of glass may be electrolytic, and also in the case of gases. On the

other hand, the process of conduction may be of a similar nature in all the five sub-divisions, and even in the metallic conductors may be electrolytic.

The action of temperature on the conductivity of the substances in the various classes is very different. In the case of metallic conductors, if they are pure metals, increasing their temperature always increases their resistivity, and lowering the temperature towards the absolute zero indefinitely decreases the resistivity. As regards metallic alloys, some of these, such as *manganin*, have a maximum resistivity at a certain temperature, above which they decrease in resistance as temperature rises, and below which they also decrease in resistivity, but not probably indefinitely, and there is no indication, as in the case of pure metals, that it would become zero at the absolute temperature.[*] In the case of the non-metallic conductors, increase of temperature lowers the resistivity. As regards *graphite* there is clear indication that it has a minimum resistivity corresponding to a certain temperature near a dull red heat.[†]

The class of dielectric conductors are distinguished by the fact that rise in temperature always reduces their electric resistivity, and this rate of decrease in some instances is very marked. Thus, in the case of glass we have an exceedingly large negative temperature co-efficient,[‡] as shown in the following tables :—

Volume-resistivity of Glasses in C.G.S. Units per Centimetre-cube.

BOHEMIAN GLASS.			FLINT GLASS.		
Temp.		Volume-resistivity.	Temp.		Volume-resistivity.
60°C.	......	6×10^{22}	60°C.	......	$1 \cdot 02 \times 10^{24}$
100°C.	......	2×10^{21}	100°C.	......	2×10^{23}
130°C.	......	2×10^{20}	120°C.	......	$4 \cdot 68 \times 10^{22}$
160°C.	......	$2 \cdot 4 \times 10^{19}$	140°C.	......	$1 \cdot 06 \times 10^{22}$
174°C.	......	$8 \cdot 7 \times 10^{18}$	160°C.	......	$2 \cdot 45 \times 10^{21}$
			180°C.	......	$5 \cdot 6 \times 10^{20}$
			200°C.	......	$1 \cdot 2 \times 10^{20}$

Another characteristic of some dielectric conductors is that the resistivity is a function of the electromotive force with

[*] For a theory as to the cause of the greater average resistivity and smaller temperature co-efficient of alloys compared with that of the metals composing them, see a Paper by Lord Rayleigh in *The Electrician*, Vol. XXXVII. p. 277.

[†] See *The Electrician*, Vol. XXXVIII., p. 835, Mr. J. C. Howell, "On the Conductivity of Incandescent Lamp Filaments."

[‡] See Mr. Thomas Gray, *Proc.* Roy. Soc. Lond., January 12, 1882.

which it is measured. This is shown by the results in the following table, obtained by measurements made at the works of Messrs. Siemens Bros. & Co.*

Resistance of Certain Dielectrics under various Electromotive Forces or Voltage after One Minute's Electrification.

Dielectric.	Voltage.	Resistance.	Dielectric.	Voltage.	Resistance.
Gutta-percha.	150	3,468	Lead-covered impregnation cable (C class).	150	2,813
	300	3,366		300	2,674
	750	3,267		750	2,541
	1,050	3,117		1,050	2,224
	1,500	3,086		1,500	2,057
	1,800	3,021		1,800	1,798
High insulation india-rubber.	150	8,054	Lead-covered impregnation cable (A class).	150	2,602
	300	7,916		300	2,492
	750	7,470		750	2,425
	1,050	7,268		1,050	2,345
	1,500	6,705		1,500	2,274
	1,800	6,245		1,800	2,212
Ordinary indiarubber.	150	513			
	300	462			
	750	419			
	1,050	355			

The above figures show beyond question that the resistance of certain dielectrics after one minute's action of the impressed electromotive force or voltage decreases as the voltage increases. The same variation has been found to exist after electrification for other intervals. Hence it must not be hastily assumed that in the case of any dielectric the current flowing through it under different electromotive forces is in a constant ratio to the electromotive force. The current flow in amperes or micro-amperes per volt of impressed electromotive force is not constant. The actual current flowing

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* The figures were given by Mr. A. Siemens at the Institution of Electrical Engineers, during a discussion on a Paper by Sir W. H. Preece on "The Specification of Insulated Conductors." See *Journal* Inst. Elec. Eng., Vol. XXI., p. 217.

through a dielectric conductor at any instant after an impressed electromotive force has been applied to it is a complicated function of the time from closing the circuit, the temperature and the impressed electromotive force, and the usual custom of denominating the ratio of the electromotive force measured in volts, to the current (in amperes) through the dielectric after one minute's application of that electromotive force, the ohmic resistance of the dielectric, is a perfectly arbitrary or conventional proceeding. The properties of dielectrics will be more fully considered in a later chapter in the section dealing with capacity measurement.

Returning to the practical test of insulation, we may add that it is necessary in these tests to provide a sufficient electromotive force in the form of a well-insulated battery In cable factories it is usual to set up in a room a battery of 600 to 1,200 Leclanché cells arranged in groups, in series, and placed on well-insulated platforms of paraffined wood standing on oil insulators. From this battery insulated wires run to the testing room, so that any required electromotive force up to 1,800 volts or so is available for testing purposes. In a laboratory it is more convenient to employ small secondary cells set up in test-tubes and arranged in trays of 10 cells. The most convenient form of cell is the lithanode secondary cell. These cells consist of glass tubes about 1 inch in diameter and 4 inches high, and each contain two plates. They should be charged when required with a small current, and no greater current than one-tenth of an ampere taken from them. The trays of cells should be arranged in a portable cupboard and terminal wires from the various sets of cells or from each cell brought out to insulated terminals on the outside.

The insulation tests of ordinary indiarubber-covered cable as used in electric lighting work are always made with an electromotive force of 300 volts to 600 volts, and the resistance taken after one minute's electrification. Previously, however, the cable must be allowed to lie in water for at least 24 hours

so that the final result is stated in the form of a certificate appended to each coil of cable that it has been tested with 600 volts applied for one minute *after* 24 hours' soaking in water.

In making a critical examination of a dielectric material the cable or insulated wire must be supplied in coils of known length, say, one-quarter or half-mile coils. The insulation tests enable us then to state the results as so many megohms per mile of cable.

It is to be noted that if, for instance, the test of 440 yards of cable showed its dielectric to have an insulation resistance of 4,000 megohms, then the insulation is at the rate of 1,000 megohms per mile.

Each coil of cable after testing should then have a certificate appended to it as follows :—

Certificate of Insulation Test.

Length of this Coil ....yards or miles.
Gauge of Copper Wire....................... ....................S.W.G.
Nature of Insulation..
Temperature of Coil when Tested ....
Insulation Resistance ....megohms
    per mile at ....°F. when tested
    with ....volts after one minute's electrification
    and........ ....hours' soaking in water.
Date of test ....
Test made at...........................Laboratory, by....................
    ....

For electric light wiring no wire should be employed which has not thus been tested, and the certificates of each coil of wire as used by the contractor should be demanded by the clerk of works or consulting engineer. No wire should be employed of less insulation than 300 megohms per mile at 75°F. after 24 hours soaking in water, the test being made with 600 volts and resistance taken after one minute.

In the case of dielectrics such as paper or jute impregnated with oil or resins or various other dielectrics, which are not in themselves waterproof, the cable must be covered with a continuous sheath of lead in order to protect them. In

this case, also, a test in water is necessary in order to ascertain whether the lead covering is perfect.

§ 20. Measurement of Dielectric Resistance by Time of Falling to Half Charge.

—A usual method of measuring the insulation or dielectric resistance of a substance of very high resistivity is to connect it between the earth and a charged conductor, which is otherwise perfectly insulated, and then observe the time taken for the conductor to fall in potential by a known fraction or, say, to half its value.

As a first simple approximation to a theory, assume that the capacity of the perfectly insulated conductor is C, and that it is charged to an initial potential V, and connected to earth through a resistance, R, of large value. After a time, t seconds, let the potential of the conductor be v, and after a further small time, dt, let it be $v - dv$; then the time rate of change of potential at that time is $-\dfrac{dv}{dt}$. If the capacity of the conductor is C, the electric quantity in it at that instant is Cv, and the time rate of change of the charge or quantity is $-C\dfrac{dv}{dt}$; hence, this also must be the rate of leakage or current flow through the dielectric resistance. If R is this resistance we have, *if Ohm's law is obeyed*,

$$\frac{v}{R} = -C\frac{dv}{dt}, \quad \text{or} \quad \frac{dv}{v} = -\frac{dt}{CR}.$$

Hence, integrating each side of the equation, we obtain

$$\log_e v = -\frac{1}{CR}t + A,$$

where A is the constant of integration. If $t = 0$, then $v = V$, where V is the initial voltage.

Hence

$$\log_e v - \log_e V = -\frac{t}{CR}$$

or

$$R = \frac{t}{C\log_e \dfrac{V}{v}}.$$

If we use ordinary logarithms to the base 10, then we have

$$R = \frac{0 \cdot 4343t}{C\left(\log_{10}\dfrac{V}{v}\right)},$$

where $0 \cdot 4343$ is the modulus or factor for converting Napierian to ordinary logarithms. If T is the time in seconds taken to fall to half potential, then $V/v = 2$, and $\log_{10} 2 = 0 \cdot 30103$, and

$$R = \frac{0 \cdot 4343T}{C \times 0 \cdot 30103} = \frac{T}{0 \cdot 693C} = \frac{10T}{7C} \text{ nearly.}$$

Hence R is obtained as the quotient of a time by a capacity. If C is measured in microfarads, then R will be measured or given in megohms.

In the above proof two things are assumed: First that the resistance strictly obeys Ohm's law, and, secondly, that the capacity is constant and independent of the voltage. In the case of actual dielectrics and condensers neither of the above assumptions is strictly in accordance with fact.

In applying this method the difficulty is to find a voltmeter or electrometer which is itself sufficiently well insulated. No electrostatic voltmeter or electrometer is so absolutely insulated that if charged and left to itself it will not lose charge, neither is any condenser or body having capacity similarly free from leakage. We may, however, proceed in practice as follows: A condenser or body having capacity is charged to a known voltage and connected to the voltmeter. The arrangement is then left insulated and the time of falling to half charge or half potential is noted. Thus a Kelvin multicellular electrostatic voltmeter, having its terminals closed by a half microfarad condenser, may be placed on a sheet of ebonite and charged, say, to 100 volts, and left insulated. We note the time T (in seconds) taken by the voltmeter to fall to 50 volts. Then the internal leakage resistance R of the voltmeter and condenser, taken together can be calculated by the

formula above given, viz., $R = \dfrac{10T}{7C}$, from which, if $C = \dfrac{1}{2}$ (microfarad), we find $R = \dfrac{20}{7}T$, where R is the insulation resistance in megohms, T the time of falling to half charge in seconds, and C the capacity in microfarads.

The insulator or dielectric whose insulation resistance is required is then joined across the terminals of the voltmeter and the experiment repeated. Let T′ be the time in seconds required then to fall to half charge. The combined insulation resistance of the insulator and the voltmeter and condenser is R′ megohms when $R' = \dfrac{20}{7}T'$. If, then, R″ is the resistance *per se* of the substance tested, we must have $\dfrac{1}{R} + \dfrac{1}{R'} = \dfrac{1}{R''}$, or $R'' = \dfrac{RR'}{R+R'}$, for the total conductivity found in the second experiment must be the sum of the conductivities of the body tested, and of the voltmeter and condenser taken together.

The above method is only practicable and useful in those cases in which the insulation resistance of the condenser and voltmeter and body tested is so high that the time required in any case for the charge of the system to fall to half its initial value is a considerable number of seconds—say 15 or 20 at the very least. If the potential falls to half value in a much less time, the power of observing it accurately is much diminished.

If, however, a highly insulated quadrant electrometer is available, and an air condenser of considerable capacity, the method may be used with success to determine the insulation resistance of glass or porcelain telegraph insulators and long lengths of highly insulated cable. The difficulty which besets this method is that the internal leakage of the voltmeter or condenser and other supports is due to films of dirt or moisture. It is not certain (in fact, very doubtful) that dirt or moisture films of this kind obey Ohm's law. The

formula for calculating resistance by this leakage method is worked out *on the assumption* that all the conductance which takes place does so in accordance with Ohm's law— in other words, that the resistance is independent of the voltage. This is not quite the case for any dielectric, and there is no proof that it is the case for surface films causing leakage. The method, therefore, must be applied with care and discrimination.

In estimating the insulating value of a dielectric it is necessary to consider not only the mere insulation resistance, but also the *dielectric strength* or voltage required to pierce the insulation. This is an important matter in connection with cables for high-pressure currents and porcelain insulators for carrying bare high-pressure electric wires.* Any report on the insulating power of a cable or dielectric should comprise the results of tests as to the average electromotive force required to pierce a known thickness of the insulation.

§ 21. Cardew's Differential Method for Measuring High Resistance.

—This method consists in comparing together the rate of leakage from the two quadrants of a quadrant electrometer. The quadrants are connected respectively to the two poles of a highly insulated battery of about 400 cells. For this purpose the most convenient battery to use is a form of small water battery devised by Lord Kelvin, consisting of a large number of copper-zinc couples attached to an ebonite slab, and so arranged that when dipped into water the drop of liquid held by capillarity between adjacent copper-zinc slips forms the exciting fluid of each couple. A battery of about 500 couples of this form provides an electromotive force sufficient for the test.

The terminals of the battery are connected to the two quadrants of the electrometer, and the needle is connected

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* *See* N. M. Hopkins on "Testing Insulators for High Pressure Overhead Service," *The Electrician*, Vol. XXXVIII., p. 432.

to the earth. The battery must be highly insulated by placing it on paraffin blocks. If, then, the quadrants are connected also to earth through two high resistances of equal magnitude, the leakage from each quadrant will be the same, and the needle will remain at zero. If one of these resistances is an adjustable one, the other may be determined in terms of it.

In starting the test, the centre of the battery should be earthed for one moment, which will bring the needle to zero. As comparison resistances lengths of silk or string have been found convenient. Thus, a white embroidery silk was found to have a resistance of 250,000 megohms per inch, and a green thread, partly silk, partly cotton, 10,000 megohms per inch, an ordinary measuring tape 1,400 megohms per inch, and a piece of wet tape 64,000 ohms per inch. These materials, however, are all more or less hygroscopic. Probably better standards would be found by using capillary glass tubes filled with liquids of definite composition, which do not act upon glass. The method, however, is of great sensibility, and useful for rapid qualitative comparisons of insulation power.

§ 22. The Practical Measurement of Electric Light Wiring Insulation. The Ohmmeter.

—In measuring the insulation resistance of the electric light wiring of a building, it is now the custom to employ some form of *Ohmmeter*. The principle of this instrument is as follows : Let there be two coils of wire, C, c, through one of which, C, flows a current passing through a conductor, R, the resistance of which is to be measured, and another coil, c, of high resistance connected to the terminals of the conductor. Let these coils be placed with their planes or axes at right angles to each other. Then, in the case of each coil there will be a magnetic field proportional at any point to the current flowing through it. Hence, at some common point in the field of the two coils (*see* Fig. 46), there will be two magnetic forces, one pro-

portional to the current through the conductor and one at right angles to it proportional to the voltage fall down the conductor. Hence the resultant magnetic force will have a direction which depends upon the relative magnitude of the components or their ratio to one another—*i.e.*, upon the ratio of the potential fall down the conductor to the current through it. In other words, the direction of this resultant magnetic force will depend upon the resistance of the conductor so connected to the coils. If, then, a small magnetic needle, *n s*, or piece of soft iron is placed at that point, its direction may be indicated on a scale, and the scale may be graduated so as to show at a glance the resistance of the conductor in ohms.
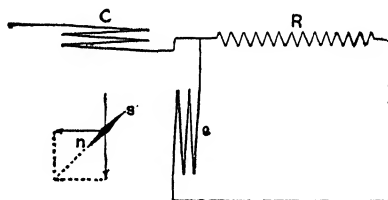
FIG. 46.—Theory of the Ohmmeter.

In order to apply this principle to the measurement of insulation resistances we require a source of high electromotive force. It is universally recognised than an insulation test is entirely insufficient if made with an electromotive force less than the working or circuit electromotive force. Hence, if the electric light wiring of a building is put in for use at 200 volts the insulation resistance must be tested with 500 volts at least.

In a well-known form of instrument by Evershed & Vignoles called a *Megger*, the principle of the ohmmeter is applied as follows : A permanent field magnet has in it two air gaps, in

one of which a direct current armature is revolved which provides an electromotive force of 500 volts. In the other gap is pivotted an iron core carrying an index needle. On this core are wound two coils, one which is placed in series with the insulation under test called the current coil, and one which is placed as a shunt across the dynamo terminals called the pressure coil. When the armature is revolved by hand the pivotted coil takes up a certain position which depends upon the ratio of the current flowing through the insulation to the electromotive force driving it. This therefore measures
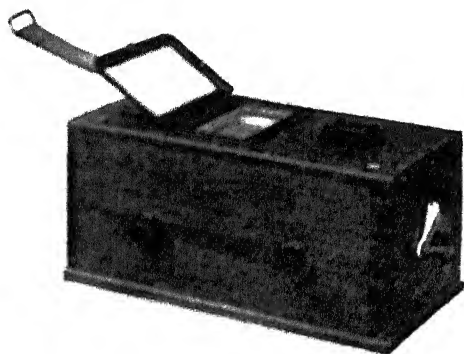
Fig. 47 - Evershed's Megger.

the resistance of the insulation. The scale can, therefore, be divided to read directly in megohms.

The pivotted coil also carries a third coil called a compensating coil

The external view of the instrument is shown in Fig. 47.

To use it one terminal is connected to the conductor of the circuit to be tested, the other terminal is connected by a wire to a water pipe or good earth. This places the insulation of the winding in series with the current coil and with the arma-

ture of the megger. When the handle of the armature is
rapidly turned the needle moves over the scale and indicates
directly the resistance of the insulation in megohms. It is
possible to make use of the service pressure itself as a testing
pressure in measuring the insulation resistance of electric light
wiring. This may be done as follows* :—

The portion of the circuit to be tested is separated from
the main circuit by removing the fuses. Let ABCD (*see*
Fig. 48) be the portion of the circuit to be tested, and let the
terminals J K be those of the dynamo or secondary battery
or street service which normally supplies the electric current.
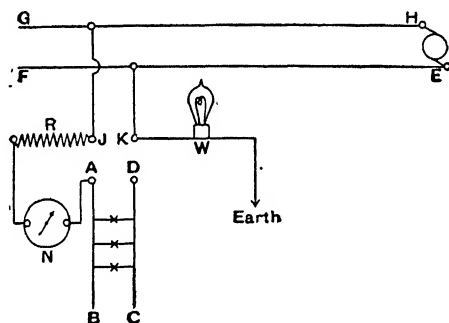
Fig. 48.

Provide a milliamperemeter or other sensitive calibrated
galvanometer and connect it in series with a resistance
between the terminals A and J. Then connect the terminal
K to earth through a lamp, W. The galvanometer or milli-
amperemeter will then show a current if there is leakage or
conductivity through the dielectric of the wiring system
ABCD, and if the galvanometer has been calibrated for
small currents the numerical value of this resistance may be

* The method here described is due to Dr. Oscar May. See *The Electrician*,
Vol. XXXVIII., p. 81 ; see also a critical letter, *ibid.*, p. 128.

determined knowing the voltage between the main GH and the earth.

It is to be noted that the testing pressure we are really here using is not the ordinary service pressure between the terminals J and K, but the voltage between J and K when this latter is connected to the earth through a lamp. Hence the determination of this voltage must always be made at the time by means of a voltmeter of sufficiently high resistance (or say by an electrostatic voltmeter) not to disturb this potential difference. The main GH should always be the positive main, because the negative main on a public supply service by continuous current is nearly always dead-earthed, especially if bare copper strip carried on glass insulators of certain descriptions is used as part of the distribution system.

The method therefore must be applied with discrimination.

§ 23. Insulation Rules for House Wiring.

—The various electric lighting companies and insurance offices have different regulations as to the minimum insulation resistance which should be indicated on test under the working pressure by electric light wiring. This is generally defined as so many *megohms per point*. If, then, there are *n* incandescent lamps (reckoned, say, in 20 watt lamps or their equivalent) in the building, the total insulation resistance between the copper conductors and the earth must not be less than this limiting number divided by *n*. It is generally specified that these tests shall be carried out with a voltage equal to double the working voltage and the reading taken after not less than one minute's application of the E.M.F., all lamps being in and switches on. Thus, for instance, the limiting values for various electric lighting corporations and fire insurance companies are as follows:—

For some calculations dealing with problems on the measurement of insulation resistance and leakage currents from insulated conductors, and especially in connection with 3-wire systems, the reader is referred to a Paper by Mr. A. Russell in *The Electrician*, Vol. XLI., p. 206 ; also Vol. XXXIV., p. 7.

Minimum Insulation Resistances allowed per Lamp in Installa-
tions of Incandescent Lamps* by various Authorities.

City of London Electric Lighting Company .... } 75 megohms per lamp.

Glasgow Corporation 60   „   „

Manchester 50   „   „

Scarborough Electric Lighting Company .... } 40   „   „

Brighton Corporation 20   „   „

Liverpool Corporation, Phœnix Fire Office, Lloyds .... } 12·5   „   „

Bradford Corporation 10   „   „

The leading Insurance Companies, including Guardian, Sun, North British, London, Liverpool and Globe, Alliance, London Assurance, Royal Exchange and others } Leakage not to exceed $\frac{1}{20000}$ part of total current $= 40$ megohms per lamp for 200-volt 20-watt lamps.

Institution of Electrical Engineers latest regulations......... { Not less than 30 megohms divided by the number of points wired.

§ 24. Measurement of the Resistance of Liquids.—

As all liquids which conduct at all do so in virtue of electrolysis, special difficulties are introduced into the resistance measurement of liquids by the counter electromotive force of polarisation due to the ions liberated on the electrodes. The possibility, therefore, of obtaining accurate information as to the resistance of a column of an electrolyte depends upon the completeness with which these disturbing effects of electrolysis can be eliminated. One obvious method is to employ an alternating electromotive force, so that in the column of liquid there is no unilateral flow of electric quantity, but the electrolytic effects due to a current for one moment in one direction may be annulled by an equal current in the opposite direction the next instant.

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* For a chart showing the regulations for insulation resistance of electric light wiring enforced by various authorities see *The Electrician,* Vol. XXXII., p. 265

A plan due to F. Kohlrausch is to employ with an ordinary Wheatstone's bridge a small induction coil instead of a simple battery, and in place of the galvanometer to use a telephone. The secondary currents from the induction coil, which are created at each make-and-break of the primary circuit, are equal in total quantity—*i.e.*, the time integrals of the two secondary currents are equal. Hence, on the whole, polarisation is absent.

The liquid resistance to be measured may be contained in a vessel of any suitable shape forming an electrolytic cell,
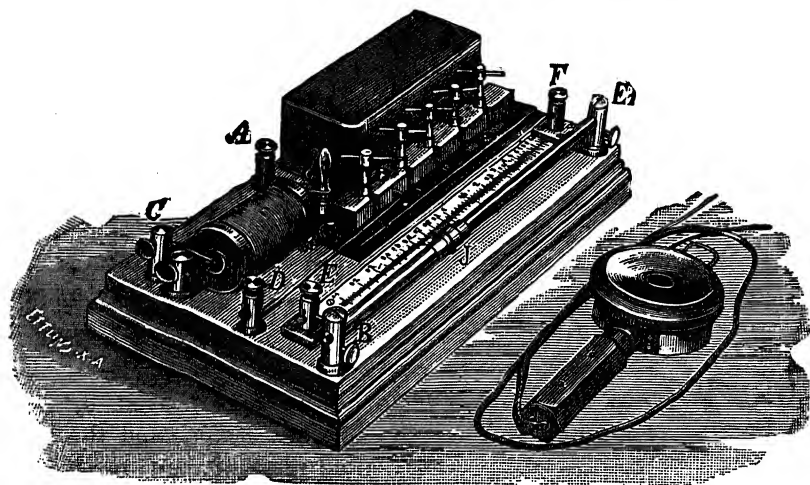
FIG. 49.—Kohlrausch's Bridge for Liquid Resistance Measurement.

the cell having two electrodes by which it is connected with the bridge circuits. These electrodes are formed of sheet platinum, which should be covered with a deposit of platinum black (*see* below). In Fig. 49 are shown diagrams of the usual arrangements. The bridge is a simple slide wire bridge, and the induction coil is set in operation by a few dry cells.

After connecting the electrolytic cell to the proper terminals of the bridge, and setting the induction coil in

action, the slider is moved until no sound is heard in the telephone. As this point of silence is not sharply marked, but reached by slow gradations of sound, the observer will, unless he possesses very sharp hearing, have some difficulty in deciding exactly the point of balance. The method, in fact, tests the acuteness of the observer's hearing as much as it tests the resistance of the electrolytic cell. The ability to distinguish an exact position of the bridge slider corresponding to silence in the telephone depends also upon the nature of the electrodes used in the electrolytic cell. If these are of bright sheet platinum they must be much larger in area than if covered with a deposit of platinum black. This is due to the deposit on them of the products of electrolytic decomposition, and is called the "polarisation of the electrodes." This polarisation creates a counter electromotive force in the circuit. It is not altogether absent even when alternating currents are employed, and in this last case it operates to produce an effect as if the inductance of the circuit in which it exists had been increased. The point of balance on the bridge when so used is often affected by the capacity and inductances of the four arms of the bridge arrangement.

The platinizing of platinum electrodes with platinum black is best done with the platinizing solution employed by Lummer and Karlbaum for their bolometer strips, and the same solution is very useful for coating platinum electrodes for use in electrolytic cells. It consists of 1 part of $PtCl_4$ to 0·008 of lead acetate and 30 of water.

The solution is used as an electrolyte in an electrolytic cell with platinum electrodes, the cathode or negative plate being the one on which a deposit of platinum black is required. The electrolysing current must be adjusted until on the chemically clean platinum cathode is deposited an adherent dull platinum black deposit. To ensure wetting the electrode so prepared with water they should be treated first with a drop of alcohol. A thick deposit of platinum black should be secured. Platinum electrodes covered with this dull

black deposit of platinum black are less easily polarisable than ordinary sheet platinum and hence give better defined readings when used with a telephone induction coil and bridge.

If ordinary bright sheet platinum is used for electrodes, these are easily polarised or covered with a deposit of liberated gaseous ions, and these polarisation films, as they are called, are strongly adherent to the platinum and create between the electrodes a potential difference which forms a counter electromotive force. Since the counter electromotive force cannot exceed 2 volts to 2·5 volts, its presence is most felt at or near the balancing point of the bridge, and it operates to prevent the development of a sharp or well-defined condition of silence in the telephone in this bridge circuit. If the electrodes are covered with a deposit of platinum black they are less polarisable, and hence better defined bridge readings can be obtained.*

A method due to Messrs. Stroud and Henderson† is, in many respects, more satisfactory than the use of a bridge with telephone and induction coil. Their method depends upon inserting in the two arms of a bridge two glass tubes, T_1, T_2, of different lengths, having terminal cups of the same size. These tubes are filled with the electrolyte the resistance of which is to be measured. In the terminal cups are placed cylinders of sheet platinum covered with a deposit of platinum black. The two tubes form, therefore, electrolytic cells identical in every way except in that one contains a longer column of liquid than the other. The dimensions of this column of liquid are easily obtained by measuring the length and mean inner diameter of the glass tubes. These electrolytic cells are arranged as the two arms of a bridge,

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* *See* F. Kohlrausch, *Annals Phys. Chem.*, 60, 2, pp. 315-322, 1897 ; or *Science Abstracts*, Vol. I., p. 338.

† " On a Satisfactory Method of Measuring the Resistance of Electrolytes by Continuous Currents." By Prof. W. Stroud and J. B. Henderson. *The Electrician*, Vol. XXXVIII., p. 49 ; also *Proc.* Phys. Soc. Lond., Vol. XV., p. 13, Oct. 30, 1896.

the shorter tube being in series with a resistance box, R (*see* Fig. 50). The two other arms of the bridge are made up with high metallic resistances, P and Q, of about 20,000 ohms each. The galvanometer used must be a sensitive galvanometer of high resistance, say 1,000 ohms or more. The battery, B, should consist of a number of cells giving an electromotive force of at least about 30 volts. It can conveniently consist of 20 dry cells in series.

The measuring operation consists in altering the resistance R until the galvanometer shows no current. When this is the case, the resistance of R must be identical with that
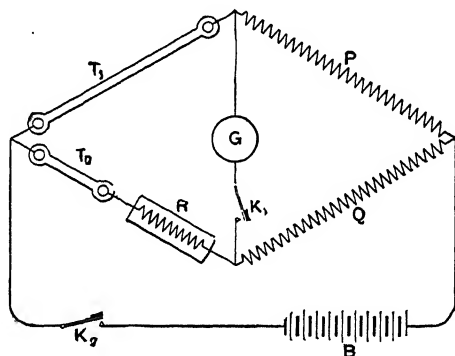
Fig. 50.—Stroud and Henderson's Bridge Arrangement for Liquid Resistance Measurement.

of a column of the electrolyte equal to the *difference* in length between the two columns forming the two cells. Since the electromotive forces of polarisation are equal, and since the two columns of liquid are traversed by equal currents, the effects of electrolysis in the two arms of the bridges exactly neutralize each other. If then the two tubes are of the same diameter and graduated in centimetres along their lengths, we can at once deduce the resistance of a column of the liquid of known length and section, and hence determine its volume-resistivity.

A modification of this method, due to Fitzpatrick (*see* Brit. Assoc. *Report*, 1886, p. 328), which has been constantly in use at the Cavendish Laboratory, Cambridge, overcomes all difficulties due to polarization. It consists in employing a revolving current-reverser, driven by a small water motor, which rapidly reverses the direction of the battery current flowing through the bridge. The direction of connection of the galvanometer is reversed about the same time, but not quite at the same instant, the galvanometer circuit being closed a little later than the battery circuit and opened a little earlier. If the galvanometer needle is made heavy, so as to have a long period of vibration, these continual reversals do not affect its steadiness, but the result is to eliminate altogether effects due to polarization, and to make it quite as easy to measure an electrolytic resistance as to measure a metallic conductor. The reverser consists of a drum on which are fixed brass sectors with wire brushes touching them. The drum resembles a dynamo commutator, only alternate segments are connected to metallic bands at each end. The segments which belong to the galvanometer circuit are rather less wide than those which belong to the battery circuit.

In measuring the absolute value of the resistivity of an electrolyte it is usual to place it in a glass containing vessel or tube having platinised platinum electrodes. Once for all a careful measurement is made of the resistivity of some pure electrolyte, such as potassic chloride, of known concentration. This is done in a rectangular or tubular vessel of such shape that the dimensions of the mass of liquid and the distribution of current through it are known. Afterwards any other electrolyte can have its resistivity determined by being placed in a cell of any form and the ratio of the resistances determined when the cell is first filled with the standard electrolyte and then with the electrolyte under test.

It is not unusual to express the conductivity of electrolytes in terms of a fraction (say 10^{-8}) of that of mercury.

For many purposes, however, it is better to express it in ohms or megohms per centimetre-cube.

In the tables on pp. 327, 329 are given the values of the volume-resistivity of electrolytes of various kinds. The aqueous solutions of the acids and many aqueous solutions of salts or hydrates have a minimum resistivity corresponding to a certain dilution or concentration. The temperature of the electrolyte very greatly affects its conductivity. It will be seen from the tables that the temperature coefficient of an electrolyte is generally about three to four times greater than that of a pure metal.

§ 25.   The Absolute Measurement of Electrical Resistance.—The determination of the value of an electrical resistance in absolute measure, or its direct recovery in terms of the units of length and time, is an operation not likely to be conducted in an ordinary electrical testing laboratory. Space cannot here be granted to review in detail all the various processes which have been suggested. The student desirous for information on this question may be referred for full information to the following advanced treatises on electricity and magnetism :—

CLERK MAXWELL. "Treatise on Electricity and Magnetism."
    Vol. II., Chap. XVIII., 2nd Ed.

E. MASCART and J. JOUBERT. "Electricity and Magnetism."
    Translated by E. Atkinson. Vol. II., Chap. VII.

A. GRAY. "Absolute Measurements in Electricity and Magnetism." Vol. II., Part II., Chap. X., p. 538.

G. WIEDEMANN. *Electricität*, Vol. IV., p. 910.

The various methods used for the absolute measurement of resistance have also been critically discussed by Lord Rayleigh, *Phil. Mag.*, Vol. XIV., 1882. and by Mr. R. J. Glazebrook, B.A. *Report*, 1890. See also *The Electrician*, Vol. XXV., p. 544.

Amongst the processes there described for the absolute determination of an electrical resistance is one due to Lorenz which, from its simplicity, has been made the starting point for the construction of an apparatus by

Prof. J. V. Jones which is likely soon to be found in the possession of every well-equipped electro-physical laboratory. This apparatus enables an observer to re-determine for himself in absolute measure the value of a low resistance, and that with not greater expenditure of time than would be incurred in making a mere comparison experiment.

The general theory of the Lorenz method of determining a resistance in absolute measure may thus be described :— If a metallic disc is caused to rotate in the mean plane of a coil concentric and co-axial with the disc, and if a current passes through the coil, an electromotive force is created in the disc. If brushes touch the centre and circumference of the disc, and if from these connections are led away to make contact with two points on the circuit, which includes the coil, then it is possible to so adjust the resistance between these contact places that no current flows through the disc—in other words, the electromotive force set up in the disc may be made to exactly balance the potential fall down the resistance in series with the coil. If, then, R is the resistance of this conductor and C the current through it and the coil, and if M is the co-efficient of mutual inductance between the coil and the disc, and n the speed of revolution, the electromotive force set up in the disc is equal to MCn, and the fall of potential down the conductor is RC. Hence, when there is equilibrium, $MCn = RC$, or $R = Mn$. The measurement of the resistance is therefore reduced to the calculation of a co-efficient of inductance from the measured dimensions of the coil and the disc and the observation of a speed of revolution.

In Fig. 51 the points O and M are the centre and circumference of the disc, and XY is the resistance of which the absolute value is to be determined. The real apparatus as designed by Prof. J. V. Jones* is represented in Fig. 52.

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* For a full description of the theory and practice of the method the reader is referred to Papers by Prof. J. V. Jones in *The Electrician* of 1890 and 1895— Vol. XXV., p. 552, and Vol. XXXV., pp. 231 and 253.

The metallic disc is driven at a uniform speed by an electric motor. Contact is made against the centre and circum-
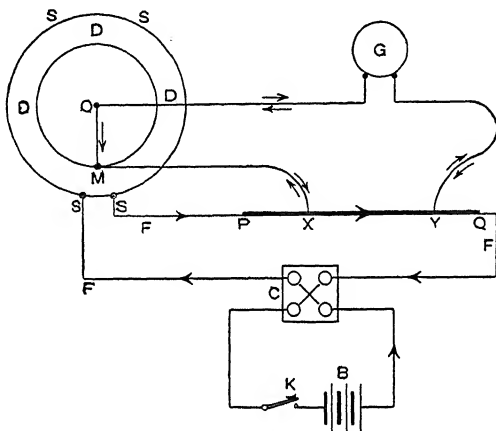
FIG. 51.
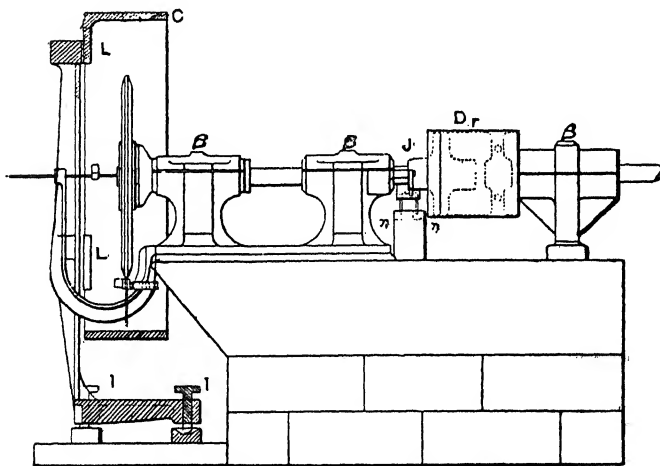
FIG. 52.—The Jones-Lorenz Apparatus for Absolute Resistance Determinations.

ference of the disc by brushes, and these are connected through a galvanometer with the ends of the resistance

being measured. The axis carrying the disc carries also a speed indicator, which is preferably an arrangement for making an electric contact every revolution of the disc. This is made to print dots on a fast-running paper tape on which also series of parallel dots are printed by a standard clock. The speed of the disc can then be accurately determined. The coil consists of a wire wound specially in grooves in a marble ring, on the outer edge of which is cut a screw-groove. The calculation of the mutual inductance of the coil and disc once effected becomes a constant of the instrument.

For the details of the machines already designed and made under the direction of Prof. J. V. Jones the reader is referred to the following Papers by Prof. Jones :—" Suggestions towards the Determination of the Ohm in Absolute Measure," a Paper read before the British Association at Leeds in 1890. (See also *The Electrician,* Vol. XXV., p. 552.)

In this Paper the details of the theory are set out. Reference is also made to a Paper, by the same author, read before the Physical Society in 1888, (see *Phil. Mag.,* Jan., 1889), in which the theory of the calculation of the mutual inductance of a disc and co-axial coil of single layer is given.

Also, by the same author, a Lecture on this subject was given at the Royal Institution, London, in May, 1895, entitled " The Absolute Measurement of Electrical Resistance." (See *The Electrician,* Vol. XXXV., p. 231.)

In this Lecture the details of an improved Lorenz apparatus are described.

A well-constructed Lorenz apparatus was made for the McGill University, Montreal, and a description of the design will be found in *The Electrician,* Vol. XXXVII., p. 267, as well as a perspective view of the machine (*see* Fig. 53). A careful determination of the absolute value of the Board of Trade standard ohm was made by Profs. Ayrton and Jones in 1897, with the above described Montreal Lorentz apparatus and the results of these experiments are recorded in *The Electrician,* Vol. XL., pp. 149-150 (see also *Science Abstracts,* Vol. I., p. 24).
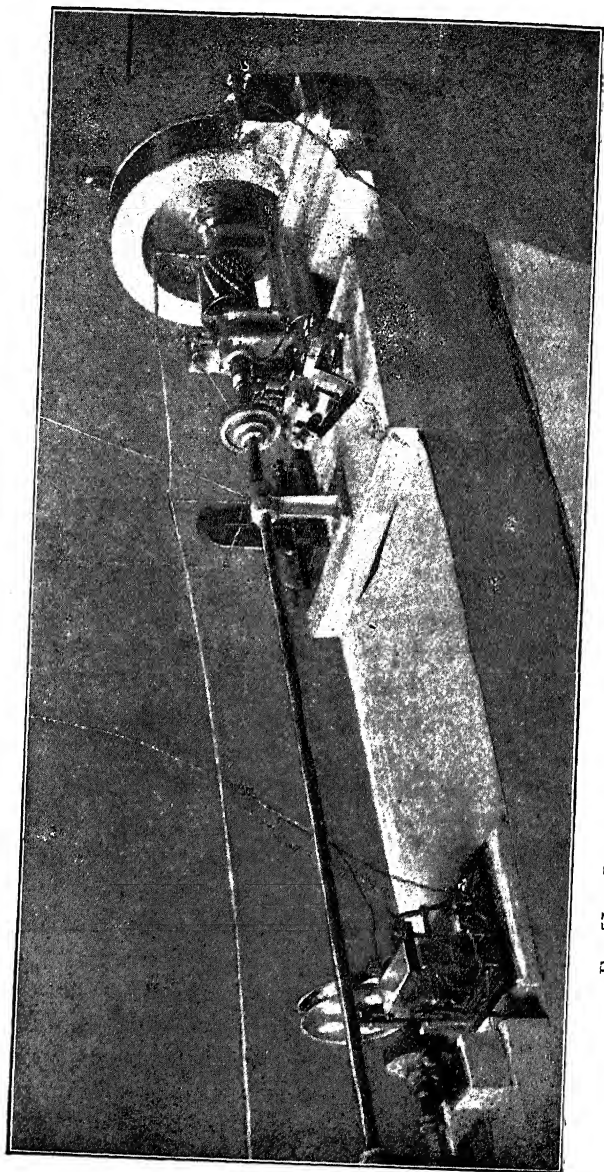
FIG. 53. —Jones-Lorenz Apparatus for the Absolute Determination of Resistance.

A Paper by Prof. Ayrton in *The Electricic.n*, Vol. XL., p. 149, entitled " Our Knowledge of the Value of a Resistance," gives the facts on which are founded the opinion that the Board of Trade standard ohm, which is intended to represent as nearly as possible the International ohm, or a resistance of 106·3cms. of mercury 1 sq. mm. in section at 0°C., may in fact be equal to 106·33cms. or may be in error by three or four parts in 10,000. Hence we cannot say that this official standard ohm really represents 10^9 absolute C.G.S. units of resistance with a greater accuracy than three or four parts in 10,000 or 0·03 per cent. The mean of nine absolute determinations of the value of the Board of Trade wire standard ohm showed that its real value is, in all probability, 1·00026 true ohms the true ohm being 10^9 absolute C.G.S. units.

The reader desirous of information on the relative value of the experimental methods which have been adopted for the absolute determination of resistance and the absolute measurement of the resistivity of mercury may be referred to two important reports made by Mr. R. T. Glazebrook on behalf of the Electrical Standards Committee of the British Association (*see* Brit. Assoc. Reports, 1890 (Leeds) and 1892 (Edinburgh); also *The Electrician*, Vol. XXV., pp. 543 and 588, and Vol. XXIX., p. 462).

Values which have been obtained by various observers for the ohm in terms of the dimensions of a column of pure mercury at 0°C. are given in Table XV. following (p. 335).

§ 26. Resistance of Conductors to Alternating Currents. —

If a metallic conductor, in the form of a wire or rod, is traversed by an alternating electric current, it is well known that the current does not distribute itself uniformly over the cross-section of the conductor, but concentrates itself more or less at the surface. The result is to make the effective resistance of the conductor greater for alternating currents than for continuous, because the periodic current, so to speak, makes less use of the conductor   Hence we need a correction

which must be applied to the true or ohmic resistance in calculating the resistance of a conductor to alternating currents. Assuming the currents to be simply periodic and the conductor a round metallic rod, Lord Rayleigh in 1886 gave a formula for calculating the resistance to alternating currents.[*] Another formula and method was given by Lord Kelvin in 1889,[†] and he gave a table calculated for conductors of round section of certain diameters, and for certain frequencies.

The above formulæ are, however, in a form which does not render them very convenient for laboratory calculation. M. E. Hospitalier has, however, reduced Lord Kelvin's formula to a very convenient table.

If we denote the resistance to alternating currents by R_A and that to continuous currents by R_c, then for round-sectioned rods as wires the two quantities are related by the general expression

$$R_A = k R_c,$$

k being a numerical constant which depends upon the frequency n and on the diameter d.

In the case of wires made of the same metal but of different diameters k has the same value for equal values of the product nd^2. Hence we can make a table showing the values of k corresponding to various values of nd^2. M. E. Hospitalier, assisted by M. A. Potier, has calculated the following table, which is correct for copper having a resistivity of 1,597 C.G.S. units.[‡]

nd^2.	k	nd^2	k
0	1·0000	1,620	1·8628
20	1·0000	2,000	2·0430
80	1·0001	2,420	2·2190
180	1·0258	2,880	2·3937
320	1·0805	5,120	3·0956
500	1·1747	8,000	3·7940
720	1·3180	18,000	5·5732
980	1·4920	22,000	7·3250
1,280	1·6778		

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* *Phil. Mag.*, May, 1886. See also "The Alternate Current Transformer," Fleming, Vol. I., third edition, p. 294.

† Presidential address to the Institution of Electrical Engineers, 1889.

‡ See *The Electrician*, Vol. XXXII., p. 277, or *L'Industrie Electrique*, 1893, p. 563.

As an example of its use, let us apply it to the following problem:—

What will be the resistance to alternating currents having a frequency of 80, of a round-sectioned copper rod 2cms. in diameter. Here $d = 2$, $n = 80$; hence $nd^2 = 80 \times 4 = 320$.

From the above table we see that, corresponding to $nd^2 = 320$, $k = 1.0805$. Hence, if the ohmic resistance of the rod is, say, 3 ohms, then its resistance to the above periodic currents is 3.2415 ohms. If the exact value of the product nd^2 required is not in the above table, it can be obtained by interpolation or by setting out the figures in a curve. In Table XVI., at the end of this chapter, are given the values of the ohmic and alternating resistance for stranded copper cables for a frequency of 100. For other *non-magnetic* metals besides copper the table in the text (p. 318) may be made available by employing a factor to find k equal to the values for copper of nd^2 multiplied by $1,597/\rho$, where ρ is the resistivity in C.G.S. units.

Hence, for a round metal rod of diameter d, and made of a material having *double* the specific resistance of copper, in order to find its resistance to alternating currents of a frequency n, we should have to find the value of k in the above table corresponding to a value $\frac{nd^2}{2}$, and not simply nd^2, as for copper. Accordingly, the higher the resistivity of the material the less different is the alternating from the continuous current resistance.

TABLE I.—Atomic Weights and Densities of Metals.

The quotient of density multiplied by 1,000 by atomic weight gives a number called the *atomic density*, and is proportional to the number of atoms per centimetre-cube.

Metal.	Symbol.	Atomic weights. *a.*		Density. Water=1. *d.*	Atomic density =1,000 ×d/a.	Atomic volume =a/d.
		H=1	O=16			
Lithium	Li	7	7	0·589– 0·598	85	11·8
Beryllium ...	Be	9·1	9·0	1·85	203	4·86
Sodium	Na	23	23·05	0·974	42	23·6
Magnesium ..	Mg	23·9	24·3	1·743	73	13·76
Aluminium ..	Al	27·0	27	2·56 – 2·583	95	10·56
Potassium ...	K	39·0	39·11	0·875	22·5	44·96
Calcium	Ca	39·9	40	1·566– 1·584	39·5	25·28
Titanium ...	Ti	48·0	48	...	...	...
Chromium ...	Cr	52·0	52·1	6·8	130	7·0
Manganese...	Mn	54·8	55·1	7·33	134	7·0
Iron	Fe	55·9	56·0	7·8 – 8·1	142	7·1
Nickel.........	Ni	58·7	58·7	8·3 – 9·0	148	6·94
Cobalt	Co	59·4	59·5	8·5 – 8·9	148	6·94
Copper	Cu	63·2	63·6	8·9 – 8·95	141	7·10
Zinc	Zn	64·9	65·3	6·9 – 7·2	108	9·12
Arsenic	As	74·9	75	...	...	12·96(?)
Rubidium ...	Rb	85·2	85·5	...	...	56·1 (?)
Strontium ...	Sr	87·3	87·66	2·54	29	34·56
Molybdenum	Mo	95·7	...	8·6	89·5	11·13
Ruthenium...	Ru	101·4	...	11·0 –11·4	110·5	9·05
Rhodium ...	Rh	102·7	103	11·0 –11·2	108	9·12
Palladium ...	Pd	106·2	106·5	11·3 –12·1	110	9·12
Silver	Ag	107·6	107·92	10·4 –10·57	97·5	10·04
Cadmium ...	Cd	111·7	112	8·54– 8·66	77	12·96
Indium	In	113·4	113·7	7·2 – 7·4	64	15·53
Tin	Sn	117·3	119	7·3	62	16·20
Antimony ...	Sb	119·6	120	6·72	56	18·16
Cæsium	Cs	132·7	132·9	1·88	14	70·5
Barium	Ba	136·4	137·4	4·0	29	34·25
Cerium	Ce	139·9	140·2	6·73	47·5	20·8
Tungsten ...	W	183·6	...	19·261	105	9·53
Osmium	Os	190·3	...	22·43	117	8·49
Iridium	Ir	192·5	193·1	22·4	116	8·6
Platinum ...	Pt	194·3	195	20·3 –22·1	106	9·12
Gold	Au	196·8	197·3	19·3 –19·5	98	10·04
Mercury	Hg	199·8	200	13·596	68	14·56
Thallium ...	Tl	203·7	204	11·8	58	17·20
Lead	Pb	206·4	206·95	11·3 –11·4	55	18·24
Bismuth......	Bi	208·0	208	9·8 – 9·9	47	21·34
Thorium ...	Th	232·0	232·6	10·97 –11·23	48	20·84
Uranium ...	U	239·8	...	18·4	77	13·03

The atomic weights under column (O=16) are the numbers published by the American Chemical Society's committee on atomic weights (*Journal American Chemical Society*, Vol. XVII.), revised to January, 1894.

The atomic weights under column (H=1) are those given by Professor T. E. Thorp ("Manual of Inorganic Chemistry," Vol. I.).

TABLE II.

Electrical Mass-resistivity of Various Metals at 0°C., or Resistance per Metre-gramme in Standard Ohms at 0°C. (Matthiessen.)

Metal.	Resistance at 0°C (in Standard Ohms) of a wire one metre long and weighing one gramme.	Approximate temperature co-efficient near 20°C.
Silver (annealed)............	0·1523	0·00377
Silver (hard drawn)	0·1657	...
Copper (annealed)	0·1421	0·00388
Copper (hard drawn)	0·1449 (Matthiessen's Standard)	
Gold (annealed)	0·4025	0·00365
Gold (hard drawn)	0·4094	...
Aluminium (annealed) ...	0·0757	...
Zinc (pressed).............	0·4013	...
Platinum (annealed)	1·9337	...
Iron (annealed)	0·765	...
Nickel (annealed)	1·058	...
Tin (pressed)	0·9618	0·00365
Lead (pressed)...............	2·2268	0·00387
Antimony (pressed)	2·3787	0·00389
Bismuth (pressed)	12·8554	0·00354
Mercury (liquid)*	12·885	0·00072

* Matthiessen's value 12·935, for the electrical mass-resistivity of liquid mercury is too high by nearly 1 per cent. The value now accepted is 12·789 international standard ohms per metre-gramme at 0°C. The values for nickel and bismuth are also much higher than are obtainable now with pure electrolytic metals. The temperature coefficients given by Matthiessen are also, in all cases, smaller than those now adopted for pure metals.

Y

TABLE III.

Electrical Volume-resistivity of Various Metals at 0°C., or Resistance per Centimetre-cube in C.G.S. Units at 0°C.

This Table is calculated from the results of experiments made by Matthiessen, employing the values given by Jenkin in his Cantor Lectures (Society of Arts, 1866) for the resistance in B.A. units of a uniform circular-sectioned wire of the metal 1 metre long and 1 mm. in diameter taken at 0°C. The figures given by Jenkin have been reduced to standard ohms and C.G.S. units by multiplying by $\frac{\pi}{4} \times 0.9866 \times 10^5 = 77,485$.

Metal.	Volume Resistivity at 0°C. in C.G.S. Units.
Silver (annealed)	1,502
Silver (hard drawn)	1,629
Copper (annealed)	1,594
Copper (hard drawn)	1,630*
Gold (annealed)	2,052
Gold (hard drawn)	2,090
Aluminium (annealed)	3,006
Zinc (pressed)	5,621
Platinum (annealed)	9,035
Iron (annealed)	10,568
Nickel (annealed)	12,429†
Tin (pressed)	13,178
Lead (pressed)	19,580
Antimony (pressed)	35,418
Bismuth (pressed)	130,872
Mercury (liquid)	94,896‡

* The value (1,630) here given for hard drawn copper is about one-quarter per cent. higher than the value now adopted, viz.: (1,626). The difference is due to the fact that either Jenkin or Matthiessen did not employ precisely the same values as at present employed for the density of hard-drawn and annealed copper in calculating the volume-resistivities from the mass-resistivities.

† Matthiessen's value for nickel is much greater than that obtained by the Author, as shown in Table IV.

‡ Matthiessen's value for mercury is nearly 1 per cent. larger than the value now adopted as the mean of the best results, viz.: 94,070.

TABLE IV.

Electrical Volume resistivity of Various Pure Metals at 0°C., or Resistance per Centimetre-cube at 0°C. in C.G.S. Units.

(Fleming and Dewar, *Phil. Mag.*, September, 1893.)

Metal.	Resistance at 0°C. per centi-metre-cube in C.G.S. Units.	Mean tempera-ture coefficient between 0°C. and 100°C.
Silver (electrolytic and well annealed),*	1,468	0·00400
Copper (electrolytic and well annealed)*	1,561	0·00428
Gold (annealed)	2,197	0·00377
Aluminium (annealed)	2,665	0·00435
Magnesium (pressed)	4,355	0·00381
Zinc	5,751	0·00406
Nickel (electrolytic)*	6,935	0·00618
Iron (annealed)	9,065	0·00625
Cadmium	10,023	0·00419
Palladium	10,219	0·00354
Platinum (annealed)	10,917	0·003669
Tin (pressed)	13,048	0·00440
Thallium (pressed)	17,633	0·00398
Lead (pressed)	20,380	0·00411
Bismuth (electrolytic)†	110,000	0·00433
Mercury (Pure)	94,070	0·00098

* The samples of silver, copper and nickel employed for these tests were prepared electrolytically by Mr. J. W. Swan, F.R.S., and were exceedingly pure and soft. The value for volume resistivity of nickel as given in the above table (from experiments by J. A. Fleming) is much less (by nearly 60 per cent.) than the value given by Matthiessen's researches.

† The electrolytic bismuth here used was prepared by Messrs. Hartmann and Braun, and the resistivity taken by J. A. Fleming. The value is nearly 20 per cent. less than that given by Matthiessen.

TABLE V.

Electrical Volume-resistivity of Pure Metals at Various Temperatures.

(From experiments by Fleming and Dewar.)

Tempera-ture (Centi-grade).	Specific Resistance in C.G.S. Units.						
	Silver.	Copper.	Gold.	Alu-minium.	Mag-nesium.	Zinc.	Nickel.
− 150°C.	580	515	960	900	1,625	2,270	2,400
− 100°C.	890	860	1,360	1,485	2,585	3,430	3,645
− 50°C.	1,195	1,210	1,770	2,075	3,485	4,585	5,130
0°C.	1,468	1,561	2,197	2,665	4,355	5,751	6,935
+ 50°C.	1,775	1,895	2,605	3,255	5,205	6,925	8,915
+ 100°C.	2,070	2,215	3,030	3,845	6,010	8,115	11,210
+ 150°C.	2,360	2,560	3,440	4,420	6,790	9,345	13,820
+ 200°C.	2,885	2,890	3,840	5,000	7,540	10,590	16,630

Tempera-ture (Centi-grade).	Specific Resistance in C.G.S. Units.						
	Iron.	Cad-mium.	Palla-dium.	Plati-num.	Tin.	Thal-lium.	Lead.
− 150°C.	2,325	4,210	4,205	4,760	5,080	6,275	8,515
− 100°C.	4,360	6,115	6,290	6,890	7,650	10,155	12,340
− 50°C.	6,590	8,045	8,300	8,945	10,350	13,930	16,390
0°C.	9,065	10,023	10,219	10,917	13,048	17,633	20,380
+ 50°C.	11,770	12,060	12,055	12,910	15,895	21,225	24,555
+ 100°C.	14,765	14,315	13,840	14,820	18,870	24,770	28,900
+ 150°C.	18,110	16,725	15,545	16,690	21,990	28,100	33,470
+ 200°C.	21,960	19,325	15,215	18,535	25,155	31,635	38,000

TABLE VI.

Electrical Conductivity of Metals at 0°C. in Absolute and Arbitrary Units.

Absolute conductivity is measured in mhos. The mho is the reciprocal of the ohm, and the mho-conductivity is obtained by dividing 10^9 by the volume-resistivity in C.G.S. units.

Metal.	Mho-conductivity at 0°C. 1 mho = conductivity of a centimetre-cube of material having a volume-resistivity of 1 ohm.	Arbitrary conductivity. Annealed electrolytic silver = 100 at 0°C.
Silver (electrolytic annealed)	681,198	100
Copper (electrolytic annealed)	640,615	94·04
Gold (annealed)	455,166	66·81
Aluminium (annealed)	375,234	55·08
Sodium (pressed at 20°C.)	253,973	37·43
Magnesium	229.616	33·71
Zinc	171,381	25·16
Calcium (at 18°C.) ...	150,818	22·14
Nickel (electrolytic)...	144,196	21·17
Potassium (at 20°C.)	141,990	20·85
Lithium (at 20°C.) ...	129,428	19·00
Indium.................	112,400	16·50
Iron	110,314	16·19
Cobalt	106,140	15·58
Cadmium..............	99,770	14·64
Palladium	97,857	14·64
Platinum	91,600	13·44
Tin	76,640	11·25
Thallium	56,712	8·32
Lead....................	49,067	7·20
Strontium (at 18°C.)	45,708	6·71
Arsenic.................	32,425	4·76
Antimony..............	31,471	4·62
Mercury	10,630	1·56
Bismuth	9,091	1·33

TABLE VII.

Volume-resistivity of Alloys of known Composition at 0°C. in C.G.S. Units per Centimetre-cube.

Mean temperature coefficients taken at 15°C.

(Fleming and Dewar.)

Alloy.	Resis-tivity at 0°C.	Tempera-ture coefficient at 15°C.	Composition in per cents.
Platinum-Silver	31,582	0·000243	Pt 33%, Ag 66%
Platinum-Iridium...........	30,896	0·000822	Pt 80%, Ir 20%
Platinum-Rhodium	21,142	0·00143	Pt 90%, Rh 10%
Gold-Silver	6,280	0·00124	Au 90%, Ag 10%
Manganese-Steel	67,148	0·00127	Mn 12%, Fe 80%
Nickel-Steel	29,452	0·00201	Ni 4·35%
German-Silver	29,982	0·000273	Cu_5 Zn_3 Ni_2
Platinoid*....................	41,731	0·00031	—
Manganin	46,678	0·0000	{Cu 84%, Mn 12%, Ni 4%
Aluminium-Silver	4,641	0·00238	Al 94%, Ag 6%
Aluminium-Copper	2,904	0·00381	Al 94%, Cu 6%
Copper-Aluminium	8,847	0·000897	Cu 97%, Al 3%
Copper-Nickel-Aluminium	14,912	0·000645	{Cu 87%, Ni 6·7%, Al 6·5%
Titanium-Aluminium	3,887	0·00290	—

* Platinoid is an alloy first produced by Martino, the composition being said to be similar to that of German silver, but with a little tungsten added. It varies a good deal in composition according to manufacture, and the resistivity of different specimens is not identical. The electrical properties of platinoid were first made known by Dr. J. T. Bottomley, F.R.S., in a Paper read at the Royal Society, May 5, 1885.

TABLE VIII.

Electrical Volume-resistivity of Various Liquids in Ohms per Centimetre-cube.

I.—FUSED SALTS.

Substance.	Resistivity.	Remarks.	Observer.
Plumbic chloride Pb Cl$_2$	0·376	...	F. Braun
Silver chloride AgCl	0·392	at 600°C.	W. Kohlrausch
Sodic nitrate NaNO$_3$	0·817	...	F. Braun
Zinc chloride ZnCl$_2$	109·3	...	F. Braun

II.—AQUEOUS SOLUTION OF ACIDS.*
Solution having maximum conductivity at 18°C.

Acid.	Resistivity.	Temperature Coefficient.	
Nitric Acid NO$_3$H	1·28	0·014	
Hydrochloric Acid HCl	1·32	0·0155	
Sulphuric Acid H$_2$SO$_4$	1·36	0·0162	as given by
Tartaric Acid C$_4$H$_6$O$_6$	100·0	0·0192	G. Wiedemann.
Acetic Acid C$_2$H$_4$O$_2$ ...	618·4	0·0174	

III.—AQUEOUS SOLUTIONS OF SALTS AND HYDRATES.*

Salt.	Resistivity.	Specific Gravity.	Temperature Coefficient.	
Potassic Hydrate KHO	1·84	...	0·0225	
Potassic Iodide KI......	2·29	1·70	0·014	
Ammonic Chloride Am Cl	2·36	1·078	0·0155	
Silver Nitrate AgNO$_3$...	4·48	2·18	0·0211	as given
Sodic Chloride NaCl ...	4·66	1·201	0·0234	by G.
Hydro-potassic Carbonate KHCO$_3$	8·54	1·15	0·0199	Wiedemann.
Copper sulphate CuSO$_4$	29·37	1·208	0·0241	
Zinc sulphate ZnSO$_4$...	21·35	1·286	...	

* The above Table contains only the resistivity values corresponding to the maximum conductivity in the case of the aqueous solutions of salts and acids.

TABLE IX.

Electrical Volume-resistivity of Various Badly Conducting Liquids in Megohms per Centimetre-cube.

Substance.	Resistivity in megohms per c.c.	Observer.
Ethylic Alcohcl	0·522	Pfeiffer.
Ethylic Ether	1·175 to 3·760	W. Kohlrausch.
Benzine	4 700	...
Water.......................	1·446 at 14°C. 5·222 at 18°C.	Pfeiffer. F. Kohlrausch.
Absolutely pure water approximates probably to	25·0 at 18°C.	Estimated value by Kchlrausch and Heydweiler.
All very dilute aqueous salt solutions having a concentration of about 0·00001 of an equivalent gramme-molecule* per litre approximate to	1·00 at 18°C.	From results by F. Kohlrausch and others.

* An equivalent gramme-molecule is a weight in grammes numerically equal to the chemical equivalent of the salt. For instance, one equivalent gramme-molecule of sodic chloride is a mass of 58·5 grammes, since NaCl =58·5.

TABLE X.

Volume resistivity of Solutions of Copper and Zinc Sulphate of Various Densities at 10°C. in Ohms per Centimetre-cube.

(Ewing and MacGregor.)*

SULPHATE OF COPPER.

Density.	Resistivity.	Density.	Resistivity.
1·0167	164·4	1·1386	35·0
1·0216	134·8	1·1432	34·1
1·0318	98 7	1·1679	31·7
1·0622	59 0	1·1829	30·6
1 0858	47·3	1·2051	29·3
1 1174	38 1	(saturated)	…

SULPHATE OF ZINC.

Density.	Resistivity.	Density.	Resistivity.
1 0140	182 9	1·2709	28·5
1·0187	140 5	1·2891	28·3
1·0278	111·1	1·2895	28·5
1·0540	63·8	1·2987	28·7
1·0760	50·8	1·3288	29·2
1·1019	42·1	1·3530	31·0
1·1582	33·7	1·4053	32·1
1·1845	32 1	1·4174	33·4
1·2186	30 3	1·4220	33·7
1·2562	29 2	(saturated)	…

* *Trans.* Roy. Soc., Edin., Vol. XXVII., 1873.

The resistivity values obtained by various observers for electrolytic conductors do not agree at all well. The above values are not quite in accord with other results.

TABLE XI.

Electrical Volume-resistivity of Dielectrics expressed in Millions of Megohms (Mega-megohms) per Centimetre-cube, and in Megohms per Quadrant-cube—i.e., a Cube whose side is 10^9 cms.

Substance.	Resistivity.		Temperature. Cent.
	Mega-megohms per c.c.	Megohms per quadrant-cube.	
Bohemian Glass	61	·061	60°
Mica..........................	84*	·084	20°
Gutta-percha	450*	·45	24°
Flint Glass at 60°C.	1,020	1·02	60°
Glover's Vulcanised India-rubber	1,630	1·63	15°
Siemens' ordinary pure Vulcanised India-rubber	2,280	2·28	15°
Shellac	9,000*	9·0	28°
India-rubber..................	10,900	10·9	24°
Siemens' High Insulating Fibrous Material	11,900	11·9	15°
Siemens' Special High Insulating India-rubber	16,170	16·17	15°
Flint Glass at 20°C.	20,000	20·0	20°
Ebonite	28,000*	28	46°
Paraffin	34,000*	34	46°

The values of the resistivity of various dielectrics given in the above Table can only be taken as approximate. In most cases the observers have not stated the time of imposition of the electric stress. Values marked (*) are those given by experiments by Profs. Ayrton and Perry : "On the Viscosity of Dielectrics" (*Proc.* Roy. Soc., March, 1878), "after several minutes' electrification."

The temperature coefficients of the resistivities of dielectric conductors are very large ; in most cases far larger than those of the pure metals, and the apparent resistivity is also a function of the value and of the time of operation of the electromotive force.

TABLE XII.

Resistances of various sizes of Platinoid Wire.

(London Electric Wire Company, Ltd.)

Size.	Diameter.		Resistance (approximate).	
L.S.G.	Inch.	m/m.	Per lb.	Per 1,000 yards.
			Ohms.	Ohms.
8	0·160	4·064	0·1241	28·852
10	0·128	3·251	0·3031	45·084
12	0·104	2·642	0·6957	68·292
14	0·080	2·032	1·9869	115·416
16	0·064	1·626	4·8520	180·338
18	0·048	1·219	15·3312	320·601
19	0·040	1·016	31·7952	461·664
20	0·036	0·914	48·4602	569·952
21	0·032	0·813	77·6480	721·368
22	0·028	0·711	132·4272	942·192
23	0·024	0·610	245·3280	1282·392
24	0·022	0·559	347·4720	1526·184
25	0·020	0·508	508·7280	1846·656
26	0·018	0·457	775·3680	2279·808
27	0·0164	0·417	1125·2160	2746·440
28	0·0148	0·376	1696·4880	3372·264
30	0·0124	0·315	3442·8000	4803·984
32	0·0108	0·274	5982·7200	6332·904
34	0·0092	0·2337	11362	8727·120
36	0·0076	0·1930	24398	12789·640
38	0·0060	0·1524	62805	20518·560
40	0·0048	0·1219	153333	32060·160
42	0·0040	0·1016	317904	46166
44	0·0032	0·0813	784280	72136
46	0·0024	0·0610	2453280	128239
47	0·0020	0·0508	5087280	184665

TABLE XIII.

Resistances of various sizes of Manganin Wire.

(London Electric Wire Company, Ltd.)

Size.	Diameter.		Resistance (approximate).	
L.S.G.	Inch.	m/m.	Per lb.	Per 1,000 yards.
			Ohms.	Ohms.
14	0·080	2·032	2·027	117·85
16	0·064	1·626	4·952	184·17
18	0·048	1·219	15·652	327·42
20	0·036	0·914	49·475	582·00
22	0·028	0·711	135·175	962·00
24	0·022	0·559	354·700	1560·25
26	0·018	0·457	791·475	2330·00
28	0·0148	0·376	1731·750	3442·50
30	0·0124	0·315	3514·250	4907·50
32	0·0108	0·274	6107·250	6467·50
34	0·0092	0·2337	11597·750	8912·50
36	0·0076	0·1930	24904·500	13060
38	0·0060	0·1524	64100	20955
40	0·0048	0·1219	156525	32875
42	0·0040	0·1016	324550	47150
44	0 0032	0·0813	792375	73675

TABLE XIV.

Sizes, Resistances and Weights of Wound Copper Wire of Annealed High Conductivity Commercial Copper Wire, according to the Standards adopted by the Committee on Copper Conductors.

Size S.W.G.	Amperes at 1000 per sq. inch at five-ninths Amperes at I.E.E. rate say approximately Amps per 100 yards	Standard Amperes at I.E.E.	Diameter Inch	Diameter Milli. metres	Area Square inches	Area Square milli-metres	Ohms per 1000 yards.	Standard resistance at 60° F. Ohms per mile.	Ohms per metre.	Maximum resistance 2 per cent. above standard.	Standard weight. Pounds per 1000 yards.	Pounds per mile.	Maximum weight 2 per cent. above standard Pounds per mile.
22	0·1658	1·7	0·028	0·7112	0·0006158	0·3973	39·05	68·72	42·70	70·09	7·120	12·53	12·28
20	1·0179	2·6	0·036	0·9144	0·001018	0·6567	23·62	41·57	25·83	42·40	11·77	20·72	20·31
18	1·8096	4·2	0·048	1·219	0·001810	1·168	13·28	23·38	14·53	23·85	20·98	36·88	36·09
16	3·2170	6·8	0·064	1·626	0·003217	2·075	7·478	13·16	8·178	13·42	37·20	65·47	64·16
14	5·0265	9·8	0·080	2·032	0·005027	3·243	4·784	8·419	5·282	8·587	58·18	102·3	100·8
12	8·4949	15·0	0·104	2·642	0·008495	5·480	2·831	4·982	3·096	5·082	98·24	172·9	169·4
10	12·863	21·0	0·128	3·251	0·01287	8·303	1·868	3·288	2·043	3·354	148·8	261·9	256·7
8	20·106	31·0	0·160	4·064	0·02011	12·97	1·195	2·104	1·307	2·146	232·5	409·2	401·0
6	28·952	42·0	0·192	4·877	0·02895	18·66	0·8307	1·462	0·9085	1·491	334·7	589·1	577·3

TABLE XIV.—continued.

Size.	Amperes at 1,000 per square inch.	Amperes at I.E.E. standard.	Diameter of each wire.		Diameter of strand.		Effective area, viz, area of solid wire having same conductivity.		Standard resistance at 60°F.			Max. resis. allowable, viz, 2% above standard ohms per mile.	Standard weight per mile.	Minimum weight allowable, viz, 2% below standard per mile.
S.W.G.			In ins.	In millimetres.	In ins.	In millimetres.	Sq. inches.	Sq. millimetres.	Per 1,000 yards.	Per mile.	Per kilometre.			
3/22	1·825	4·258	0·028	0·7112	0·060	1·524	0·001825	1·177	13·18	23·19	14·41	23·65	37·05	36·31
3/20	3·016	6·444	0·036	0·9144	0·078	1·981	0·003016	1·946	7·972	14·03	8·718	14·31	62·92	61·67
3/18	5·364	10·81	0·048	1·219	0·108	2·616	0·005364	3·461	4·482	7·889	4·902	8·049	111·8	109·6
7/22	4·266	8·543	0·028	0·7112	0·084	2·134	0·004266	2·752	5·636	9·920	6·164	10·11	88·63	86·87
7/20	7·152	12·90	0·036	0·9144	0·108	2·743	0·007152	4·550	3·410	6·001	3·729	6·121	148·7	146·6
7/18	12·54	20·68	0·048	1·219	0·144	3·658	0·01254	8·090	1·918	3·375	2·097	3·443	260·5	255·3
7/16	22·27	33·12	0·064	1·626	0·192	4·877	0·02227	14·37	1·080	1·900	1·181	1·938	463·1	453·9
7/14	34·88	47·80	0·080	2·032	0·240	6·096	0·03488	22·47	0·6903	1·215	0·7550	1·239	728·6	709·1
7/12	58·84	73·47	0·104	2·642	0·312	7·925	0·05884	37·96	0·4086	0·7192	0·4469	0·7336	1223	1199
7/10	89·17	108·3	0·128	3·251	0·384	9·754	0·08917	57·53	0·2697	0·4746	0·2950	0·4841	1853	1816
19/22	11·57	19·36	0·028	0·7112	0·140	3·556	0·01157	7·464	2·079	3·659	2·2736	3·732	240·8	236·0
19/20	19·12	29·23	0·036	0·9144	0·180	4·572	0·01912	12·34	1·257	2·213	1·3751	2·257	398·3	390·3
19/18	33·99	46·85	0·048	1·219	0·240	6·096	0·03399	21·93	0·7074	1·245	0·7736	1·270	707·9	693·7
19/16	60·39	75·06	0·064	1·626	0·320	8·128	0·06039	38·96	0·3981	0·7007	0·4354	0·7147	1258	1233
19/14	94·42	108·3	0·080	2·032	0·400	10·16	0·09442	60·91	0·2547	0·4482	0·2785	0·4572	1966	1927
19/12	159·5	166·4	0·104	2·642	0·520	13·21	0·1595	102·9	0·1507	0·2653	0·1649	0·2706	3323	3257
19/10	241·7	234·0	0·128	3·251	0·640	16·26	0·2417	155·9	0·09949	0·1751	0·1088	0·1786	5034	4933
37/20	37·22	50·47	0·036	0·9144	0·252	6·401	0·03722	24·01	0·6460	1·137	0·7065	1·160	775·8	760·3
37/18	66·19	80·91	0·048	1·219	0·336	8·534	0·06619	42·70	0·3653	0·6394	0·3973	0·6522	1379	1352
37/16	117·6	129·6	0·064	1·626	0·448	11·38	0·1176	75·87	0·2045	0·3599	0·2236	0·3671	2451	2405

TABLE XV.

The Value of the Ohm.

Observer.	Date.	Method.	Value of B.A.U. in ohms.	Value of 100 centimetres of mercury in ohms.	Value of ohm in centimetres of mercury.
Lord Rayleigh	1882	Rotating coil	0·98651	0·94133	106·24
Lord Rayleigh	1883	Lorenz method	0·98677	...	106·21
G. Wiedemann	1884	Rotation through 180°	...	...	106·19
Mascart	1884	Induced current	0·98611	0·94096	106·33
Rowland	1887	Mean of several methods	0·98644	0·94071	106·32
Kohlrausch	1887	Damping of magnets	0·98660	0·94061	106·32
Glazebrook	1882 1888	Induced currents ...	0·98665	0·94074	106·29
Wuilleumier	1890	...	0·98686	0·94077	106·27
Duncan and Wilkes	1890	Lorenz.................	0·98684	0·94067	106·32
Jones	1891	Lorenz	...	0·94067	106·31
		Mean	0·98653	...	...
Strecker	1885	...	...	0·94056	106·32
Hutchinson	1888	An absolute determination of resistance was not made. The value 0·98656 has been used	...	0·94074	106·30
Salvioni	1890	...	...	0·94054	106·33
Salvioni	..	...	...	0·94076	106·30
		Mean	...	0·94076	106·28
H. F. Weber	1884	Induced current	...	...	105·37
H. F. Weber	...	Rotating coil	...	...	106·16
Roiti	1884	Mean effect of induced current	Absolute measurements compared with German silver wire coils issued by Siemens or Strecker		105·89
Himstedt	1885	...	...		105·98
Dorn	1889	Damping of a magnet	...		106·24
Wild	1883	Damping of a magnet	...		106·03
Lorenz	1885	Lorenz method	...		105·93

TABLE XVI.

Table of Resistance of High Conductivity Round Copper Conductors to Alternating and Continuous Currents.

Size of stranded cable.	Area in square inches.	Resistance in ohms per 1,000 yards.	
		To continuous currents.	To alternating currents. Frequency 100 ∿ per second.
7/18	0 0126	1·974	1·974
7/17	0·0172	1·452	1·452
7/16	0·0225	1·108	1·108
7/15	0·0285	0·878	0·878
7/14	0·0351	0·712	0·712
19/18	0·0351	0·712	0·712
19/17	0·0477	0·524	0·524
19/16	0 0624	0·401	0·401
19/15	0·0789	0·318	0·318
19/14	0 0973	0·257	0·257
19/13	0·1289	0·194	0·195
19/12	0·1645	0·153	0·155
19/11	0·2048	0·122	0·1247
19/10	0·2500	0·100	0·1034
37/16	0·1227	0·204	0·2041
37/15	0·1551	0·162	0·104
37/14	0·1913	0·131	0·1334
37/13	0·2534	0·099	0·1024
37	0·3000	0·083	0·087
37/12	0·3235	0·077	0·081
37/11	0·4000	0·063	0·068
37/10	0·4905	0·051	0·057
61/15	0·2582	0·097	0·1004
61/14	0·3185	0·078	0·082
61/13	0·4218	0·060	0·065
61	0·5000	0·050	0·056
61/12	0·5385	0·046	0·052
61	0·6000	0·041	0·048
61/11	0·6476	0·039	0·0465
61	0·7000	0·0357	0·0435
61/10	0·8167	0·0305	0·0391
91/13	0·6354	0·0385	0·0458
91	0·7500	0·0330	0 0412
91/12	0·8111	0·0305	0·0391
91	0·9000	0·0277	0·0370
91/11	1·0000	0·0250	0·0350

TABLE XVII.

Materials Used for Electrical Resistances.

Material.	Resistance in microhms per centimetre-cube at 0°C.		
Copper (hard drawn) ...		1·626	microhms.
Iron (annealed)		9·0	,,
German Silver	From	20·0 to 30·0	,,
Platinoid	About	40·0	,,
Manganin		42·0 to 46·0	,,
Resista		76·0	,,
Beacon	About	80 0	,,
Eureka	,,	40·0	,,
Cast Iron.......		80·0 to 100·0	,,
Steel		15·0 to 50·0	,,
Graphite		300·0 to 400·0	,,
Arc Lamp Carbon		3,000 to 4,000	,,
Plumbago and Stourbridge Clay, about equal parts mixed and well baked.	,,	100,000	,,
Saturated Solution of Sulphate of Copper.		$29·3 \times 10^6$	,,
Saturated Solution of Sulphate of Zinc.		$83·7 \times 10^6$	,,
Dilute Sulphuric Acid, 1·2 density.		$1·24 \times 10^6$	,,
Sodic Sulphate Solution, 15 per cent. salt.		$11·37 \times 10^6$	,,

CHAPTER III.

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THE MEASUREMENT OF ELECTRIC CURRENT.

1. Classification of Electric Currents.—Electric currents may be either *unidirectional*—that is, flowing continually in one and the same direction, or *alternating*—that is, periodically changing their direction.

The character of a current in a conductor is determined by the nature of the field of magnetic force associated with the conductor when it forms part of a circuit in which a current exists. If the direction of the field when tested in any manner at any point outside the conductor is always in the same direction, as indicated, say, by the behaviour of a small magnet held in the field, the current is said to be *unidirectional* or *continuous*. If the direction of the field periodically changes, the current in the conductor is said to be *alternating* or *periodic*.

A continuous current may be either *uniform* or *unvarying*, or it may be *intermittent* or *pulsatory*. An alternating current may, in the same manner, be *steadily periodic*, or it may be *variable* and *periodic*.

Alternating currents are furthermore divided into *monophase* and *polyphase*. In the first case the periodic current exists in a single circuit; in the second case the circuit in which the current is created is a complex circuit, each elementary circuit of which is traversed by a periodic current, these various currents differing in phase—that is to say, not changing their directions at the same instant, but preserving a constant phase difference with respect to each other.

z 2

In the measurement of electric currents we are concerned with the determination of a quantity capable of having direction as well as magnitude. The magnitude—or, as it is sometimes called, the strength—is measured by the degree to which some measurable physical effect is produced. The three most important physical effects accompanying or constituting an electric current are : (i.) The production of heat in the conductor; (ii.) the production of a magnetic field around and in the conductor; and (iii.) the production of an electrolytic effect in liquid conductors of a certain kind forming part of the circuit.

Other things being equal, the heating effects of an unvarying current increases as the square of the current when measured in either of the other two ways. That is to say, if there be two currents which, when respectively passed through the same circuit, produce magnetic forces at any one point in the ratio of $1 : x$, they produce in the same time chemical decompositions of a given electrolyte inserted in the circuit which, measured by the masses of a liberated ion, are in the ratio of $1 : x$. These two currents, however, will produce in any part of the conducting circuit quantities of heat which are in the ratio of $1 : x^2$. Two currents are said to have the same *mean square* value if they produce in the same time and in the same conductor the same total quantity of heat.

If a current is periodic or alternating, its *effective* or *virtual* value, estimated in its equivalent of continuous current, is that of the unvarying and unidirectional current which will produce in the same conductor the same heat in the same time. This effective value is also called the *root-mean-square* (R.M.S.) value, because it is equal to the square root of the mean of the squares of the instantaneous current values taken at equidistant and very near intervals of time throughout one complete cycle. The *true mean* (T.M.) value of a current is the arithmetic mean of its instantaneous values taken at equidistant intervals of time throughout any period.

We have, therefore, three methods of measuring a current, which may be called respectively the *thermal,* the *magnetic,* and the *chemical* methods.

The definition of the absolute unit of current in magnetic measure is thus given:—

Let a thin conducting wire be bent into a circle having a radius of r centimetres, and let the pole of a very long linear magnet of strength m be placed at the centre, the other complementary pole being at a considerable distance. Then, if the mechanical force on the magnetic pole placed at the centre is measured in absolute units of force (in dynes), and is represented by f, the absolute magnetic measure of the current (C) is determined by the relation

$$C = \frac{fr}{2\pi m}.$$

The absolute electromagnetic (C.G.S.) unit of current is therefore the current which, when circulating in a circuit of unit radius, exerts on a unit magnetic pole placed at the centre a force of 2π dynes. The practical unit of current (the ampere) is one-tenth of the above electromagnetic unit in magnitude.

The Standard or *International Ampere,* as already explained, has been officially defined as the current which, when passed through a solution of nitrate of silver, made according to a certain specification, deposits 0·001118 of a gramme of silver per second on the cathode.* As previously pointed out, there is reason to believe that the unit of current called the International ampere is smaller by about 1 part in 1,000, or 1 part in 800, than the unit of current or ampere defined as one-tenth of the absolute C.G.S. unit of current. This view is confirmed by the discrepancy between the values of the mechanical equivalent of heat, or, as it should be called, the dynamical value of the specific heat of water when determined mechanically and electrically. In the determination of the above equivalent by electrical methods, values obtained by Griffiths exceed those of Rowland, made by mechanical or frictional methods, by about 1 part in 400 at all temperatures between 15deg. and 20deg. on the nitrogen gas thermometer scale. Those of Schuster and Gannon exceed those of Rowland at 19·1deg. on the same scale by 1 part in 500. Since the current enters as a square, it follows that the above discrepancies would be reconciled by the substitution of 0·0011191 or 0·0011194 for the electro-chemical equivalent of silver, instead of 0·001118, as now adopted.

* For the electro-chemical equivalents of other metals see Table II., p. 420, at the end of this chapter.

A careful series of experiments by Patterson and Guthe at Michigan in 1898,[*] showed that the most probable value of the ampere-second silver equivalent is 0·0011192 gramme, and hence a corresponding change must be made in the E.M.F. at 15°C. of the Clark cell, which is thereby reduced to 1·4327 International volts, instead of 1·4342, as now accepted.

Dr. K. Kahle[†] has investigated carefully the silver voltameter, and finds that the electro-chemical equivalent obtained for silver varies slightly with the age of the nitrate of silver solution. He states that with a freshly prepared solution the ampere-second equivalent is very nearly 0·0011182 gramme and with an old solution 0·0011193 gramme. Some small degree of uncertainty therefore exists as to the actual relation between the International and the True or theoretical ampere, similar to that which occurs in the case of the International and True ohm.

§ 2. The Measurement of Current by the Electrolysis of a Solution of Copper Sulphate. Standardisation of an Ammeter.

—The employment of a solution of sulphate of copper as the electrolyte enables an electrochemical determination of a current to be made with less initial outlay than when silver is used. With certain precautions a high degree of accuracy can be obtained. In order that any great degree of exactness may be reached in the measurement of a current, it is essential that the current shall be as nearly as possible constant. To standardise an ampere-balance or ammeter— that is to say, to determine the true value in international amperes corresponding to an observed scale reading—we proceed as follows :—

The current should be provided from large secondary cells which have been slightly discharged—that is, about 10 per cent. of their full charge taken out. The ammeter or ampere-balance to be standardised should be joined in series with the cells and with a regulating wire resistance and a carbon rheostat (*see* page 81), by which to make very small variations in the resistance of the circuit.

This circuit must also include an electrolytic cell containing a solution of sulphate of copper. The electrolytic cell found most convenient is a round glass jar about 30cm. high

and 20cm. in diameter. This is placed on a slab of wood at the sides of which are two vertical wooden rods carrying brass forked strips which project over the top of the jar (*see* Fig. 1). One of these forks may have three prongs and the other one two. To these prongs are clamped by brass clamps the copper anode and cathode plates. Each plate can be separately removed from the solution. It is convenient to make the two-prong brass fork the cathode. To these forks large screw terminals are attached. The solution placed in the electrolytic cell is made by dissolving
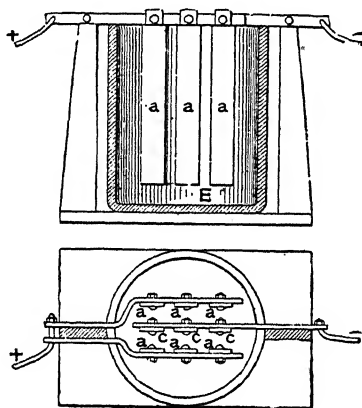
FIG. 1.—Voltameter for Current Measurement by the Electrolysis of Copper Sulphate.

pure re-crystallised sulphate of copper in distilled water until a density of 1·15 or 1·18 is obtained. One per cent. by volume of pure sulphuric acid is then added. This addition of free acid is absolutely necessary to obtain good results. No satisfactory determinations can be made with neutral solutions of sulphate of copper. In any case the density of the solution should be between 1·1 and 1·2. The copper plates should be cut from high-conductivity pure electrolytic copper, and should be 15cm. long and 5cm. wide. The corners of the plates should be neatly

rounded off and all burr on the edges removed. Each plate should be stamped with a number in one corner for recognition, and have a hole near the top edge by means of which the plate can be handled with a wire hook.

The number and size of these plates to be employed in each experiment is determined by the current to be measured. To obtain an adherent and regular deposit of metal on the cathode plate, it is necessary that the surface exposed should exceed 20 sq. cm. per ampere. An exposed cathode surface of 50cm. per ampere gives excellent results.*

If, then, the cathode plates are of the dimensions above given and are placed in the electrolyte 10cm. deep, the exposed surface on each plate, assuming both sides used, will be 100 sq. cm., and each plate will be good for 2 amperes. The plates selected as cathodes have then to be clamped to the cathode fork so that each row is included between two rows of anode plates, and the number so selected that the above-mentioned current density is not exceeded. These preparations being complete, the cathode plates have to be very carefully cleaned and weighed. The cleaning is best performed by placing the plates in a flat porcelain dish and covering them with strong commercial nitric acid. This immediately evolves copious nitrous fumes, which are very deleterious, and hence the process should be conducted in the open air or in a well-ventilated fume cupboard. The experimentalist should carefully avoid inhaling the nitrous vapours. The plates having been left in the acid for a few minutes, or until the acid boils violently, are fished out by the aid of a stout copper wire and dropped into a large jug of clean water. Each plate should then present a clean, bright salmon-coloured surface, without a trace of brown oxide upon it on either side. If the plates are newly cut from sheet copper it will generally be necessary to give them a good scouring with emery-cloth and water before treatment with the nitric acid.

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* *See* Mr. A. W. Meikle, "On the Electrolysis of Copper Sulphate in Standardising Electrical Instruments." *Proc.* Phys. Soc. of Glasgow University, Jan. 27, 1888.

Unless the plates are chemically clean the electrolytically deposited copper will not adhere to them.

The plates, having been cleansed and washed copiously are then to be dried between clean white blotting paper as quickly as possible, and completely dried in an air oven or in a dessicating vessel. When thoroughly dry, each plate must be weighed on a good chemical balance, and its weight recorded. In so doing, the plate must not be touched with the fingers, but be handled by a clean copper wire thrust through the hole in the plate. When dried and weighed the plates should, as soon as possible, be placed in the electrolytic cell and the experiment commenced. The circuit must be completed so that the same uniform current flows through the ammeter to be calibrated, and the electrolytic cell and the weighed plates must be made the cathode or negative pole of the cell so that the current deposits copper upon them. The time when the circuit is closed must be noted on a good chronometer, and the current must be kept perfectly constant, as indicated by the observed reading of the ammeter, for a time varying from one to four hours. The circuit is then opened, and the time of so doing is noted.

The weighed plates should then at once be removed from the electrolytic cell, be washed copiously in water, and be again dried and weighed. The deposit of electrolytic copper should be a bright, clean and adherent film of metal. The increase in weight of each plate is noted, and the total deposit of copper in an observed time thus found. The increase in weight in grammes per second can then at once be calculated. This last figure, divided by the electro-chemical equivalent of copper, gives the mean value in amperes of the current, or true time average of the current during the experiment.

It was found by experiments conducted in Lord Kelvin's laboratory at Glasgow University that the electro-chemical equivalent of copper varies with the temperature of the electrolyte and with the current density. Hence, in reckoning

out results the proper electro-chemical equivalent must be employed as given by Mr. Meikle. The electrochemical equivalent in grammes per ampere-second is a number not far from 0·000328, and it diminishes very slightly both with rise of temperature and increase of cathode surface per ampere.

The Electro-chemical Equivalent of Copper in grammes per ampere-second.

Area of cathode surface in square centims. per ampere.	Temperature of electrolyte.				
	2°C.	12°C.	23°C.	28°C.	35°C.
50	·0003288	·0003287	·0003286	·0003286	·0003282
100	·0003288	·0003284	·0003283	·0003281	·0003274
150	·0003287	·0003281	·0003280	·0003278	·0003267
200	·0003285	·0003279	·0003277	0003274	·0003259
250	·0003283	·0003278	·0003275	·0003268	·0003252
300	·0003282	·0003278	·0003272	·0003262	·0003245

If accurate results are to be obtained, the following precautions must be taken in the employment of the copper voltameter as a means of measuring the time integral of a continuous current :—

No satisfactory results can be reached unless the copper sulphate solution contains free sulphuric acid. On the other hand, copper plates placed in this solution lose in weight at a rate depending on their immersed surface. The plates must, therefore, be most carefully freed from all traces of oxide before being used as the weighed plates. The anode plates should also be cleaned, but it is not necessary that they should be of such pure copper as the cathode plates. If, however, the electrolyte is to be frequently used, it is desirable to employ electrolytic copper for both sets of plates. It is important to round off the edges and make the cathode plates very smooth, so as to avoid producing a rough deposit of copper or one nodulous or uneven. The plate is then not so easily dried, and there is a risk of small particles of copper being detached.

It need hardly be said that in conducting the observations a good and correct watch or chronometer must be used for determining the time during which the experiment lasts.

The copper voltameter may be advantageously employed in evaluating a steady continuous current and in standardizing an ampere-balance, but it is not so well adapted for current measurement when the current cannot be kept constant for considerable periods of time.

With the above precautions, however, there is not the slightest difficulty in determining the true value of a steady unidirectional current of the order of 10 amperes to within one quarter per cent., and with a little care to within one part in a thousand.

§ 3. **The Measurement of Current by the Electrolysis of Silver Nitrate.**—The measurement of the time integral of a current when the current is of the order of about one ampere or so, is best effected by the employment of a solution of nitrate of silver, as silver has a higher atomic weight than copper, and, moreover, is a monad element. Hence the mass of silver deposited by a given current is nearly three times that of the copper deposited in the same time. Accordingly there is more mass to weigh, and hence, on that account, a greater possibility of accuracy when dealing with small currents.

On the other hand, the materials used are expensive compared with the copper method, and there is considerably greater difficulty in washing and drying the silver deposit. From a neutral or nearly neutral solution of silver nitrate the deposit of silver on a platinum cathode is apt to be nodulous, crystalline or non-adherent. This irregular deposit occludes small particles of the salt or solution and it is not easy to wash the deposit so as to secure perfect removal of the salt without washing away some of the metal.

The full detailed specification for performing the operation of electrolysis of silver nitrate in current measurements has already been given (*see* Chap. I., p. 57), hence there is no need to repeat the details.

A very careful investigation by Dr. K. Kahle (see *Science Abstracts*, Vol. II., p. 41, or *Zeitschr. Instrumentk.*, 18, p. 229, 1898), was conducted with the special object of seeing how far the silver voltameter can be relied upon for standardising current. In the course of 115 measurements the amount of silver deposited by the same current acting for 40 minutes varied from 0·97134 grammes to 0·97473 grammes. With great care an accuracy of 1 part in 10,000 can be obtained. A clean platinum surface receives rather less deposit than an existing surface of silver, and fresh solutions of silver nitrate deposit less easily than old ones. To free the silver deposit of all silver nitrate solution repeated cold water washing and one final washing in water at 80°C. is necessary.

Dr. Kahle's value for the mass of silver deposited per ampere-second is 0·0011193 from an old solution and 0·0011182 from a freshly-prepared solution.

The reader is referred for further information to the following original Papers :—

Lord Rayleigh and Mrs. Sidgwick.   "The Electrochemical Equivalent of Silver."   *Phil. Trans.* Roy. Soc. Lond., 1884.

K. Kahle.   "The Silver Voltameter and Standard Cells." *Zeitschrift für Instrumentenkunde*, 18, 1898, pp. 229, 267.

H. S. Carhart.   "Standards of Measurement."   *Science*, 8 p. 326, 1898.

G. W. Patterson and K. E. Guthe.   "Electrochemical Equivalent of Silver."   *Proc.* Amer. Assoc., 47, p. 154, 1898.

T. W. Richards, E. Collins and G. W. Heimrod.   "Electrochemical Equivalents of Silver and Copper."   *Proc.* Amer. Acad., 85, p. 123, 1899.   *Science Abstracts*, Vol. III., p. 332.

G. F. C. Searle.   "The Silver Voltameter."   *The Electrician*, Vol. XXIX., p. 111., 1892.

Mr. Searle here discusses the various advantages and disadvantages in using different salts of silver.
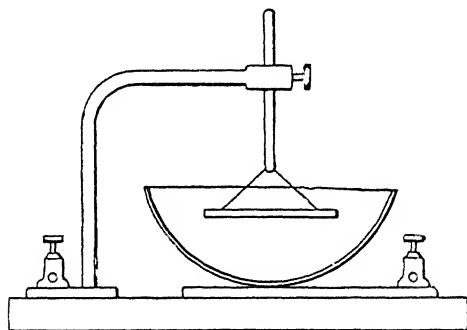
Fig. 2.—Silver Voltameter for Current Measurement.

The practical details to which attention must be directed in making a current determination by the silver voltameter are referred to in the above-mentioned article by Mr. Searle. The deposit is best made upon the internal surface of a carefully cleaned platinum bowl (*see* Fig. 2).   The anode should be a plate of pure silver wrapped in white filter paper.

The washing, drying and weighing of the silver deposit are to be conducted in accordance with the specification given on p. 58, Chap. I. of this volume.

§ **4. Standard Current Measuring Instruments.**—Although the official or practical method of determining the value of a current is based upon an electrolytic definition, it is more convenient in ordinary work to rely upon a standard current measuring instrument which operates in virtue of the magnetic properties of a current-conveying conductor. These instruments may be of such a form that the absolute measurement of a current can be made when the geometrical form of the circuits is determined. In this case it is called *an absolute standard* current-measuring instrument. Of this type are the absolute tangent galvanometer, or the absolute electro-dynamometer or absolute current balance.

On the other hand, the instruments may be so constructed that, whilst the same current invariably gives the same indication, the ampere or absolute value of the current cannot be determined until the instrument has been standardised by passing through it a current the value of which is electro-lytically determined. To this latter class belong the various forms of ampere-balance and standard electro-dynamometer already described in Chap. I.

It will seldom happen that it is necessary in an ordinary electro-technical laboratory to make an original re-determination of the unit current by means of an absolute instrument, but it may be convenient to collect here a few elementary principal facts involved in the construction of absolute galvanometers.

Magnetic Fields of Current-conveying Conductors of Various Forms.—(i.) *Single Circular Conductor.*—If a very thin wire is bent into a circle of one turn, the mean radius (r) of the circle being large compared with the diameter of the wire, and if a uniform current is sent through the conductor,

the magnetic force (F) in C.G.S. units at the centre is given by the expression

$$F = \frac{2\pi A}{10r},$$

where A is the current in amperes through the conductor and r is the mean radius of the circle in centimetres. If the point P selected is not at the centre of the plane of the circle, but is a point on the axis or line drawn through that centre at right angles to the plane of the circle, and distant x centimetres from the centre, then the magnetic force there is given by the expression

$$F = \frac{2\pi A}{10} \frac{r^2}{(r^2 + x^2)^{\frac{3}{2}}}.$$

If we write u for $\sqrt{r^2 + x^2}$, the above becomes

$$F = \frac{2\pi A}{10} \frac{r^2}{u^3},$$

where A is the current in amperes through the coil. The above formula may also be written

$$F = \frac{2\pi A}{10r} \sin^3 \theta,$$

where θ is the angle subtended by the radius r at the point P. The expression for the magnetic force due to a circular current at a point not on the axis is more complicated, and for the detailed proof of the following formulæ the reader must be referred to other sources of information. (*See* Mascart and Joubert's "Electricity and Magnetism," English translation by Atkinson, Vol. II., p. 90, § 736 *et seq.*)

Let r be the radius of a thin circular wire carrying a current, and x and y the co-ordinates of a point P outside its plane, the centre of the circle being the origin and the axis through the centre perpendicular to the plane of the circle taken as the axis of x. Let X and Y be the components of the magnetic force at P parallel respectively to the axes of x and y, when one absolute or C.G.S. unit of current (= 10 amperes) flows through the wire. Let $u^2 = r^2 + x^2$, as above. Then it can be shown that, if y is small compared with u, we have, approximately,

$$X = 2\pi \frac{r^2}{u^3} \left[1 - \frac{3}{4} \cdot \frac{(4x^2 - r^2)}{u^2} \cdot \frac{y^2}{u^2} + \frac{45}{64} \cdot \frac{(r^4 - 12r^2x^2 + 8x^4)}{u^4} \cdot \frac{y^4}{u^4} \right]. \quad \text{(i.)}$$

$$Y = 3\pi \frac{r^2 x y}{u^5} \left[1 + \frac{5(3r^2 - 4x^2)}{8} \frac{y^2}{u^2} \cdot \frac{y^2}{u^2} \right]. \quad \cdots \cdots \quad \text{(ii.)}$$

The above expressions hold good on the assumption that powers of the ratio y/u above the fourth may be neglected.

The resultant magnetic force at the point P outside the axis is given by the value of $\sqrt{X^2 + Y^2}$, and the inclination θ to the x-axis by $\tan \theta = Y/X$.

If the point P is taken in the plane of the circle, then the force at any point not far removed from the centre by a distance y is given by making $x = 0$ in the above expressions for X and Y. We have then, approximately,

$$X = \frac{2\pi}{r}\left[1 + \frac{3}{4}\frac{y^2}{r^2} + \frac{45}{64}\frac{y^4}{r^4}\right].$$
$$Y = 0.$$

The above formulæ are only true when y is very small compared with r. When this is not the case Y is no longer zero.

(ii.) *Solenoid.*—Let insulated wire be wound closely in one layer over a cylinder of length $2l$, so as to cover the whole cylinder. Let r be the mean radius of one circular turn. The magnetic force at any point on the axis of the cylinder may be considered to be the sum of the actions due to a number of circular currents. Take the centre of the cylinder
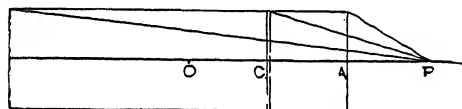
Fig. 3.

as origin, and consider the force at a point P (*see* Fig. 3) on the axis due to one single circular turn at C occupying a length δx on the cylinder. Let the distance $CP = x$. Suppose there are N turns of wire on the cylinder ; then the number of turns per unit of length is $N/2l$, and the number in a length δx is $\dfrac{N\delta x}{2l}$. Hence the magnetic force in absolute units due to this single turn when a current of A amperes traverses the wire is given by the expression

$$d\mathrm{F} = \frac{2\pi N A}{20 l}\frac{r^2 \delta x}{(r^2 + x^2)^{\frac{3}{2}}}.$$

Accordingly, the magnetic force at P due to the whole solenoid is

$$\mathrm{F} = \frac{2\pi N A}{20 l}\int_{x = a}^{x = a + 2l}\frac{r^2 \delta x}{(r^2 + x^2)^{\frac{3}{2}}},$$

where $a=$ the distance of P from the nearest face of cylinder $= AP.$

Omitting the constant of integration the value of the integral $\int \frac{r^2 \delta x}{(r^2 + a^2)^{\frac{3}{2}}}$ is $\frac{x}{\sqrt{r^2 + x^2}}.$

Let us call the above integral the cosine of an angle, say ϕ. Hence,

$$F = \frac{2\pi NA}{20l} \left(\frac{a+2l}{\sqrt{r^2 + (a+2l)^2}} - \frac{a}{\sqrt{r^2 + a^2}} \right) = \frac{2\pi NA}{20l} (\cos \phi - \cos \phi'),$$

where ϕ and ϕ' are the angles subtended by the radii of the two circular ends of the cylinder at P.

Suppose we take the point P in the centre of an infinitely long solenoid, and call the length of the cylinder L. Then $\cos \phi$ approximates to 1 and $\cos \phi'$ to -1. Hence, at the centre of the long solenoid we have a magnetic force F given by the equation

$$F = \frac{4\pi NA}{10L} = \frac{1\frac{1}{4} \text{ ampere-turns}}{\text{length}}.$$

Or, the magnetic force in the centre is equal to $4\pi/10$ times the ampere-turns per unit of length of the solenoid.

The above equation holds true approximately for a solenoid the length of which is, say, 20 times its diameter except at regions close to the ends.

It is easily seen that, in the case of a long solenoid, the force at the centre is just double that at the mouth or entrance to the solenoid, taking the points on the axial line.

The above formula is also very nearly true for a self-closed or endless solenoid. The calculation of the magnetic field at points not on the axis outside cylindrical solenoids or circular conductors conveying currents involves mathematical processes of a difficult kind, and the reader must be referred to advanced treatises on the subject.

§ 5. **Absolute Galvanometers.**—An absolute galvanometer consists of a coil of insulated wire wound in such a form that

the field due to a current through it can be calculated at an assigned point. If this coil is arranged so that the coil field at the stated point is at right angles to a known and constant field, such as that of the earth, at the same point, then the ratio of these field strengths can be determined by placing at that point a small magnetic needle and observing its direction.

Let C (Fig. 4) be a circular coil of wire, and let F be the magnitude and direction of its field at the point P. Let the coil be arranged so that F is at right angles to the magnetic field H due to the earth; then if a small magnetic needle is suspended freely at P, it will set its axis in a direction inclined to the magnetic meridian by an angle θ, such that
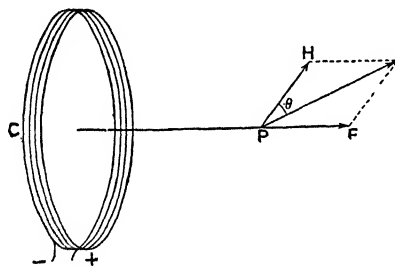
FIG. 4.

$F/H = \tan \theta$. The current through the coil, reckoned in amperes, is proportional to the magnetic force F at any point. Hence, if G is a constant depending on the geometrical form of the coil, we may write

$$F = GA.$$

Hence, $$A = \frac{H}{G} \tan \theta.$$

The quantity H/G is called the *galvanometer deflectional constant.*

Accordingly, if the space distribution of magnetic force due to the current in the coil is of such a nature that everywhere in the region occupied by the magnetic needle it is at right angles to the magnetic force due to the earth and of

constant value, the deflections are strictly proportional to the tangents of the angles of deflection, and the instrument is called a *tangent galvanometer*.

Suppose, for instance, that an exceedingly large circular coil of wire of one turn has a very small horizontal magnetic needle hung exactly at the centre of the coil. Let the needle not exceed in length one-hundredth of the radius (r) of the coil. Let the coil be placed with its plane in the magnetic meridian. Then the magnetic force F at the centre of the coil due to a current of A amperes flowing in the wire is $F = \dfrac{2\pi}{10}\dfrac{A}{r}$. Hence, for this coil $G = 2\pi/10r$; and if the magnetic needle takes a deflection θ under a terrestrial horizontal force H we have $A = \dfrac{10rH}{2\pi}\tan\theta$.

An arrangement of this kind constitutes an absolute tangent galvanometer, because we can determine the value of the current absolutely from measurements of r H, and θ.

Consider next the case when the magnetic needle is placed with its centre at a point on the axis outside the plane of the circular coil of one turn, and has
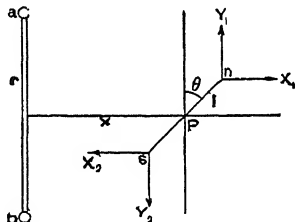
Fig. 5.

a length not very small compared with the radius of the coil. Let the current in the wire be 10 amperes or a unit (C.G.S.) current. Let ab (Fig. 5) be the coil seen in section and let the magnetic needle ns have a length $2l$ and be placed with its centre at P, at a distance x from the centre of the coil. Let each pole of the needle have a strength, m. Then each pole is acted upon by two forces mX_1 mY_1, mX_2 mY_2, the values of X and Y being determined by the co-ordinates of the pole, and are as stated in equations (i) and (ii) in § 4 on p. 350. This system of forces resolves itself into one resultant force and one resultant couple. If, however, the centre of the needle is constrained to remain at P, then we need only consider the couple. This couple may be called D, and it has a value such that

$$D = ml\,(X_1 + X_2)\cos\theta - ml\,(Y_1 + Y_2)\sin\theta.$$

By substituting the proper values of X_1 Y_1, &c., obtained from the equations above mentioned on p. 350, it is not difficult to show that

$$D = 2\pi\frac{r^2}{u^3}M\cos\theta\left[1 + \frac{3}{4}\frac{r^2 - 4x^2}{u^2}(1 - 5\sin^2\theta)\frac{l^2}{u^2}\right.$$
$$\left. + \frac{45}{64}\frac{r^4 - 12r^2x^2 + 8x^4}{u^4}(1 - 14\sin^2\theta + 21\sin^4\theta)\frac{l^4}{u^4}\right].$$

In the above equation r is the radius of the coil and $u^2 = r^2 + x^2$, and it is assumed also that l is so small compared with u that we may neglect powers above the fourth of the ratio l/u. Also, M is written for $2ml$ or the moment of the magnet.

If the coil is placed with its plane in the magnetic meridian, then the opposing or controlling couple due to the earth is MH $\sin \theta$. If the coil consists of many turns of insulated wire wound on a square-grooved circular frame, then in place of the factor $2\pi \dfrac{r^2}{u^3}$ we have to write a more complex function G, which is obtained as follows :—

Let the windings of the coil be in a square groove having a length $2b$ parallel to the axis of x and a depth $2c = r'' - r'$ parallel to the radius of the coil. Let there be n turns of wire per unit of length of the groove. Consider first, one single layer forming a short solenoid or diameter $2r$. It has already been shown that the magnetic force at P due to unit current in this coil would be expressed by

$$2\pi n \left[\frac{x+b}{\sqrt{r^2 + (x+b)^2}} - \frac{x-b}{\sqrt{r^2 + (x-b)^2}} \right].$$

To obtain the magnetic force due to the whole of the windings we have to integrate the above expression between the limits $r = r''$ and $r = r'$ and obtain the value of G from the expression.

$$G = 2\pi n^2 \int_{r''}^{r'} \left[\frac{x+b}{\sqrt{r^2 + (x+b)^2}} - \frac{x-b}{\sqrt{r^2 + (x-b)^2}} \right] dr$$

$$= 2\pi n^2 \left[\overline{x+b} \left(\log \frac{r'' + \sqrt{r''^2 + (x+b)^2}}{r' + \sqrt{r'^2 + (x+b)^2}} \right) - \overline{x-b} \left(\log \frac{r'' + \sqrt{r''^2 + (x-b)^2}}{r' + \sqrt{r'^2 + (x-)b^2}} \right) \right].$$

Returning to the expression for the value of the couple D exerted by the unit current on the needle placed with its centre at P. It is seen that if the ratio l/u is so small that we can neglect powers of it above the second, then we can make the correcting factor in the square bracket due to the finite length of the needle reduce to unity, either by making $4x^2 = r^2$ or by making $1 - 5 \sin^2 \theta = 0$.

The first condition is complied with by placing the centre of the needle at a distance from the plane of the coil equal to half the radius r, and the second by making the reading as nearly as possible in the neighbourhood of the angle $\sin^{-1} \theta = \dfrac{1}{\sqrt{5}} = 26° 4'$.

We may, then, summarise the above results as follows :—

If a coil of insulated wire wound in a square-sectioned groove is placed with its plane in the magnetic meridian, and is traversed by a current of A amperes, it exerts a magnetic couple on a needle placed with its centre at a point on the axis of the coil which is determined by the equation for D given at the bottom of p. 354. Let this equation be written

$$D = \frac{AGM \cos \theta}{10} [1 + K],$$

where G is the coil constant or value of the magnetic force due to unit C.G.S. current (10 amperes) in one turn of the coil at a point on the axis, and K is a correcting factor for the distribution of the force at points not on the axis. This couple is balanced against the couple MH sin θ due to the action of the terrestrial magnetic force on the needle. Hence we have

$$A = \frac{10H \tan \theta}{G(1 + K)}.$$

If the needle is placed with its centre at a distance from the plane of the coil equal to $r/2$, then the correcting factor K is reduced to a value

$$\frac{27}{1000} \frac{l^4}{r^4} (1 - 14 \sin^2 \theta + 21 \sin^4 \theta),$$

on the assumption that powers of l/r above the fourth may be neglected. If the length of the needle is not greater than say one-twentieth of the mean radius of the coil, then K becomes sensibly zero and the tangents of the deflections of the needle are proportional to the currents flowing through the coil.

If the tangent galvanometer consists of a single circular coil of mean radius r, and having the wire wound in n turns in a square-sectioned groove of width $2a$ and radial depth $2b$, also having a magnetic needle of length $2l$ placed at its

centre, the value for the current in amperes (A) creating a deflection θ is approximately given by the equation

$$A = \frac{10r\mathrm{H}}{2\pi n}\left(1 + \frac{1}{2}\frac{a^2}{r^2} - \frac{1}{3}\frac{b^2}{r^2} - \frac{3}{4}\frac{l^2}{r^2}\right)\left(1 + \frac{15}{4}\frac{l^2}{r^2}\sin^2\theta\right)\tan\theta.$$

In the above equation it will be sufficient to take $2l$ as equal to 0.82 of the full length of the magnetic needle. (*See* F. Kohlrausch, "Physical Measurements.")

A tangent galvanometer of the above type is employed generally in the Postal Telegraph Department.

Helmholtz Standard Tangent Galvanometer.—A better and more practically convenient form of tangent galvanometer is that devised by Von Helmholtz. In this instrument there are two large circular coils of insulated wire, the wire being either wound in a square groove in the edge of a wooden ring or wound in one layer on the edge of a ring which forms the frustrum of a cone. These coils are fixed at a distance apart equal to the mean radius of either coil of wire. If the wire windings lie on the frustrum of a cone the cone angle is selected so that the apex of the cone is the midpoint between the coils. Every turn of wire then complies with the condition that it is separated from another equal circular turn in the other coil by a distance equal to the radius of either. This latter arrangement of the winding is most suitable for an absolute instrument. These coils are fixed to a base so that, whilst remaining at the fixed distance apart, they can be turned round on axes perpendicular to and passing through the mid-point. At the middle point is placed a compass-box containing a short magnetised needle having attached a long aluminium index needle moving over a circular scale of degrees. Otherwise the magnetic needle may be suspended by a fibre of cocoon silk and have attached to it a mirror. If the compass needle has the ordinary jewel centre suspension some device should be added to lift the needle off its pivot when not in use. The length of the needle should not exceed one-twelfth of the radius of either coil.

To use the instrument it is placed at a distance from all iron or magnets, and the current to be measured is conveyed to it through a length of concentric cable, or through insulated wires twisted together. The current then flows through the coils in such a manner as to produce a uniform field in the space between the coils. The needle then takes a deflection, the tangent of which is proportional to the current. The instrument must be so oriented that if the current is reversed the angular deflection of the needle from the plane of the meridian is in both cases the same in amount though opposite in direction.

If, then, the dimensions and disposition of the windings is known, the value of the current can be calculated when the magnitude of the earth's horizontal magnetic force H is known at that place. Very roughly speaking, this has a value lying between 0·15 and 0·18 C.G.S. units in Great Britain; but its value at any given spot in a laboratory may be greatly affected by the neighbourhood of iron pipes or masses of iron. Hence, a standard tangent galvanometer can only be employed, for purposes where accuracy is required, in a special room set apart for its use and where facilities also exist for determining the value of H as often as required. This constant can, however, be determined with a fair degree of approximation as follows :—

Provide a cylinder of steel carefully magnetised longitudinally and measure (in cms.) its length l and mean diameter d. The moment of inertia I of this cylinder round an axis through its centre and perpendicular to its own axis of symmetry is

$$I = W\left(\frac{l^2}{12} + \frac{d^2}{16}\right),$$

where W is the weight of the cylinder in grammes. If this cylinder is suspended in a paper stirrup by a few threads of floss silk and set in vibration round a vertical axis, it is easy to determine, from the time taken to execute, say, 50 complete vibrations, the time t of one vibration. If, then, M is the

magnetic moment of the magnet, it can be shown that

$$t = 2\pi\sqrt{\frac{I}{MH}}.\ *$$

Hence,

$$MH = \frac{4\pi^2 I}{t^2}.$$

Then let this magnet be placed in a position with its axis in a horizontal line passing at right angles to the meridian, through the centre of the tangent galvanometer needle, and with its centre at a distance D centimetres from the needle-pivot. Observe the deflection θ produced on the galvanometer needle. It can be shown that

$$\frac{M}{H} = \frac{D^3}{2}\left(1 + \frac{1}{2}\frac{L^2}{D^2}\right)^{-1}\tan\theta,$$

where L is the "magnetic length" or distance between the poles of the cylindrical magnet and D the distance of the centre of the deflecting magnet from the centre of the galvanometer needle; and the assumption is made that L/D is a quantity so small that its squares and higher powers can be neglected.

From the two equations for MH and M/H we can find at once the value of H, by taking two sets of observations with the magnet at different distances, D_1 and D_2, from the galvanometer needle and then eliminating L from the two equations so obtained, viz. :

$$\frac{M}{H} = \frac{D_1^3}{2}\left(1 + \frac{1}{2}\frac{L^2}{D_1^2}\right)^{-1}\tan\theta_1,$$

$$\frac{M}{H} = \frac{D_2^3}{2}\left(1 + \frac{1}{2}\frac{L^2}{D_2^2}\right)^{-1}\tan\theta_2,$$

from which L may be eliminated.

The above equations may be written

$$\frac{M}{H} = \frac{D_1^5}{2D_1^2 - L^2}\tan\theta_1,$$

and

$$\frac{M}{H} = \frac{D_2^5}{2D_2^2 - L^2}\tan\theta_2,$$

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* *See* the Appendix of a book entitled "Magnets and Electric Currents," by J. A. Fleming.

from which we obviously have

$$\frac{M}{H} = \frac{1}{2} \frac{D_1^5 \tan \theta_1 - D_2^5 \tan \theta_2}{D_1^2 - D_2^2}.$$

Hence the ratio M/H is easily calculated when D_1, D_2, θ_1, θ_2 are found.

Having, then, the value of the ratio M/H and that of the product MH, we obtain at once the value of H.

The values of G (the coil constant) having been calculated as already explained, and H determined as described for the locality, we can obtain the ampere value of a current creating any observed galvanometer deflection.

Standard galvanometers in which fixed coils and magnetic needles are used are, however, almost useless as exact measuring appliances unless placed in buildings set apart for the purpose, into the construction of which no iron enters. In ordinary laboratories warmed with iron hot-water pipes or near engines and boilers or machinery, the value of the local terrestrial magnetic force is so constantly varying in amount and direction that the standardization of the instrument changes almost from moment to moment. Hence the use of the tangent galvanometer under these circumstances as an instrument of precision is as impossible as would be the use of the balance if the mass of the standard weights of comparison were changing from moment to moment.

§ 6. The Electro-dynamometer.

§ 6. **The Electro-dynamometer.**—A most valuable substitute for the standard galvanometer in the precise measurement of current is the standard electro-dynamometer. In this instrument, as in the commercial form, there are two coils of wire, one fixed and the other movable. In standard instruments both these coils take the Helmholtz form—that is to say, they consist of two equal separate circular coils placed with their planes parallel to one another and fixed at a distance equal to the radius of either. The movable coil is suspended by a bifilar suspension consisting generally of the

two wires by which the current enters and leaves this movable coil. The coils are held normally with their axes at right angles and centres coincident. When one and the same current is passed through the coils, the electrodynamic forces tend to turn the coils so that their axes are more in the same direction, and this torque is resisted by the bifilar torque.

The theory of the electro-dynamometer has been given by Clerk Maxwell (*see* " Electricity and Magnetism," Vol. II., p. 337, 2nd edition). The reader is also referred to an excellent series of explanatory articles by Mr. G. F. C. Searle,
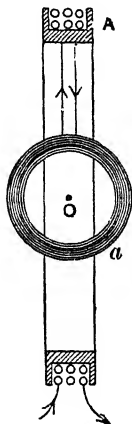
FIG. 6.

on the determination of current in absolute measure in *The Electrician*, Vol. XXVII. and Vol. XXVIII., for 1892. From these sources the following abbreviated analysis has been taken :—

Consider, in the first case, the electro-dynamometer to consist of two coils only (*see* Fig. 6).

Let the inner coil *a* be the suspended coil, and let its diameter be small compared with the outer or fixed coil A.

Let G be the galvanometer constant of the large coil, *i.e.*, the magnetic force at the centre due to unit absolute current

flowing in it. Hence, if i is the current flowing in both coils when joined in series, the magnetic force over the central region of the large coil is nearly equal to Gi.

Let g be the total area enclosed by all the windings of the small coil; then gi is the magnetic moment of the small coil. Hence, when the latter is held so that the angle between the axes of the coils is ϕ, the magnetic couple or torque on the small coil is $i^2 Gg \cos \phi$. If the large coil is placed with its plane in the magnetic meridian there is also a couple due to the earth's force acting on the small coil and equal to $giH \sin \phi$.

The bifilar suspension produces a restoring couple, which may be represented by $\mu \sin \phi$, since for small angular displacements it is proportional to the sine of the displacement. Hence the equation of equilibrium of the small coil, hung in a uniform magnetic field H and supported by a bifilar suspension, is

$$i^2 Gg \cos \phi = igH \sin \phi + \mu \sin \phi,$$

or

$$\tan \phi = i^2 Gg / (igH + \mu).$$

It is always possible to make th eterm igH negligible compared with μ; so that approximately we have

$$\tan \phi = \frac{i^2 Gg}{\mu} - \frac{i^3 HGg^2}{\mu^2}.$$

If we then take four observations of the deflection of the small coil, first by reversing the direction of the current through the small coil alone and then through the large one alone, and call the several observed angular displacements of the small coil ϕ_1, ϕ_2, ϕ_3, ϕ_4, it is easy to see that, since

$$\tan \phi_1 = i^2 Gg / \mu + i^3 HGg^2 / \mu^2,$$

and

$$\tan \phi_2 = i^2 Gg / \mu - i^3 HGg^2 / \mu^2,$$

we have

$$i^2 = (\tan \phi_1 + \tan \phi_2) \mu / 2Gg \; ;$$

and therefore, also,

$$i^2 = (\tan \phi_1 + \tan \phi_2 - \tan \phi_3 - \tan \phi_4) \mu / 4Gg$$

Hence, i is determined in terms of the tangents of the deflections and the constants of the instrument, viz., μ, G and g.

The constants for any instrument can be found as follows :—Let M be the mass of the suspended coil. Then $\mu = \lambda M$. Let T be the time of a small vibration of the suspended coil, and K its moment of inertia. Then

$$T = 2\pi \sqrt{\frac{K}{\lambda M}}.$$

Affix to the movable coil a bar of mass M', of which the moment of inertia K' is known, and observe the time T' of a small vibration of the new system.

Then,

$$T' = 2\pi \sqrt{\frac{K + K'}{\lambda (M + M')}}.$$

Hence, from the two above equations, we have

$$\mu = \frac{4\pi^2 K'M}{M(T'^2 - T^2) + M'T'^2}.$$

In the next place, we have to determine G and g. Suppose the fixed coils have the Helmholtz form, and consist of two coils of mean radius A fixed with centres at a distance B = A/2. Then the magnetic force at the central region for unit current in the coils = G is such that

$$G = \frac{4\pi n A^2}{(A^2 + B^2)^{\frac{3}{2}}},$$

or, if B = A/2,

$$G = \frac{32 \pi n}{5 \sqrt{5} A}.$$

Also, g stands for the total area included by all the windings of the small coil. If a is the mean radius of the small coil—that is, the radius from the centre to the centre of gravity of the windings, it can be shown that

$$g = n' \pi a^2 \left\{ 1 + \frac{h^2}{12 a^2} \right\},$$

where h is the radial depth of the windings supposed to be placed in a rectangular groove, and n' is the number of turns.

If we substitute the above values of μ, G and g in the equation for i, we arrive at the following result:—

$$i^2 = \frac{\dfrac{x}{d} \left(1 + \dfrac{B^2}{A^2} \right)^{\frac{3}{2}} \dfrac{h'^2}{A^2}}{2 n n' \left\{ \left(\dfrac{T'^2}{T^2} - 1 \right) + \dfrac{M'}{M} \dfrac{T'^2}{T^2} \right\} \dfrac{a^2}{A^2}} \cdot \frac{M' A}{T^2}$$

where the symbols have the meanings below :—

$x =$ observed scale deflection.
$d =$ distance of scale from mirror on movable coil.
$B =$ half distance of fixed coils.
$A =$ mean radius of fixed coil.
$a =$ mean radius of movable coil.
$M =$ mass of movable coil.
$M' =$ mass of inertia bar.
$K' =$ moment of inertia of the above bar.
$k' = \sqrt{M'} / \sqrt{K'}$.
$T, T' =$ times of vibration of movable coil with and without inertia bar.
$n, n' =$ number of turns of wire on fixed and movable coils respectively.

Maxwell gives (" Electricity and Magnetism," Vol. II.) the full theory of the action of one circular coil upon another, and shows that if there be two circular coils whose axes intersect at an angle θ, then the co-efficient of mutual induction M can be expressed in a series of zonal harmonics, such that $M = G_1 g_1 P_1(\theta) + G_2 g_2 P_2(\theta) + \&c.$, where the constants are G_1, G_2, &c. The quantities P_1, P_2, called zonal harmonics, are functions of $\cos \theta$ of the form $P_0(\theta) = 1$, $P_1(\theta) = \cos \theta$, $P_2(\theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$, $P_3(\theta) = \frac{1}{2}(5 \cos^3 \theta - 3 \cos \theta)$, &c., and have been tabulated and calculated out numerically for various values of θ.[*]

The quantities G_1, G_2, &c., are found as follows :—Maxwell shows (*loc. cit.*) that if any point be taken on the axis of a circular current, the magnetic force

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[*] See Prof. John Perry, *Phil. Mag.*, Dec., 1891.

at that point, F, due to unit current in the coil can be expressed in terms of the distance x of the point from the centre and certain functions of the radius of the coil and the distance of a point of reference on the axis. Consider the case of a circular current of radius A, and take any point O as origin on the axis at a distance B   Let $C^2 = A^2 + B^2$. Then the magnetic force F at any point at a distance x from the centre of the circle where x is *small* compared with C may be expressed by

$$F = G_1 + 2G_2 x + 3G_3 x^2 + \&c. ;$$

or, if x is *large* compared with C, by

$$F = \frac{2g_1}{x^3} + \frac{3g_2}{x^4} + \frac{4g_3}{x^5} + \&c.$$

We have already shown that the magnetic force at a point at distance x from the centre on the axis of a circular current is given by the formula

$$F = \frac{2\pi I A^2}{(x^2 + A^2)^{\frac{3}{2}}}.$$

Hence, if we expand the above expression in ascending or descending powers of x and equate the co-efficients to those of the above series, we have the values of G and g as follows :—

$$G_1 = 2\pi A^2/C^3, \qquad\qquad G_3 = 4\pi A^2(B^2 - \tfrac{1}{4}A^2)C^7,$$
$$G_2 = 3\pi A^2 B/C^5, \qquad\qquad G_4 = 5\pi A^2 B(B^2 - \tfrac{3}{4}A^2)/C^9.$$
$$g_1 = \pi A^2, \qquad\qquad g_3 = 3\pi A^2(B^2 - \tfrac{1}{4}A^2),$$
$$g_2 = 2\pi A^2 B, \qquad\qquad g_4 = 4\pi A^2 B(B^2 - \tfrac{3}{4}A^2).$$

Now in the Helmholtz pattern electro-dynamometer, when we are considering the magnetic force at the centre, we have $B = A/2$, and, as a consequence, all the terms vanish in the expansion for M between the first and fifth. In other words

$$M = G_1 g_1 P_1(\theta) + G_5 g_5 P_5(\theta) +$$

We need not generally take account of terms beyond the fifth. Hence also if the deflection ϕ of the movable coil is small, and since $\phi = \pi/2 - \theta$, it is easy to show that $P_5(\theta) = \frac{15}{8} \cos \theta$ nearly. Under these circumstances $G_5 = -\frac{144}{625} G_1/A^4$, and if a single suspended coil is placed at the centre of the pair of fixed coils, then $g_5 = \frac{5}{8} a^4 g_1$. Then we have

$$M = G_1 g_1 \sin \phi \left\{1 - 27a^4/100A^4\right\},$$

and the couple or torque experienced by the movable coil when both are traversed by a current i is

$$i^2 \frac{dM}{d\phi} = G_1 g_1 \cos \phi \left\{1 - 27a^4/100A^4\right\}.$$

Hence the factor $(1 - 27a^4/100A^4)$ comes in as a correcting factor to the term $Gg \cos \phi$ in the equation of equilibrium of the movable coil.

Space does not permit us to enter more fully into the detailed theory of the electro-dynamometer, but the

mathematical reader is referred to the following sources of information :—

For the full mathematical treatment of the action of circular currents on each other see Maxwell, " Electricity and Magnetism," Vol. II., Chap. XIV., 2nd edition.

Also Mascart and Joubert, "Treatise on Electricity and Magnetism," translated by Atkinson, Vol II., Chap. IV.

And A. Gray, "Absolute Measurements in Electricity and Magnetism," Vol. II., part 2.

In addition, the above-mentioned series of articles by Mr. Searle on " Current Measurements " in *The Electrician*, Vol. XXVII. and XXVIII., may be consulted.

As an instance of the use of the electro-dynamometer and calculations connected therewith the reader may consult a paper by Mr. Dugald McKichan "On the Number of Electrostatic Units in one Electromagnetic Unit," *Phil. Trans.* Roy. Soc., 1873.

§ **7. Current Balances.**—Dr. Joule was one of the first persons to construct an amperemeter in which a current was measured by observing the apparent increase or decrease in weight of a coil carrying a current produced by the mutual electrodynamic action of another coil conveying a current placed parallel to it. The much more elaborate current balances of Lord Kelvin have already been fully described (*see* Chapter I.). In these latter instruments the movable coil is placed between two fixed coils.

In order that stability may be secured, it is necessary that a displacement of the movable coil from its position of rest should not decrease the electrodynamic forces acting upon it. If we place two fixed circular coils parallel to each other, as on the Helmholtz galvanometer, and a smaller movable coil is held between them so that the planes of all three are parallel, then, by suitably arranging the direction of the currents, we can create a force tending to move the movable

coil parallel to itself. Maxwell points out that if the diameter of the fixed circular coils is to the distance between their planes as $2 : \sqrt{3}$, then they will produce a nearly uniform force on a much smaller circular coil placed between them and with its plane parallel to those of the fixed coils.
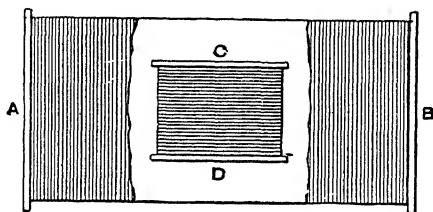
Fig. 7.
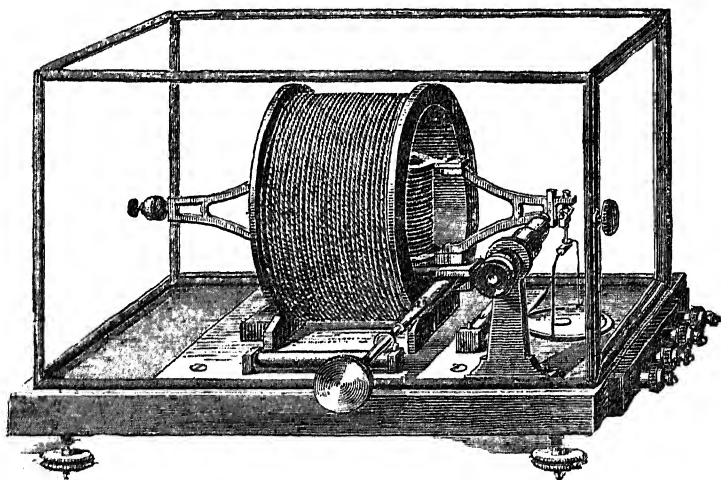
Fig. 8.—Pellat's Ampere Balance.

A form of standard ampere balance has been designed by M. Pellat for the Laboratoire Central d'Electricité in Paris. It consists (*see* Figs. 7 and 8) of a fixed horizontal solenoid and an enclosed smaller solenoid attached to the beam of a balance. When a current is passed through the two coils in

series it tends to turn round the movable solenoid so as to bring the axes of the coils more into colineation. This torque is resisted by weights put upon the scale pans of the balance. If H is the value of the earth's horizontal magnetic field parallel to the axis of the fixed coil, and if A is the current flowing through the coils, then the resultant magnetic force in the interior of the large coil parallel to its axis is

$$F = RA + H.$$

If the magnetic moment of the movable coil is M, then

$$M = R'A.$$

Hence the couple acting on the movable coil when it is in equilibrium is

$$C = MF = RR'A^2 + R'HA.$$

If the current in the fixed coil is then reversed, the couple becomes

$$C' = -RR'A^2 + R'HA.$$

Hence $\qquad C - C' = 2RR'A^2, \quad$ or $A = \sqrt{\dfrac{C - C'}{2RR'}}.$

If the couples are produced by weights W and W' put in the scale pans attached to the movable coil, then these weights are proportional to the couples, and we have

$$A = K\sqrt{\frac{W - W'}{2}},$$

where K is some single constant. If the coil dimensions are measured K can be calculated. The value of K for the Pellat balance in the Laboratoire Central in Paris is $K = 0.217682$ in terms of the ampere and gramme.

§ 8. **Working or Laboratory Amperemeters.**—It would be of little use to describe the multitudinous forms of commercial ampere or ammeters which have appeared and disappeared in the last 20 years. Broadly speaking, in addition to the standard instruments already described, the practical electrician has need of three classes of current-measuring

direct-reading instruments called respectively galvanometers, table or portable ammeters, and switchboard ammeters. Table or portable ammeters are employed for the numerous occasional measurements of current in practical units made anywhere or everywhere in the laboratory, and those of the third class only in fixed positions and on certain circuits.

It is necessary that the above two classes of instruments should give at once, by a direct-scale reading, the approximate value of the current in amperes. The first class, or galvanometers, are not usually direct-reading. They are mostly employed to indicate the mere presence or absence of a current in a circuit, and when the ampere value of
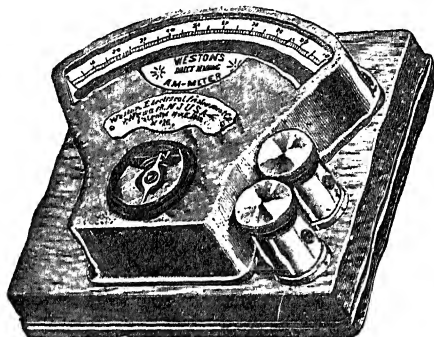
FIG. 9.—The Weston Portable Ammeter.

their indications is required they have to be standardised. The necessary qualifications for a good portable or table ammeter are (i.) that it should be dead-beat—that is to say, its indicating needle must come immediately and without oscillations to the scale reading corresponding to the current passing; (ii.) it should have no dead or undivided part of the scale; and, (iii.) if possible, the scale divisions for equal increments of current at various portions of the scale should be equal—in other words, the scale should be equi-divisional. No laboratory instruments fulfil the above requirements, as far as continuous currents are concerned, so well as the Weston instruments. In these ammeters (*see* Fig. 9) there

is a well-aged magnet which produces a constant magnetic field. In this field is held a circular coil of wire carried on an axis revolving in jewelled centres. The control is produced by a steel spiral spring, like the hair-spring of a watch.

In all but the ammeters for very small currents the greater portion of the current in the circuit containing the ammeter passes through a shunt, so that the above-mentioned coil only carries a very small portion of the current. The terminals of the instrument are marked + and — so as to
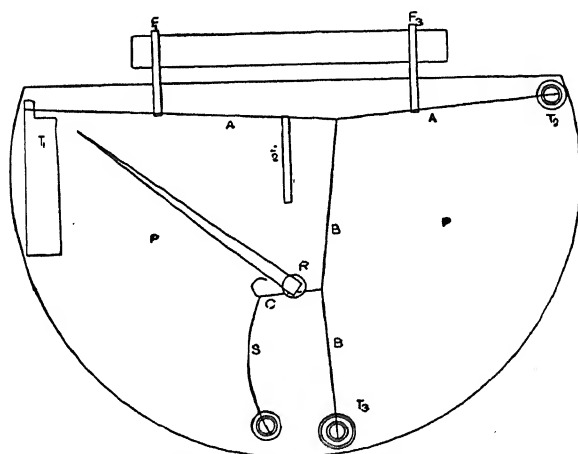
Fɪɢ. 10.—General Design of Hartmann and Braun Hot Wire Ammeter.

show how the connections should be made. The instruments are made in various grades, to read from milliamperes to hundreds of amperes, the readable range in each instrument being about 1 to 1,500. These instruments are not, however, available for alternating-current measurement. For this last purpose, a convenient form of ammeter is that depending on the heating of a wire, and therefore called a hot-wire ammeter.

A good form of hot-wire ammeter is that of Hartmann and Braun (*The Electrician*, Oct. 6, 1899). In this instrument (*see* Figs. 10 and 19) a fine wire is stretched between two fixed

points, and is heated by the whole or part of the current to be measured. The wire, therefore, extends and "sags," and the indicating portion of the instrument is a device for accurately measuring this sag. The advantage of this arrangement is that the sag is greater than the mere longitudinal extension of the wire. This is easily proved as follows :—

Let an inextensible thread of length $2l$ be fastened between two supports at a distance $2l$, and held, therefore, in a straight line. Let one of these supports move towards the other by a small distance, $2x$, which is equivalent
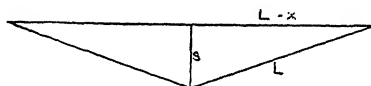
FIG. 11.

to assuming a small increment of length, $2x$, to be made in the thread. The thread, therefore, will sag down. Let s be the amount of the sag. Then, as seen from Fig. 11, we have the equation

$$l^2 = s^2 + (l - x)^2,$$

or

$$s^2 + x^2 - 2lx = 0.$$

If x is small compared with l, we may neglect x^2 in comparison with $2lx$, and we have

$$s = \sqrt{2xl}.$$

Hence it will be seen that s is very much greater than x provided l is much greater than x. For instance, if $l = 100$mm. and $x = 1$mm., then $s = 14\cdot1$mm. nearly.
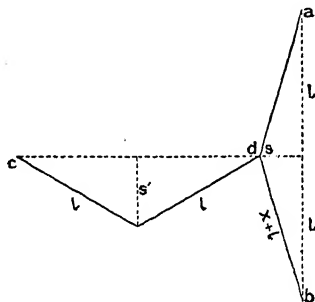
FIG. 12.

A means of still further multiplying the extension of a wire may be found by allowing the sag of one wire to create a still greater sag in a second. Thus, suppose two wires, each of length $2l$, are arranged as in Fig. 12, connected to three fixed points, a, b, c.

Let the wire cd have its end d attached to the middle point of ab.   Let the wire ab increase by a small length $2x$.   Then, if s is its sag, we have

$$(x+l)^2 = s^2 + l^2,$$

or

$$2xl + x^2 = s^2,$$

or, if x/l is small,

$$s = \sqrt{2xl}.$$

Then, in the same way, for the second wire we have, if s' is its sag,

$$s'^2 = l\sqrt{2xl} - \frac{xl}{2},$$

or, if xl is small compared with xl^3,

$$s' = .2xl^3)^{\frac{1}{4}}.$$

Hence, s' is very much greater than x.

Thus, if $x = 0·1$mm. and $l = 100$ $s' = 21$mm. nearly, or the extension is multiplied 210 times.

In the Hartmann and Braun hot-wire ammeter the wire is mounted between fixed insulated pivots, on a plate of metal having the same coefficient of expansion for heat as the wire. The sag of the wire is measured by causing it to create rotation in an indicating needle, the movement of the needle being resisted by a steel spiral spring which brings it back to zero.

In the case of ammeters for small currents the whole current passes through the wire and heats it.   In the case of ammeters for larger currents, the main portion of the current passes through a shunt.   The instruments are very dead beat and the scale reading is most open at the point at which greatest accuracy is required.   The vibrations of the needle are checked by attaching to the needle shaft a light circular disc of copper which moves between the closely placed poles of a strong horse-shoe magnet, thus creating "magnetic friction" by reason of the eddy currents set up in the disc whenever rapid rotation of the disc takes place.

These instruments are available both for direct and alternating currents, and have many practical advantages. There is, however, a dead portion of the scale below which no scale divisions are engraved; the lower limit of the scale reading is approximately 10 per cent. of the upper limit. Thus an ammeter reading to 100 amperes would be useless

for measuring less than 10 amperes (see *The Electrician*, Vol. XLIII., p. 839).

For switchboard purposes very popular instruments are those called the *Edgewise* instruments (*see* Fig. 13). In this case the scale is a curved circular band, and the needle end projects through a slit in it and has its end turned over so as to be seen against the scale. The instruments take up less room on a switchboard than the circular type, and are more easily read from the floor level.
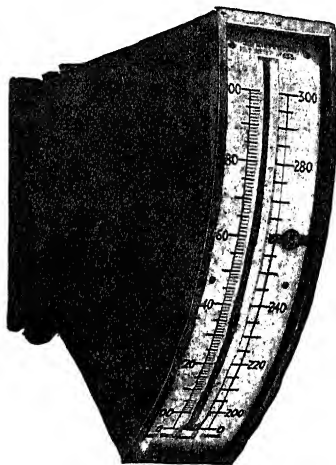
FIG. 13.—Kelvin Edgewise Switchboard Ammeter.

For a full discussion of the advantages and disadvantages of various forms of ammeter for switchboard purposes the reader is referred to a paper by Mr. Blakie on " Instruments for Switchboards," in *The Electrician*, Vol. XLI., p. 209.

§ 9. **Calibration of Laboratory Ammeters.**—No commercial instruments can be taken to be absolutely correct in their scale indications no matter what the makers of them may assert. The practical electrician must proceed first to standardise them in order to obtain the true or most probable

real ampere value of any scale-reading by an independant measurement. For this purpose no appliance is so convenient as the potentiometer already described (*see* Chapter I., § 13 p. 134) aided by appropriate standard current-carrying resistances. For by this means the measurement of a current is reduced to the measurement of a resistance and the electromotive force of a standard cell, both of which are tolerably permanent values for given well-made instruments. Hence, the standardisation of an ammeter is conducted as follows :—

The ammeter A (*see* Fig. 14) must be joined up in series with a low resistance standard R of such type that it can
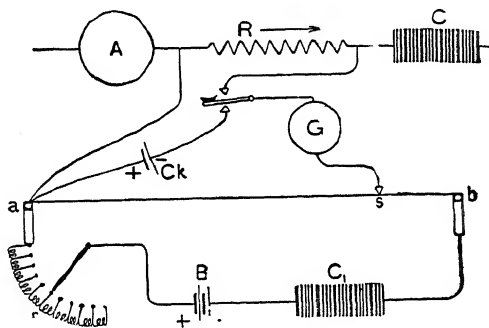
Fig. 14.

carry, without injurious heating, the maximum currents to be passed through the ammeter. In series with this should be another adjustable rheostat for varying the current passed through the ammeter, and also a carbon rheostat C for making very small adjustments in the current. From the potential terminals of the standard low resistance are brought two potential wires, which are connected with the potentiometer wire *a b*, as shown in the diagram The potentiometer is first *set* by means of a Clark or Weston cell so that the fall of potential down the slide-wire is of known amount, obtained by regulating the current flowing from the working battery through the slide-wire.

If, for instance, a Clark cell is employed, the temperature of which is 15°C. and the corresponding E.M.F. 1·434 volts, the sliding contact is set to make contact at that division on the slide-wire marked 14,340, and the current down the slide-wire regulated until the reading galvanometer shows no deflection. When this is done, the potential wires connected to the ends of the low resistance are substituted for the connections to the Clark cell, and the fall of potential down the low resistance is read off directly on the potentiometer slide-wire. Hence, knowing the value of this low resistance, we have at once the true current through the resistance, and,
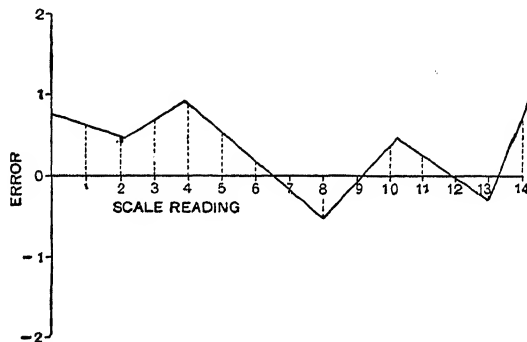
Fig. 15.—Ammeter Error Curve.

therefore, that through the ammeter. We can, therefore, write down two columns of figures, one of which gives the observed scale-reading of the ammeter and the other the true ampere value of the corresponding current. The difference between these two figures is the *error* of the ammeter corresponding to that scale-reading.

We can then make an error curve for the instrument as follows :—Let the various scale readings be plotted out as a horizontal line (*see* Fig. 15) and, corresponding to each, let a line be erected the length of which is the error of the ammeter at that reading, the said line being drawn upwards

when the error is positive and downwards when the error is negative. Thus if, corresponding to an abscissa 20, denoting a scale-reading of 20 amperes, the error is $+2\cdot2$, we draw the line representing $2\cdot2$ upwards, and the true ampere value of the current when the ammeter needle points to 20 on the scale is $20+2\cdot2=22\cdot2$. If, on the other hand, the error is negative, the line is to be drawn downwards, and the true ampere value corresponding to 20 divisions would be $20-2\cdot2=17\cdot8$ amperes.

Every working ammeter in the laboratory should in this manner have an error curve drawn for it which should be repeatedly verified. The error curve should be taken with ascending as well as descending values of the currents, since, in certain types of ammeter, particularly those containing soft iron masses moved in a magnetic field, there is often considerable hysteresis error. The ammeter will not give the same reading for the same current value if it has previously been used with a different and much larger or much smaller current.

Hence, in checking an ammeter it is not sufficient to determine its scale inaccuracy for one cycle of current values: we must know how far the instrument gives the same scale-readings for the same current values independently of the values of the currents previously passed through it. At the same time, its zero-keeping quality and dead-beatness can be noted.

If the ammeter is one intended for use with alternating currents, an investigation must be made to determine how far the readings are affected by change in frequency of the current.

There is no better method of doing this than by calibrating, first of all, a Siemens' electro-dynamometer and delineating for it a curve showing the torsion in scale degrees corresponding to various continuous currents passed through it. In so doing, it is essential to place the instrument in such a position that the movable coil when in its zero position

shall not be affected by the earth's magnetic force. In order that this may be the case the instrument should be so oriented that the axis of the movable coil when in its zero position is in the direction of the local magnetic meridian. To test this try reversing the direction of the current through the movable coil alone, and ascertain if the torsion required to restore the movable coil to the sighted position is the same in both cases. When the instrument has been calibrated for direct currents, it may be used with alternating currents of various frequencies; and employed in series with the ammeter to be tested to calibrate it and to detect if the ammeter reading varies with the frequency.

§ **10. Direct Measurement of Current by the Potentiometer**.—The majority of commercial ammeters are so liable to change of errors that, in order to avoid the expenditure of time in constantly calibrating them, it is better in many cases to rely on the potentiometer directly as a means of measuring current directly. Thus, in the case of the measurement of current through an incandescent lamp, it is necessary to be able to determine a current which may have a value approximately of half an ampere or so to within 1 per cent. at least. No commercial ammeter can be depended upon to measure or indicate correctly a current of this order with the above-mentioned accuracy. Hence incandescent lamp currents are far better measured by inserting in series with the lamp a resistance adapted for carrying 1 ampere to 10 amperes and measuring with the potentiometer the fall of potential down this resistance. If the resistance has a value of 1 ohm and is made of manganin, we can, by its aid, determine the current value with great ease to within one-tenth of 1 per cent.

These measurements may take a little longer than when made with a direct-reading ammeter, but they can be depended upon when made to be accurate within the above-named limits.

In the same manner, in measuring large currents, such as those given by large dynamos, it is much better to pass the current through a resistance strip of one-thousandth of an ohm resistance and determine the current by the fall of potential down the strip.

A stock of low-resistance current-carrying resistance strips and a potentiometer is thus a more profitable investment of capital for an electrical laboratory than the purchase of a large number of commercial ammeters of different kinds.

The measurement of the large currents sent out from continuous-current stations is most accurately effected by the direct measurement of the fall of resistance down a low-resistance strip or strips, the resistance values of which have been carefully determined in absolute measure by a Jones-Lorenz apparatus.

If a low-resistance strip of known value is not at hand, then a resistance can be constructed by joining in parallel a known number of wires of measured resistance. Thus, if a resistance capable of carrying 1,000 amperes is to be made, it can be constructed by joining in parallel 100 wires each having a resistance of one-tenth of an ohm and each capable of carrying 10 amperes. In making these arrangements a large margin must be allowed in the carrying capacity of the single wires. It does not follow that, if one single bare wire, say of platinoid, No. 36 S.W.G. size, will carry, without undue heating, one-quarter of an ampere, that 100 of these wires closely laid in parallel will carry 25 amperes. Owing to the diminution in the radiative power of each wire by its proximity to others, the final temperature of each wire will be much higher, and therefore a less total safe current-carrying capacity results.

§ **11. Current-carrying Capacity of Wires.**—In the construction of resistances used for the purpose of causing a definite fall in voltage in a current passing through them it is necessary that the rate of generation of heat shall be so

related to the rate of dissipation that the wire does not rise in temperature by an amount sufficient to seriously affect its resistance, or else one of the factors required for the current measurement—viz., the resistance of the measuring portion of the circuit—is uncertain. The heat generated per second in a bare wire in air by the passage through it of a steady current having a value of A amperes is mechanically equal to A^2R joules, where R is the resistance of the wire. This energy is dissipated or conveyed away from the surface by radiation and convection, and every surface at a certain temperature has a certain *emisssivity*, which is defined as the energy passing out per second per square centimetre. The true emissivity is a function of the difference of temperature of the surface of the wire and that of the surrounding vessel or enclosing surface. If the wire is in a vacuum, the dissipated energy is wholly radiant, except in so far as there is conduction out of the ends of the wire. In the case of wires exposed in the air, energy is removed by air convection as well as by radiation, and convection in this case forms an important part of the heat loss.

Let w stand for the total energy removed from the wire per second per square centimetre of surface, whether by radiation or convection. Then, when a state of thermal equilibrium is reached, we have the equation

$$A^2R = w\pi DL,$$

where D is the diameter and L the length of the wire, which we will suppose to have a circular section. Also, if ρ is the electrical resistivity of its material, we have

$$10^9 R = \frac{L}{\frac{\pi}{4}D^2},$$

where R is the whole resistance of the wire in ohms and ρ its resistivity in C.G.S. units.

If the diameter d of the wire is measured in millimetres, we obtain from the above two equations another, viz.,

$$A^2 = \tfrac{1}{4}\frac{w}{\rho}\pi^2 10^6 d^3, \quad \text{or } A = 1{,}570\sqrt{\frac{wd^3}{\rho}}.$$

This equation tells us the value of a current in amperes (A) which will bring a circular-sectioned wire of diameter d millimetres to a temperature at which it will be losing energy at a rate equal to w joules per second from each square centimetre of its surface.

Suppose, for instance, that $w = 0\cdot 1$, or that the surface rate of loss of energy is one-tenth of a watt per square centimetre, and that the wire has a diameter of 1 mm.; then we should have

$$A = 1{,}570\sqrt{\frac{1}{10\rho}}, \quad \text{or } \rho A^2 = 246{,}490.$$

Experience shows that when a bare metallic wire is radiating $0\cdot 1$ watt per square centimetre, its surface temperature would be about 60°C. if surrounded by air at 15°C. If we assume the wire to be of copper, then at 60°C. its resistivity ρ would be about 2,000 C.G.S. units. Hence the current carried would approximate to 11 amperes.

It is generally the custom to specify the diameter of wires in *mils,* the mil being one-thousandth of an inch. If the resistivity r is measured in microhms per cubic centimetre at the final steady temperature, and d is the diameter in *mils,* then the above formula for the current carried can be transformed into

$$A = \frac{1{,}570}{8{,}000}\sqrt{\frac{wd^3}{r}}.$$

If the rate of loss of energy w reaches one watt per square centimetre of surface for a bare wire, it is far too hot to touch, and is probably at a temperature of about 400deg. If w has a value of $0\cdot 1$, then the wire is just hand-hot, or, perhaps, at about 60°C. At the above temperature (60°C.) the resistivity r of copper will be 25 per cent. greater than its resistivity at

0°C. and that of iron 36 per cent., but for platinoid and most resistance alloys, only about 2 per cent. greater.

Inserting in the formula these values for r and w we have the following convenient but *rough* empirical formulæ for the *safe currents* in amperes A, carried by round bare wires, of the stated materials, having a diameter of d mils, viz. :—

For copper,

$$A = \sqrt{\frac{d^3}{500}}.$$

For iron,

$$A = \sqrt{\frac{d^3}{4,000}}.$$

For platinoid or German silver, $A = \sqrt{\dfrac{d^3}{5,000}}.$

The above formulæ only apply to bare wires in air, the wires being either straight or coiled into very open spirals.*

The following table, embodying experimental results, shows the safe current-carrying capacity of some common sizes of bare wires of copper, iron, brass and German silver in loose or open spirals :—

Safe Current-carrying Capacity in amperes of Bare Wires, wound into Open Spirals.

Size of wire in B.W.G.	Diameter of wire in inches.	Safe currents carried. Final temperature not exceeding 60°C.			
		Copper.	Brass.	Iron.	German silver.
10	0·134	50 amps.	30	19	16
12	0·109	38 ,,	19	16	13
14	0·083	27 ,,	15	10	8
16	0·065	20 ,,	10	8	6
18	0·049	15 ,,	7	5	4
20	0·035	9 ,,	5	4	3

It will, therefore, be seen that the current density or amperes per square inch of section can be very much greater in fine wires than in large ones. In all cases, however, the surface

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* These formulæ cannot be applied to extreme cases, such as that of a wire *one* mil in diameter, but apply generally to ordinary laboratory sizes of wires.

of a bare wire in air should approximate to 10 sq. cm. per watt expended on it, if the temperature is not to rise above the point at which it is possible to touch the wire with the hand.

The result of investigation has been to show that the *emissivity* of a bare cylindrical wire is not merely a function of the temperature difference between its surface and that of the surrounding envelope, but is also dependent on its form. Prof. Ayrton and Mr. H. Kilgour have deduced the following laws from observations on nine platinum wires varying between 1·2mm. and 14mm. in diameter.[*]

1. For any temperature the emissivity is higher the finer the wire.

2. For each wire the emissivity increases with the temperature, the rate of increase being greater the finer the wire.

3. The effect of surface on the total loss of heat per square centimetre per second per 1°C. excess of temperature increases as the temperature rises.

These authors find that for platinum wires of different diameters the emissivity e, or loss in heat in calories per square centimetre of surface per second per 1°C. excess of temperature above the enclosure, can be expressed by the formulæ—

$$\text{At } 100°\text{C.,} \qquad e = 0·0010360 + 0·120776 \, d^{-1},$$
$$\text{At } 200°\text{C.,} \qquad e = 0·0011113 + 0·0143028 \, d^{-1},$$
$$\text{At } 300°\text{C.,} \qquad e = 0·0011353 + 0·016084 \, d^{-1},$$

where d is the diameter in mils. Hence, whilst the simple theory above given, in which the emissivity is taken as constant, leads to the conclusion that the current required to maintain a wire of given material at a given temperature varies as the diameter of the wire raised to the power of three-halves, as a matter of fact, for very thin wires, the current is more nearly proportional to the diameter simply.

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[*] *See* Ayrton and Kilgour, "On the Thermal Emissivity of Thin Wires in Air," *Proc.* Roy. Soc., November 19, 1891 ; or *The Electrician*, Vol. XXVIII., page 119.

In 1884, Sir W. H. Preece gave some measurements of the currents required to fuse fine bare platinum wires 6in. in length and of various diameters. These currents were found to be as follows:

Diameter of the Wire in inches.	Fusing Current in amperes.
0·00050	0·277
0·00075	0·356
0·001	0 437
0·002	0·790
0·003	1·150

It will be seen that these fusing currents are much more nearly proportional to the first power of the diameter of the wire than to the three-halves power of the diameter. For wires of copper varying in diameter (d) from 8 mils to 30 mils he found the observed values of the currents (**A**) required to make the wires just visibly red hot in the dark (temperature about 500°C.) were in very fair agreement with the values calculated from the formula

$$A = 0\text{·}14587 \sqrt{d^3}.$$

For Swedish wrought-iron wires of diameters varying from 10 mils to 60 mils the same current could be calculated from the formula

$$A = 0\text{·}035755 \sqrt{d^3},$$

and for German silver wires between 8mm. and 25mm. in diameter from the formula

$$A = 0\text{·}056376 \sqrt{d^3}.$$

These experiments made it evident that, whilst for round bare wires in air of diameters between 8 mils and 60 mils the currents required to keep them at constant temperatures could be calculated from a formula of the form *current* $= (constant) \times (diameter)^{\frac{3}{2}}$. this law does not hold good for wires of much smaller diameter, such as 1 mils to 3 mils, for in this latter case the current varies more nearly *as* the diameter.

In connection with this subject a knowledge of the current required to fuse short bare wires of different materials and various diameters in air is practically important. Sir W. H. Preece has given a table (*see* Table I., end of this chapter) of fuse currents for wires of different sizes of iron, copper, lead and tin.

The reader may be referred to the following papers for additional information on the heating of conductors by electric currents :—

PROF. G. FORBES. British Association *Report*, 1882.

SIR W. H. PREECE. " On the Heating Effects of Electric Currents." *Proc.* Roy. Soc., April, 1884, or *The Electrician*, Vol. XII., p. 518.

PROF. G. FORBES. " On the Diameter of Wires to Prevent Over-heating by Electric Currents." *Proc.* Inst. Elec. Engineers, London, April, 1884 ; also *The Electrician*, Vol. XIII., pp. 16, 39, 63, 82.

J. T. BOTTOMLEY, *The Electrician*, Vol. XII., p. 541.

A. E. KENNELLY. " On the Heating of Conductors by Electric Currents." *The Electrician*, Vol. XXIV., p. 142.

§ 12. Calibration of a Galvanometer by the Potentiometer. The Measurement of Very Small Currents. —When using a sensitive galvanometer for many purposes it becomes necessary to know the absolute value in amperes of the small current causing an observed deflection. The most simple method of calibrating any form of mirror or deflectional galvanometer is as follows :—

Unless the galvanometer G has a high resistance, say 2,000 ohms and upwards, it must have a resistance, R, of known value joined in series with it, and the resistance of the galvanometer coil itself must be measured. A potentiometer wire, *a b*, is then set or adjusted by means of a Clark cell, Ck, so that there is a known fall of voltage down the wire per centimetre. In the absence of a potentiometer it is always possible

to improvise an equivalent by stretching a fine uniform platinoid wire along a 2-metre scale and attaching to the ends of the wire a single secondary cell. The galvanometer may then be employed in connection with a Clark cell to discover the length of the wire down which there is a fall in potential equal to the voltage for the moment of the Clark cell. Thus, suppose the connections made as in Fig. 16, and that the Clark cell has an E.M.F. of 1·434 volts at 15°C. at that temperature, and that we find the fall of potential down 80cm. of the wire is equal to 1·434 volts; then the fall of potential per millimetre of the wire is 1·434/800 volts.
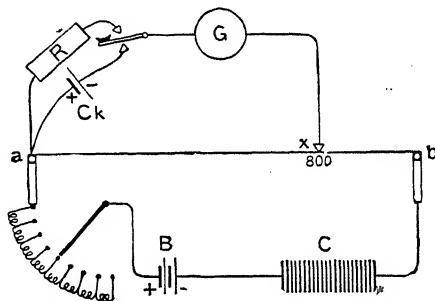
FIG. 16.

This done, remove the Clark cell, attach in series with the galvanometer the known resistance R, and join up the ends of the galvanometer to any two points on the slide-wire separated by such a distance that the resulting galvanometer deflection is the one the current value of which is required.

Let the galvanometer have a resistance G ohms and be joined in series with a resistance R ohms; let L be the length $a\,x$ of the slide-wire down which there is a fall of potential equal to the voltage V of the Clark cell; and let l be the length of slide-wire separating the terminals of the galvanometer circuit when the galvanometer deflection is δ. Then the value of A

the current in amperes through the galvanometer which produces this deflection δ is

$$A = \frac{Vl}{L(R+G)}$$

Hence the galvanometer constant k is equal to A/δ.

The conditions of success are that the value of the resistance $R + G$ must be very large compared with the resistance of the length l of the slide-wire, so that the addition of the galvanometer in parallel with a section of the slide-wire does not sensibly affect the current flowing through the slide-wire.

The galvanometer constant should, in the above manner, be determined for a number of different deflections, and it must not be taken for granted without proof that the *constant* is constant. If, however, the deflectional constant has been found, the galvanometer may be employed for the measurements of large currents by being *shunted.*

If the terminals of a galvanometer having a resistance G ohms are joined by a resistance of S ohms, called a *shunt,* then the combined resistance of the galvanometer and shunt is $\dfrac{SG}{S+G}$ ohms. If a steady current A amperes is sent through the system, it is divided in the inverse ratio of S and G, and the galvanometer is traversed by a current α, such that

$$a = A\frac{S}{S+G}.$$

Hence,
$$A = a\frac{S+G}{S}.$$

But if the current a produces a deflection, δ, and corresponding to this deflection we have a constant, K, we then have

$$A = K\delta\frac{S+G}{S}.$$

A galvanometer is usually provided by the maker with a set of shunt coils so adjusted for use with it that, by

connecting each shunt coil in turn across the terminals of the galvanometer, one-tenth, one-hundredth, or one-thousandth of a current is sent through the galvanometer when the shunted galvanometer is connected in a circuit.

A universal shunt-box has been designed by Prof. W. E. Ayrton and Mr. Mather which is capable of being applied to *any* galvanometer. The principle may be explained by reference to the diagram (Fig. 17).

Let G be a galvanometer having its terminals *ab* shunted by a resistance consisting of a long fine wire. Let B be a battery having one terminal connected to *a* and the other to a point, *x*, on the shunt wire. Then, if the
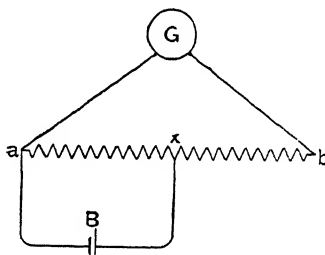
FIG. 17.—Diagram illustrating the principle of the Ayrton-Mather Universal Shunt-Box.

resistance of the galvanometer is G, and that of the wire *ab* is *n*G, and the resistance of the fraction of the wire *ax* is *m*G, and if C is the current flowing out of the battery and *c* the current flowing through the galvanometer, we obviously have

$$c = C\frac{m}{n+1}.$$

Hence the fraction of the current flowing out of the battery which passes through the galvanometer is independent of the resistance of the galvanometer. Accordingly, if we make $n=9$ and $m=1$, $c=0.1C$; if we make $n=9$ and $m=0.1$, $c=0.01C$, and so on.

If, then, the shunt wire *ab* remains fixed, but an arrangement exists by which the point *x* can be shifted so as to make *ax* successively equal to one-tenth, one-hundredth, &c., of *ab*, we shall have a shunt which can be applied to any galvanometer. It must be noticed, however, that the movement of the point *x* alters the total resistance between *a* and *x*, and therefore affects the total resistance of the circuit, and, as a consequence, the value of the main current, unless some adjustment is made to keep it constant. In the actual arrangement, the shifting of the
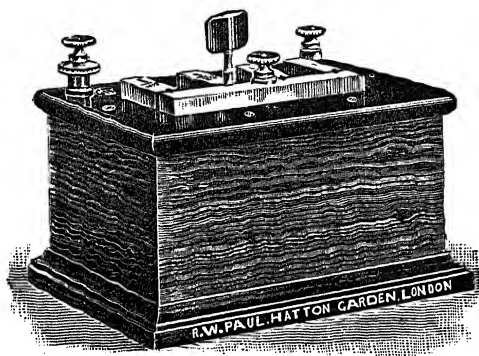
FIG. 18.—Ayrton-Mather Universal Shunt-Box.

contact point *x* from place to place along the shunt line is achieved either by a plug and block arrangement or by a rubbing contact (*see* Fig. 18).

In connection with this matter it may be well to point out that in the ordinary shunt arrangement, in which the galvanometer is provided with a series of shunts, the resistances of which are respectively $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, &c., of that of the galvanometer, we must not assume that, because steady currents are divided in decimal ratios by the shunts, that therefore sudden impulsive currents or instantaneous discharges, as when the galvanometer is used ballistically, are

c c 2

likewise so divided. Mr. Hockin showed many years ago that the application of the shunt affected the logarithmic decrement or damping of the galvanometer, and hence the instantaneous swing or throw is less than it should be if the shunt merely acted to divide the discharge in the inverse ratio of the resistance of the galvanometer and shunt.

§ **13. Alternating-Current Measurement.**—It has already been explained that in the case of alternating or periodic currents we are concerned with either the *instantaneous* value (i) of the current, or with its maximum value (I), or with its root-mean-square (R.M.S.) value, and, occasionally, with the true-mean (T.M.) value. Generally speaking, the measurement of an alternating current implies the determination of its root-mean-square value, and most alternating-current ammeters and methods give us this value. In order, therefore, that an instrument may be useful for alternating-current measurement, it must be based on some principle such that the actual mechanical force, couple or displacement measured varies as the square of the instantaneous value of the current. We may avail ourselves therefore of the heating power of the current, or of the mechanical force between two conductors traversed by the same current, or of the mechanical force between conductors connected to the ends of the circuit traversed by the current, as a means of constructing an ammeter for periodic current measurement, because in each of these cases the observed effects, whether heat production or mechanical force, varies as the square of the current at any instant, and is therefore independent of its direction.

We have already described the construction of the instruments of the electro-dynamometer class, such as Siemens' electro-dynamometer or the Kelvin ampere balances, which depend for their action upon the mechanical forces existing between fixed and movable conductors traversed by the same current. In using these types of instruments for alternating-

current measurement the condition of success is that the periodic time of the alternating current must be small compared with the free time of vibration of the movable part of the instrument.

There is, however, another very important matter concerned in the proper construction of an instrument of this class in order that it may be available for alternating-current measurement. The instrument must have no metal parts or plates or case near to the coils traversed by the alternating currents. For if there are such metal parts, then the alternating currents in the fixed coils will set up local or eddy currents with these metal parts, and these eddy currents will be nearly in opposition as regards phase with the currents producing them. Hence the eddy currents will react upon the movable coil, and will make the mechanical force upon it different from that which it would be if the eddy currents were absent. This defect may be masked by the opposing action of the currents in the fixed and movable portions of the circuit, and it will in general be more marked if there is a difference in phase between the currents in the fixed and movable circuit. In any case, it is a possible source of serious error, and any electro-dynamometer instrument intended to measure alternating currents which is included in a brass case or has brass or metal parts near the coils should be viewed with suspicion as regards its instrumental accuracy. The Kelvin ampere balances intended for alternating current measurement have their coils wound on cores of slate, marble or porcelain, and the bases on which the instruments are mounted are also non-metallic.

The most suitable form of Siemens electro-dynamometer is that called the "Workshop" form, in which the coils are mounted on a wooden base. Even here the instrument maker often fixes the coils to the base by metal straps. These should be removed and ebonite straps used to hold the fixed coil in place. Instrument makers are far too fond of enclosing electrical measuring instruments in lacquered brass

cases, which look nice but often injuriously affect their performance and accuracy.*

The measurement of a large alternating current is best effected by some suitable instrument of the electro-dynamometer type. This must previously have been calibrated by passing through it a continuous current of known value, as determined by the potentiometer method, and an error curve constructed for the instrument as above described. The instrument may then be used to measure the R.M.S. value of an alternating current since the numerical value of the latter is that of the continuous current which produces the same dynamometer reading.

The most useful type of hot-wire alternating-current ammeter is that in which the current to be measured passes through a strip resistance of negligible inductance, and from the ends of which connections are made to a very fine wire which is heated by the small portion of the current shunted through it. If the wire is in an enclosure of constant temperature it soon comes to a final steady temperature, which produces a definite elongation of the wire. This expansion is measured by measuring the sag of the wire, its ends being attached to fixed points. Instruments of the above kind are manufactured by Hartmann and Braun (*see* Fig. 19), and a self-recording hot-wire ammeter is in use, designed by Holden and Pitkin, in which the current to be measured passes through and heats a number of fine wires of platinum or platinum silver arranged in parallel, the sag or extension of these wires being detected and measured by a multiplying gear which actuates a pen moving over a revolving drum.

The measurement of small alternating currents, such as the magnetising currents of transformers, is best accomplished by

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* As an instance of the kind of error that may thus result, the reader is referred to a Paper by the Author in the *Proc.* of the Institution of Electrical Engineers, Vol. XXI., p. 666, 1892, entitled, "Experimental Researches on Alternate Current Transformers," in which reference is made to the results of measurements made with a certain brass-cased wattmeter which were found to be in some cases in error by nearly 50 per cent.

measuring the fall of potential down a known resistance by means of an electrostatic voltmeter or quadrant electrometer which has been previously standardised by continuous voltage. Thus, for instance, if we have to measure an alternating current, say, of about the magnitude of 0·25 ampere, a resistance can be constructed of four strands of No. 36 bare platinoid wire capable of carrying without sensible heating 0·25 ampere, the total resistance being, say, 400 ohms. This resistance may be in the *cage* form described on p. 80. Then the fall in potential down this resistance produced by the current will be nearly 100 volts, and can be measured to

Fig. 19.—Hartmann and Braun Hot-Wire Ammeter.

within one-tenth per cent. by means of a standardised Kelvin multicellular voltmeter, the only condition necessary for so doing being that we must be able to afford to drop 100 volts in the voltage of the current used. In those cases in which this cannot be done an air-core alternating current transformer must be employed to effect a transformation.

If a transformer is constructed without an iron core, the two windings of which have their turns intermingled and in the ratio of N_1 to N_2, then if to the secondary terminals we connect an electrostatic voltmeter, the scale reading of this

will be N_2/N_1 times or fractions of the voltage on the primary terminals. If, then, the fall of pressure down the resistance through which the current to be measured is flowing is too small to be measured, say on a Kelvin multicellular voltmeter, we can, by employing an air-core transformer, raise this voltage to a readable value by using a transformer of the kind mentioned having its primary and secondary turns in a known ratio. This transformer should be made with circuits having as large an inductance as possible. It is desirable, however, to calibrate the transformer directly by taking a series of readings, within the range of the same voltmeter if possible, of the observed voltage on the primary and secondary terminals.

The same arrangement—viz., an air-core transformer—may be employed as shown by Mr. A. Campbell[*] to measure currents directly by transformation. In an air-core transformer the ratio of the maximum value of the primary current I_1 to that of the secondary current I_2 is

$$\frac{I_1}{I_2} = \frac{\sqrt{R_2{}^2 + p^2 N^2}}{pM},$$

where R_2 and N are the resistance and inductance of the secondary circuit and M the mutual inductance, and $p = 2\pi$ times the frequency. Hence, if pN is large compared with R_2, the ratio of the currents is always N/M, and independent of frequency.

By using an air-core transformer with the secondary circuit, made of thick wire and a large number of turns, so that the ratio R_2/N is at least 50, then, for a frequency of even 50 $\sim$ the ratio of the currents is at most 1 per cent. different from that which it would be for infinite frequency. This ratio of current transformation may, however, be found experimentally.

Thus, for instance, suppose it is required to measure an alternating current of about 1,000 amperes, and we have

only a Kelvin ampere balance and a Siemens' electro-dynamometer available, each reading up to 100 amperes. We may construct an air-core transformer having an approximate transformation ratio of 10 : 1 by making it with a thick wire core capable of carrying 1,000 amperes, and a coil of 10 times as many turns capable of carrying 100 amperes. The transformer has then to be calibrated by sending through the dynamometer and the thinner circuit a current, say, of 10 amperes, and joining the thicker circuit up in series with an adjustable resistance and the Kelvin balance. Suppose that, when the reading of the dynamometer is 10, the Kelvin ampere balance reads 98·2, we then know that the current transformation ratio is 9·82 to 1, and we can read the value of any current passing through the thick coil by taking the readings of the dynamometer placed in circuit with the thinner coil and multiplying by 9·82. Mr. Campbell has shown that iron ring core transformers may be used in the same way provided that the resistance of the secondary circuit is sufficiently low.

Convenient forms of electrostatic voltmeter for measuring alternating currents by means of the fall of potential down a resistance have been devised by Mr. G. L. Addenbrooke.* He constructs a quadrant electrometer with two double quadrant plates each formed of a pair of quadrant-shaped plates. A paddle-shaped aluminum needle is suspended between the plates by a very fine flat phosphor bronze strip (*see* Fig. 20). The needle carries a mirror as usual. One quadrant is connected to the needle and the two quadrants are connected to the ends of the resistance through which passes the current to be measured. The instrument can be rendered so sensitive that a deflection of 100mm, of a spot of light at 2 metres scale distance, can be obtained with only 1 volt fall of potential down the resistance. The instrument, therefore, can be used as an electrostatic voltmeter

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* *See* a Paper read before the International Congress of Electricians at Paris, 1900 ; also *The Electrician*, Vol. XLV., p. 901.

for the measurement of small potential differences, and also by association with a resistance strip it becomes an alternating current ammeter.

For the measurement of the small alternating currents passing through a transformer on open secondary circuit (magnetiz-
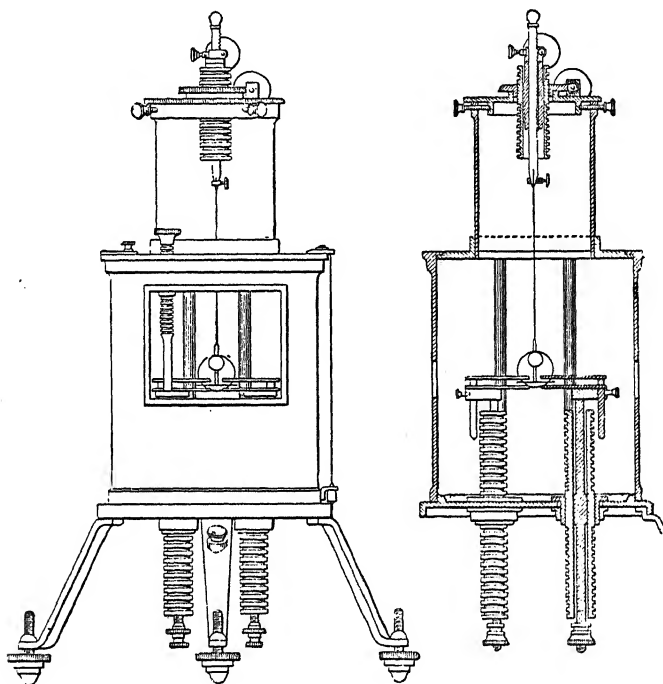
Fig. 20.—The Addenbrooke Electrostatic Voltmeter.

ing currents) the instrument is especially useful. It can be calibrated by the employment of a battery of Clark cells. Since the deflection is approximately proportional to the square of the potential difference at the terminals of the instrument, the deflections increase very rapidly with the voltages. Some form of low-potential electrostatic voltmeter, such as the one

above described, is a valuable acquisition in an electrical testing laboratory

§ **14. Wave Form Measurements.**—One of the most valuable methods of investigating the effects due to alternating currents consists in delineating the *wave form*—that is, drawing a curve to represent the mode of variation of the periodic current or currents and their relative phase difference. This process is called alternate current wave form tracing, or wave delineation. The results are generally expressed by taking a horizontal line to represent the uniform flow of time, and at distances measured from an origin representing the time or epoch, ordinates are set up representing to scale the instantaneous value of the current strength. The curve defined by their upper ends is the current wave form. Innumerable methods have been devised for drawing this wave form curve, but there are two which must be particularly described. These are called the *point-by-point* method and the *oscillograph* method.

The first-mentioned method originated with M. Joubert in 1880, and can be carried out in any laboratory possessing an alternator, or at any place to which alternating current is supplied, provided a synchronous alternating-current motor is at hand. The second method necessitates the possession of a more expensive and special device, called an oscillograph.

If we desire to know the current wave form of an alternator which is supplying a certain circuit, we may attach to the shaft of the alternator an insulating disc preferably made of a material called *stabilit*. This disc has a narrow brass strip inserted in its periphery. The strip may be in width about 1 per cent. to 2 per cent. of the periphery of the disc. Against this disc two insulated springs or narrow brushes press, so that when the disc revolves the springs are conductively connected during the moment the brass strip is passing under them The brushes are carried on a rocking lever which moves over a divided

scale, so that they can be shifted round through a known
and measured angle. This arrangement is called a revolving
contact-maker. It can easily be fitted to any alternator
shaft. The following arrangement of circuits are then made.
In the circuit, the current in which is to be delineated, is
inserted a resistance, R, of known value (*see* Fig. 21), and one
end of this resistance is connected to one of the brushes *a*
of the contact-maker *c*, and the other, *b*, to one terminal of
an electrostatic voltmeter, V. The other terminal of the
voltmeter is connected to the other brush, but arrangements
are made for inserting in between a variable number of
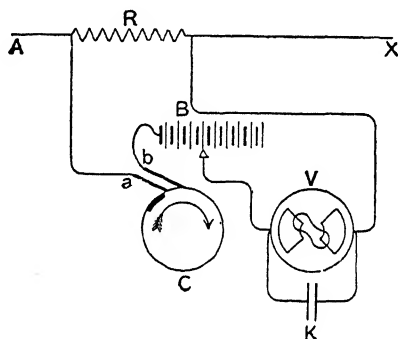
Fig. 21.

small (lithanode) secondary cells B. The needle of the
voltmeter is thus caused to give a reading at any required
part of the scale according to the number of cells in use
whenever the brushes are connected together. The terminals
of this voltmeter are connected to a condenser K having a
capacity of about 0·5 microfarad, to act as a reservoir or
flywheel.

If now the alternator is set in operation, and the brushes
occupy a fixed position; every time the brushes are inter-
connected by the revolving brass strip, the voltmeter is put
momentarily in connection with the ends of the resistance,
and after a few revolutions the condenser becomes charged

to a potential depending upon the phase of the current at the instant when this contact takes place. The voltmeter reading therefore indicates this potential, *plus* or *minus* that due to the cells of the battery B, according as they are connected to act with or against the fall of potential down the resistance. By taking the voltmeter readings as the springs or brushes are shifted over degree by degree through an angle covering the whole period, and correcting each observation by adding or subtracting the cell voltage, we obtain a series of values for the fall in pressure down the
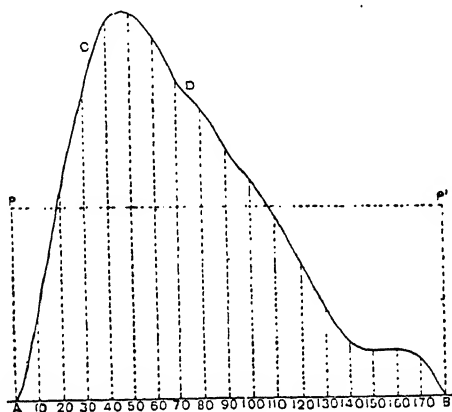
FIG. 22.

resistance corresponding to regular time intervals during one complete period.

Generally speaking, at least 20 observations should be taken at equal intervals during the period. These are then plotted out in a curve and give the wave form (*see* Fig. 22). They may also be plotted out in the form of a polar curve (*see* Fig. 23) in which radial lines are drawn at angular intervals representing the time intervals of the observations and lengths set off along these proportional to the instantaneous value of the current. The polar curve defined by the ends

of these radii vectores has the property that, if we take its
area and find the radius of a semi-circle having the same
area, this last radius is to the same scale as the root-mean-
square (R.M.S.) value of the current.*

If the alternator supplying the current is not accessible,
the same process may be carried out by employing a small
synchronous motor to drive the contact-disc in step with
the distant alternator.   A small synchronous alternating
motor is practically a small alternator.   A good type of
machine to employ for this purpose is the one represented

Fɪɢ. 23.

in Fig. 24.   The motor is a small Kapp alternator having
eight magnet poles and an armature consisting of an iron
wire core wound over with eight armature bobbins.   The
field magnets are excited by an independent continuous
current, and the armature is connected through a resistance
with the circuit supplying the alternating current under
investigation.   The motor is brought up into step with the
supply circuit by passing a strap round its shaft and
spinning it up to speed like a top.   When in step it will
continue to run, and will drive the contact-disc on its shaft

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* For the proof of this propostion see " The Alternate Current Transformer."
Fleming, Vol. I., 3rd edition, p. 193

so that the circuit of the brushes is closed for an instant at an assigned moment during the phase once during every
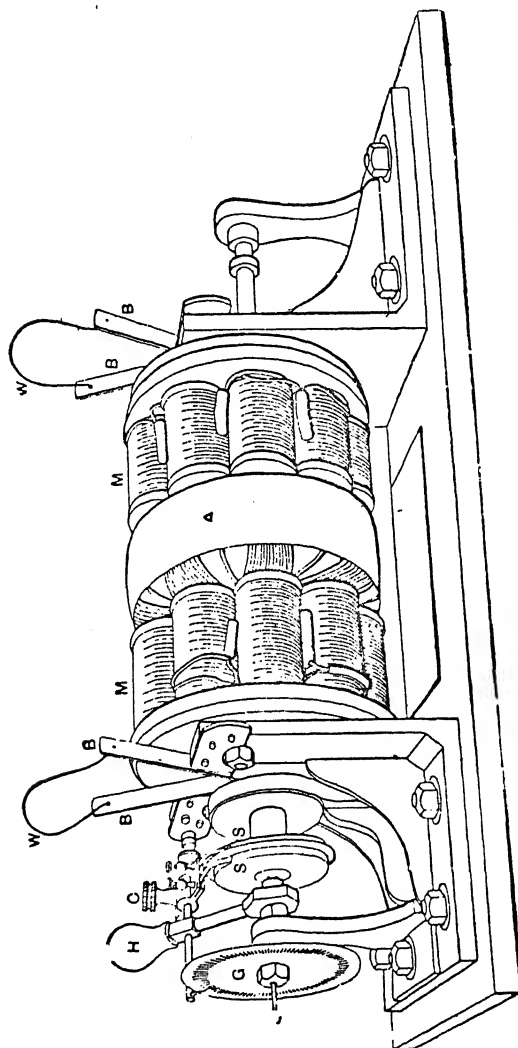
FIG. 24.—Synchronous Alternating-Current Motor fitted with Curve Tracer.

revolution. The process of obtaining graphically the wave form of the alternating current supplied by the circuit

is then carried out as above described, using, however, the contact-disc on the motor instead of one on the inaccessible alternator.

An example of an alternating current wave form plotted out by the point-by-point method is given in Fig. 25.

The great labour involved in slowly delineating a current wave form by this method led many inventors to devise methods which are susceptible of greater rapidity.   In some
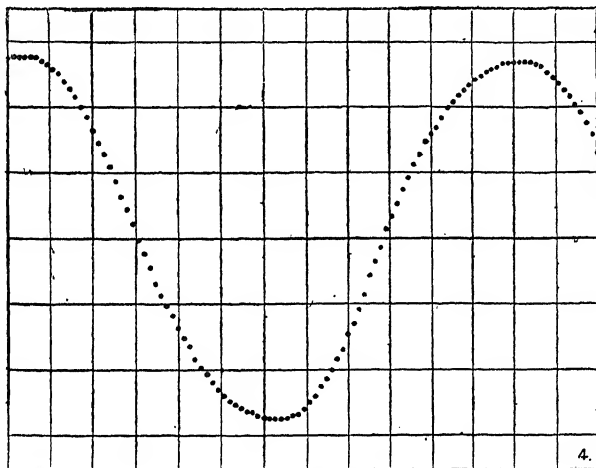
Fig. 25.

cases these take the form of self-acting potentiometers, in which the instantaneous value of the periodic electromotive force or difference of potential being studied is balanced against the fall of potential down a slide-wire.   In this case the galvanometer in the potentiometer circuit is replaced by a very sensitive relay, which operates a mechanism moving the contact point on the slide-wire one way or the other until the current in the relay circuit is zero.   A very ingenious instrument of this kind has been devised by Prof. Callendar, and another by Prof. Rosa.

In the Callendar instrument the essential parts are a form of potentiometer, the contact along the wire being shifted one way or the other by a couple of small motors. The contact-piece carries a pen which records a curve on a slowly revolving drum covered with paper. The contact is driven by the action of a delicate relay, which starts either one motor or the other in revolution unless the current
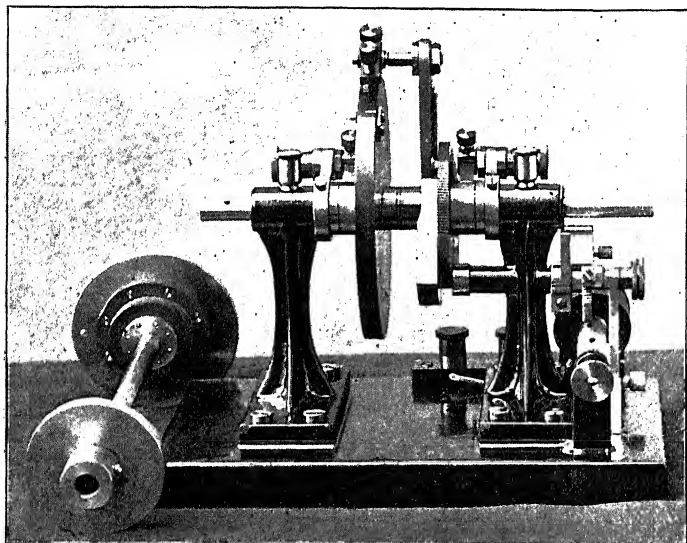
Fig. 26.—Rosa's Alternating Current Curve Tracer. Revolving Contact.

through the relay is zero. The relay takes the place of the usual galvanometer used with the potentiometer. Hence, if the instantaneous voltage of a periodic current is balanced against the fall of potential down the potentiometer slide-wire the mechanism will make the pen follow the variations of the periodic voltage as this last varies. The alternator or synchronous motor is provided with a contact-disc and brushes as above described, and these brushes are slowly shifted over by an automatic gearing through an angle equal

to one complete period. The pen then draws on the drum the curve corresponding to the wave form of this periodic voltage.*

The instrument designed by Prof. E. B. Rosa† is not very dissimilar. In it the fall of potential down a potentiometer wire is balanced against the difference of potential between the ends of a resistance traversed by the alternating current and by means of a revolving contact maker with displacable brushes (*see* Fig. 26). A single value of this periodic potential is selected at an assigned instant during the period for balance. The shifting contact on the potentiometer is coupled to a pantograph, which draws the wave form curve on a uniformly revolving paper-covered drum (*see* Fig. 27.)

The other method, making use of an instrument called the Oscillograph, designed for giving a complete view of the whole curve at once, is due originally to M. A. Blondel, who first described it in 1892.

The oscillograph has been given a very practical and improved form by Mr. Duddell. An oscillograph may be defined as a galvanometer, the needle or coil of which has such a small periodic time of vibration that it is able to follow accurately the variations of a periodic current which runs through its cycle of values in as little as one-hundredth of a second ; the deflections of the needle being proportional to the current. As a laboratory instrument the Duddell oscillograph‡ has many advantages. It is made as follows :— On a base board is fixed a nearly circular electromagnet (*see* Fig. 28) the coils of which are preferably excited by current at a pressure of 100 volts. This magnet creates a strong magnetic field between two pole pieces. In this field are stretched two loops of flattened phosphor bronze wire, which

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* *See* Prof. H. L. Callendar on " An Alternating Cycle Curve Recorder," *The Electrician*, Vol. XLI., p. 582.

† "An Electric Curve Tracer," by Prof. E.B. Rosa, *The Electrician*, Vol. XL. p. 126.

‡ Made by the Cambridge Scientific Instrument Company.
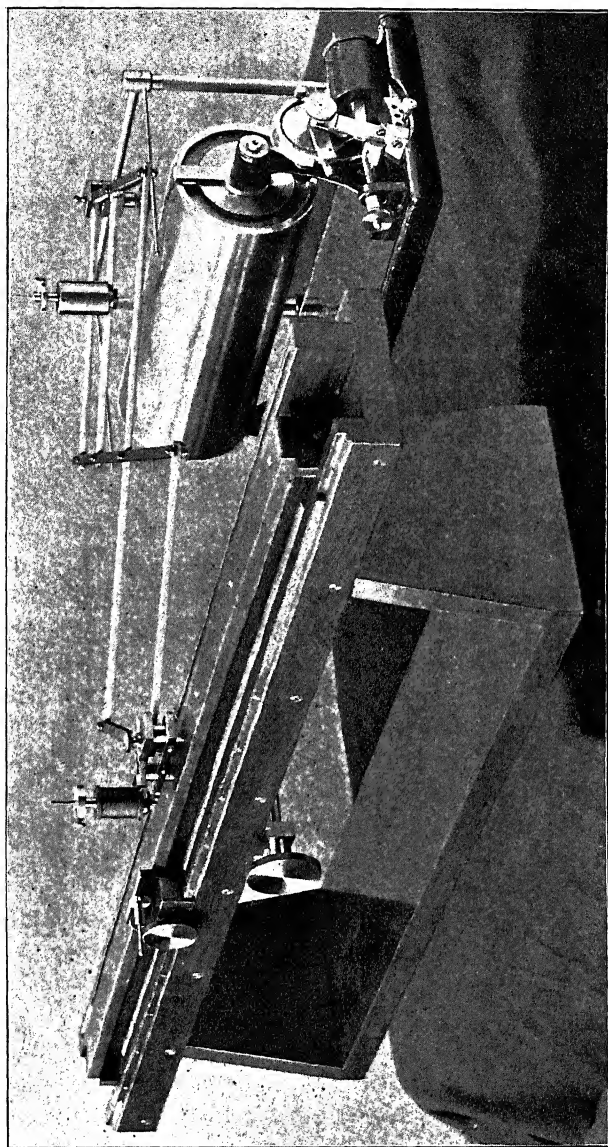
FIG. 27.—Rosa's Alternating Current Curve Tracer. Potentiometer Portion.

are kept in tension by a spring. These loops are fixed in the
field so that, if traversed by a current, one wire is forced
forward by the magnetic force and the other forced back.
The straps have attached to them a very small mirror, and
the space in which they move is filled with oil. The time of
free vibration of the mirror is either $\frac{1}{3000}$th of a second in
one form of the instrument, or $\frac{1}{10000}$th of a second in another
form. The first form of instrument is called the projection
form. In it a ray of light from an arc lamp is made to fall
upon the mirrors attached to the strips, and is reflected back
on to another mirror, which is given a rocking action by a
small synchronous motor, the rocking taking place round an
axis perpendicular to the reflected ray and parallel to the
plane of movement or vibration of that ray. The result of
the combined action of the mirrors is to reflect a spot of light
on to a screen which has a motion in two directions, its
displacement horizontally from the zero position being
proportional to the phase angle and its displacement
perpendicularly being proportional to the current in the
oscillograph strips. Accordingly, the spot of light executes a
motion on the screen which, in consequence of the persistence
of vision, is seen as a luminous curve delineating the wave
form of the periodic currents passing through the strip of the
oscillograph. The oscillograph is furnished with two strips
or loops in order that we may delineate two periodic curves
at once. It has also a fixed mirror to project a horizontal
datum line on the screen. In the high-frequency instrument
this reflected ray of light from the mirrors on the oscillograph
wires is received on a uniformly moving photographic film or
plate and photographs upon it an exact reproduction of the
wave form of the current through the strip.

In using the instrument to delineate the forms of current
and potential difference of the carbons of an alternating
current arc lamp, or motor, or other appliance, we attach the
terminals of one strip through appropriate resistances to the
points between which exists the periodic potential difference
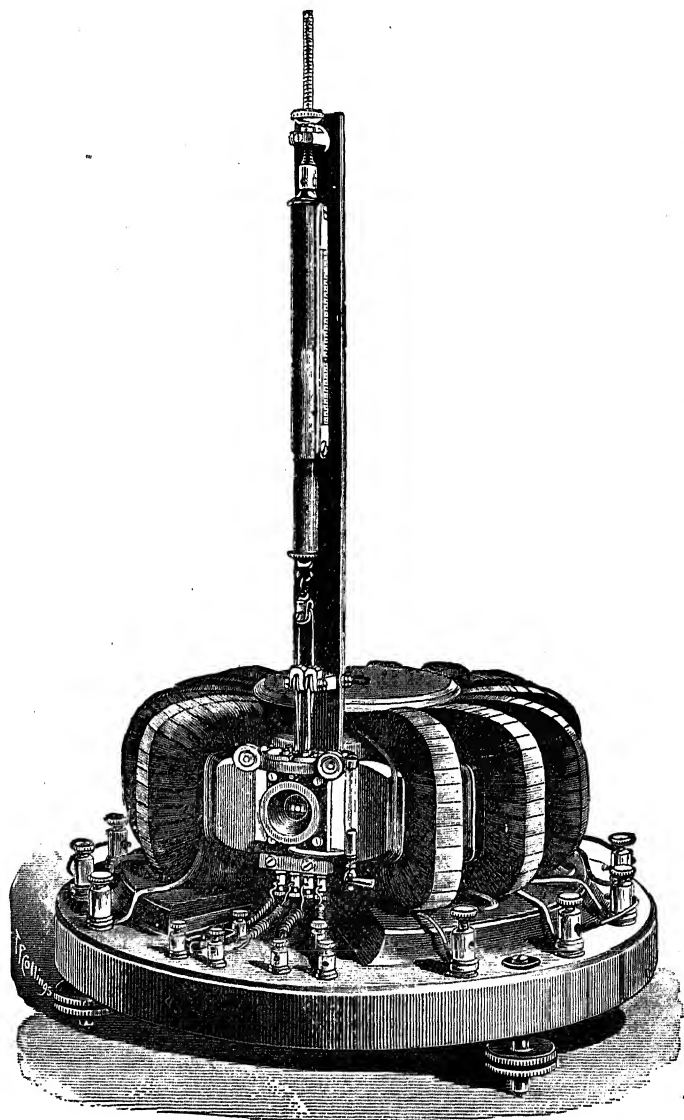
FIG. 28. — The Duddell Oscillograph.

to be delineated, and we attach the terminals of the other strip through resistance to the ends of a non-inductive resistance in series with the circuit of the apparatus, which is traversed by the current to be studied. We then obtain on the oscillograph screen or plate two curves, which represent the periodic current and potential difference in question.

The projection pattern of the Duddell oscillograph has sufficiently large mirrors attached to the wires that the light reflected from them will project distinct curves upon a screen at a distance of about 6ft. In the high-frequency instrument the mirrors are necessarily much smaller, and it can only be used as a visual or photographic instrument.

The following are the approximate data of the two types of Duddell oscillograph:—

—	High-frequency pattern.	Projection pattern.
Resistance of the field coils at 15°C....	360 ohms.	180 ohms.
Magnetising current	0·25 amp.	0·5 amp.
Working temperature of oil for correct damping	30° to 35°C.	30° to 35°C.
Periodic time	0·0001 sec.	0·0005 sec.
Normal working current in strip	0·05 to 0·1 amp.	0·5 amp.
Resistance of each strip	2 ohms.	1 ohm.
Ditto with fuses	10 ohms.	3 ohms.

As it is impossible to recapitulate here a description of more than a fraction of the methods which have been suggested or employed in alternating-current curve tracing, the reader may be referred for further information to the following original Papers and Memoirs, most of which references are taken from a comprehensive Paper on oscillographs by Mr. W. D. B. Duddell.* The references are, for the sake of convenience, divided into groups according to the methods which they describe, and they have, as far as possible, been made complete.

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* Read at the British Association Meeting at Toronto, 1897. *See* B.A. *Report*, 1897 ; or *The Electrician*, Vol. XXXIX., p. 636.

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W. E. AYRTON. British Assoc. *Report*, 1895.

BARR, BURNIE and RODGERS. *The Electrician*, Vol. XXXV. p. 719.

MANJAN LUTOSLAWSKI. *Electrotechnische Zeitschrift*, Vol. XVII., p. 582.

H. L. CALLENDAR. *The Electrician*, Vol. XLI., p. 582.

E. B. ROSA. *The Electrician*, Vol. XL., pp. 126, 221, 318.

III. Optical or Projection Methods

(*a*) With metal diaphragm as the moving part :—

Q. FRÖHLICH. *Electrotechnische Zeitschrift*, Vol. VIII., p. 210 , Vol. X., pp. 65 and 345.

E. THOMSON. *La Lumière Electrique*, Vol. XXVII., p. 339.

C. ROLLEFSON. *Ecl. Electrique*, Vol. I., p. 461.

(b) With a soft iron needle as the moving parts :—

A. Blondel. *Comptes Rendus*, Vol. CXVI., pp. 502, 748

H. J. Hotchkiss and F. E. Mills. *Physical Review*, Vol. III., pp. 49, 358; Vol. IV., p. 128.

F. J. A. McKittrick. *Trans.* Am. Inst. Elec. Eng., Vol. XIII., Nos. 6 and 7.

(c) With a coil of wire carrying the observed currents as the moving part :—

E. Gérard. *Bull.* de l'Acad. de Belgique, 1888; *Phil. Mag.*, Vol. XXIX.

H. Becker, G. Lahmeyer and G. D. Picard. *La Lumière Electrique*, Vol. XXXI., p. 16.

G. S. Moler. *Physical Review*, Vol. I., p. 214.

(d) With the plane of polarization of a beam of light as the moving part :—

A. C. Crehore. *Physical Review*, Vol. II., p. 122; Vol. III., p. 63.

J. Pionchon. *Comptes Rendus*, Vol. CXX., p. 872.

(e) With cathode ray in vacuo as the moving part :—

F. Braun. *Weid. Ann.*, Vol. LX., p. 552.

(f) With stream of mercury as the moving part :—

E. L. Nichols. *Jour.* Amer. Assoc. Ad. Science, Vol. XLII., p. 57.

(g) Miscellaneous methods :—

Lippman. 1875. *The Electrician*, July 17, 1896. (Uses a capillary electrometer.)

P. Janet. *Comptes Rendus*, Vol. CXVIII., p. 862; Vol. CXIX., pp. 58, 27, 399. (Uses a chemical method.)

IV. Oscillograph Methods.

A. Blondel. *Proc.* French Association for the Advancement of Science. 1898.

H. Abraham. *Comptes Rendus*, Vol. CXXIV., p. 758.

W. D. B. Duddell. *The Electrician*, Vol. XXXIX., p. 636.

Duddell and Marchant. *Journal* Inst. E.E., Vol. XXVIII., p. 1.

The most convenient instrument for ordinary laboratory work will be found to be the projection form of Duddell oscillograph.   In using it, a powerful parallel beam of light from a lantern containing a hand-regulated arc lamp with good vertical and horizontal slow-motion screws is thrown on the mirrors of the oscillograph, and a cylindrical lens fixed in front of the mirrors focuses the ray when reflected on to a sheet of tracing paper laid over a curved glass screen.   On this tracing paper is then seen, when the oscillograph is in operation, one or two bright sinuous lines of light, which are the wave forms of the currents passing through the oscillograph wires.   If one of these currents is proportional to the electromotive force in, or the potential difference of, the ends of a circuit, and the other is part or the whole of the current in that circuit, then the two sinuous lines will properly represent the variation of current and voltage, and the curves can be traced on the paper and investigation can be made of their form and phase difference.   The oscillograph method has one enormous advantage over the point-by-point method, in that the observer is enabled to see a transient change in the wave form.   The oscillograph shows at a glance the whole of one wave, and a change anywhere is instantly visible.

§ 15. The Use of Transformers in Alternating Current Measurement.—The measurement of alternating currents of moderate value may be carried out as already described by electrodynamic or hot-wire instruments.   It often happens however that measurements are required of alternating currents having very large or very small values.   It is then possible to bring these within the compass of instruments found in every well-equipped laboratory by employing transformers.   Thus, for instance, a large alternating current may be measured by passing it through a low resistance strip of known value and measuring the potential difference of the ends of the strip by means of a step-up transformer and an

ordinary electrostatic voltmeter reading, say, from 60 to 100 volts.

It has been shown by Mr. A. Campbell* that for the purpose it is necessary to use a transformer with a well closed iron circuit and a primary coil of low resistance and large inductance. This primary has its terminals connected to the ends of the low resistance strip through which the current to be measured is passing, and its secondary terminals connected to an electrostatic voltmeter. The transformer may conveniently have a transformation ratio of 1:100.

As an example of the details of an appropriate transformer Mr. Campbell (*loc. cit.*) gives the following specification :— The primary coil may consist of 100 turns of No. 16 wire, and the secondary of 10,000 turns of No. 40 wire, the coil consisting of iron stampings having a total weight of 1·5 kilos.

In the case of the above transformer the variation of the transformation ratio from the value 10 was 1·9 per cent. at a frequency of 39 and 0·8 per cent. at a frequency of 86.

Such a transformer can, of course, be adjusted to have any required transformation ratio at a fixed frequency. If it is to be used for varying frequencies, then compensation may be introduced by the employment of a very small additional transformer having its primary in series with that of the main transformer and its secondary joined in reverse direction with that of the secondary of the main transformer. In this case the small transformer can have its turns so adjusted that an increase in frequency causes it to deduct a little from the secondary voltage of the main transformer, and keep the resulting voltage transformation ratio constant.

The above device enables us to make use of an electrostatic voltmeter, reading from, say, 60 to 150 volts, to measure alternating currents covering a wide range in value.

The advantage of the method is that it introduces no sensible inductance into the circuit of the current to be measured, and that it is applicable to very large currents,

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* *Proc.* Inst. Elec. Eng., April, 1901.

such as those which would have to be measured in testing a large alternator.

We have already mentioned the use of an air-core transformer to transform alternating currents in known ratios (*see* page 392), but the method of measuring the volt fall down a low resistance strip by means of a suitable transformer combination and an electrostatic (preferably reflecting) voltmeter is more convenient. It need hardly be said that the electrostatic voltmeter used should be carefully calibrated in position by means of a potentiometer, a Clark cell and a divided resistance.

§ 16. Measurement of the Frequency of an Alternating Current. Frequency Tellers.—In many alternating current investigations it is necessary to know the periodicity of the current. If the alternator is accessible this can easily be done by ascertaining the speed. If, however, the alternator is in a distant station some appliance is necessary which will measure the frequency of the current as it is supplied on the circuit. Instruments for doing this are called *frequency tellers*. For this purpose several devices have been invented. In Campbell's frequency teller* a soft iron strip is so held in a frame that it can be caused to protrude more or less from a holding block. The end of the iron strip projects over an electromagnet with laminated core, through the coils of which passes the alternating current to be estimated. The iron strip has a natural free period of vibration, and if the attractive impulses which act on it, due to the period magnetisations of the electromagnet, agree in frequency, the amplitude of vibration of the strip becomes considerable.

The instrument includes a slow-motion arrangement for pushing the iron strip more or less in or out of the clip which holds it at one end and an indicating needle which magnifies the motion. The observer makes the movement until the sound or visible oscillations of the vibrating iron strip is a

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* See *The Electrician*, Vol. XXXVII., p. 437.

maximum, and then he reads off on the scale of the instrument the frequency of the current passing through the coils of the electromagnet. The instrument is graduated by applying to it currents of various frequencies, and is said to read frequency to within 0·2 per cent.

In another modification due to C. Kinsley[*] the free period of the strip is altered, not by varying its length, but by sliding along it a non-magnetic rider or weight.

A third variety of frequency teller, suggested originally by Profs. Ayrton and Perry,[†] consists of a stretched wire placed between or over the poles of an electromagnet. The wire may be of iron, and the magnet coils may be traversed by the current to be examined. In this case the periodic attractive impulses set the wire in vibration if its free period is in concordance with that of the currents or is some harmonic of it. The free period of the wire is determined by its length, diameter, density and tension, and is expressed by the equation

$$t = 2l\sqrt{\frac{\pi r^2 \delta}{\mathrm{T}}},$$

where $t =$ the periodic time, $l =$ the length of wire, $r =$ the semi-diameter, $\delta =$ the density of the wire, and $\mathrm{T} =$ its tension in dynes. Hence, if T is varied by using different stretching weights, the free period of vibration of the wire may be made equal to that of the exciting current of the magnet, and when this is the case the vibrations of the wire become a maximum.

The periodic current may be sent along the wire and the magnet may be a permanent magnet. In this case, however, the determination of the frequency is complicated by the variable rise in temperature of the wire produced by the current. Hence it is better to pass the current examined through the coils of the magnet and use an iron wire or wire with small iron armature attached to it, varying the tension

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* See *The Electrician*, Vol. XL., p. 258.

† See "Laboratory Notes on Alternate Current Circuits," *Proc.* Inst. E. E. Vol. XVIII., p. 310.

on the wire until unison is produced between its free period of vibration and that of the current through the magnet coil.

A fourth method depends upon the use of a telephone If an alternating current is passed through a magneto telephone receiver its diaphragm is set in vibration, and it emits not only a fundamental note corresponding to the frequency of the current, but a series of higher harmonics. The note of the telephone may be examined by testing it with a sliding resonator tube and varying the length of the column of air until resonance is produced. The resonator tube length is altered until two lengths are found which both exalt the note of the telephone. The periodicities corresponding to these lengths are then known from the formula

$$V = n\lambda,$$

where λ is the wave-length and V the velocity of sound at the then temperature of the air. The length of the resonator tube which then resounds is obviously one-quarter of a wave-length.

Thus, if the length of resonator tube is 20cm., we know that the wave-length is 80cm., and hence the frequency n is $\dfrac{33,240 + 60t}{80}$, where t is the temperature of the air. Hence, at 15°C. $n = 427$.

If, for instance, two resonator tube lengths are found, one 19·1cm. and the other 16·5cm., corresponding to frequencies of 420 and 480, then these are harmonics, and the greatest common divisor—viz., 60—is the frequency of the fundamental.

For most electrical laboratory purposes, a carefully calibrated Campbell frequency teller will be found to be a most useful instrument, and will give the frequency of usual alternating periodicities within at least 0·5 per cent.

§ 17. The Measurement of the Phase-difference of Periodic Current Phasemeters.

—An important measurement in connection with alternate-current working is the determination of the phase difference of two currents. If

two equi-periodic currents follow a simple sine law of variation, then the angular interval between their maximum or zero values expressed in degrees, taking 360° to represent the complete common period, is called their *phase difference.* If the two currents can be passed in part or in whole through the two wires of a double oscillograph, then a simple inspection suffices to give the required information.   It may be, however, that the observer does not possess an oscillograph, and then he may proceed by other means to obtain the desired knowledge.   One method is by the use of a soft iron needle, as suggested by Lord Rayleigh.*   Let two circular coils of wire be prepared suitable for carrying the two alternating currents which are to be examined as regards difference of phase or lag.   Let us, in the first place, assume that these currents are simple periodic currents, or that they are sine-curve curves ; in other words, let the instantaneous value i of either current be represented by an expression of the form,

$$i = \text{I} \sin pt,$$

where I is the maximum value and $p = 2\pi n$, n being the frequency and t the time reckoned from the instant when i is zero.   Let the two currents have the same frequency but different maximum values, and let their phase difference be represented by an angle ϕ. It is required to determine ϕ. The two currents are then represented by the equations

$$i = \text{I} \sin pt,$$
$$i' = \text{I}' \sin (pt - \phi).$$

Hence, the root-mean-square (R.M.S.) value of i is $\text{I}/\sqrt{2}$, and that of i' is $\text{I}'/\sqrt{2}$.

Suppose, then, that the two currents are passed through the above-mentioned coils, and that these are placed with axes in one line and paralled to each other.   At some point between them is placed a short soft iron needle (say 2cm. in length), suspended by a torsion wire or glass fibre and

* *Phil. Mag.,* **May,** 1897 ; or *The Electrician,* Vol. XXXIX., p. 180.

carrying a mirror so that its deflections can be observed by a mirror and scale method as usual. The upper end of the torsion fibre should be carried on a torsion head so that it may be twisted round through a known angle. The needle must be so placed that its axis makes an angle of 45deg. with the line joining the centres of the coils. First let one current, i, be passed through one coil. It will create an alternating magnetic field, and this field will exert a couple or torque on the soft iron needle proportional to I^2. This can be measured by the twist necessary to be given to the torsion head to bring the needle back to the original position, or, if the deflection is small, by observing simply the deflectional angle of the needle.

Next, let the other current be passed alone through its coil, and again observe the angle of deflection or torque acting on the needle. It is proportional to I'^2.

In the third place, let both currents be passed at once through their coils, and let the joint field act on the soft iron needle. Then the deflecting couple is proportional to

$$I^2 + I'^2 \pm 2II' \cos \phi.$$

Hence, if the first observed deflection or torsional angle is θ, the second θ', and the third θ'', we have

$$I^2 = a\theta,$$
$$I'^2 = a\theta',$$
$$I^2 + I'^2 \pm 2II' \cos \phi = a\theta'',$$

where a is some constant depending on the torsion value of the suspension. Hence, if the fields due to each current oppose each other,

$$\cos \phi = \frac{\theta + \theta' - \theta''}{2\sqrt{\theta\theta'}}.$$

Thus, suppose $\theta = \theta' = 40$ and $\theta'' = 26\frac{1}{2}$, then $\cos \phi = 0\cdot67$, $\phi = 48$deg. If the two equi-periodic currents considered do not follow a simple sine law of variation, then their zero points have not the same phase-difference as their maximum

values. Lord Rayleigh shows, in the original Paper, that even if the currents do not follow the sine law, or are not simple periodic currents, the above expression for cos ϕ gives us what is called the *power-factor* of one current with respect to the other.

If, then, one of the currents is proportional to the voltage of a current and the other to the current flowing in it, the value of cos ϕ obtained as above is a factor which, if multiplied by the R.M.S. values of the current and voltage, will give the true power being taken up electrically by the circuit.
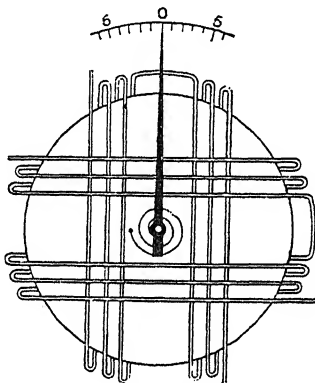
Fig. 29.

Instruments for indicating the phase difference of two simple periodic or harmonic currents are called *phasemeters.* One convenient form is that invented by Von Dolivo-Dobrowolsky.*

If two circuits are wound on a frame at right angles to each other (*see* Fig. 29), and if a disc of soft iron having an indicating needle attached is suspended at the centre, then, as is well known, if there is any difference of phase between periodic currents of equal frequency passing through the two coils, then there will be a torque tending to rotate the iron

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* See *The Electrician*, Vol. XXXIII., p. 610.

disc. This torque can be resisted by the action of a spiral spring, and the instrument can be graduated by trial to show on a dial (*see* Fig. 30) the phase difference of the currents. If i and i' are the instantaneous values of the equi-periodic currents, and if ϕ is their difference of phase, then the torque T acting on the disc is proportional to $ii' \sin \phi$. If one of these coils is a high resistance inductionless coil, and is connected to the terminals of any alternating current circuit, then the current through that coil will be proportional to the potential difference of the ends of the said circuit and in step
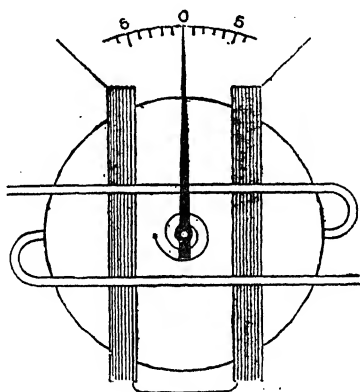
FIG. 30.

with it. If the other coil is traversed by the current entering the above-named circuit, then the indications of the phase-meter are proportional to $ie \sin \phi$, where ϕ is the angle of phase difference of the current i and the voltage e of the circuit. If the voltage is always the same, then the indications of the instrument are proportional to the *idle current*, $i \sin \phi$, passing into the circuit, and the direction of deflection shows whether this idle current lags or leads.

The phase-meter may be graduated for a given frequency and R.M.S. value of the voltage to read directly on the dial the ampere value of the idle current passing through it.

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Hence such an instrument is useful, in an alternating current station, to indicate at a glance the so-called *wattless current* passing out into a circuit.

References to appliances and methods for measuring phase difference will be found in the following Papers :—

J. TUMA.   *Science Abstracts*, Vol. I., pp. 148 and 266.

The author employs a soft iron needle and an arrangement of two coils at right angles carrying the two currents. Otherwise the principle of the apparatus is the same as in the instrument of Lord Rayleigh above described.

MOLER and BEDELL.   *The Electrician*, Vol. XXXIII., p. 210, Describes an optical phase indicator.

VON DOLIVO-DOBROWOLSKY.   See *The Electrician*, Vol. XXXII., p. 40, and Vol. XXXIII., p. 610, for description of soft iron disc phase meter.

R. ARNÒ.   *Ecl. Electr.*, Vol. XX., 1899, p. 225; also *Science Abstracts*, Vol. III., p. 71.

This writer describes an instrument which is a combination of a Weber dynamometer, having separate coils, and an instrument to produce a rotating magnetic field, having separate coils fixed at right angles, and a closed movable circuit placed inside them.

C. V. DRYSDALE.   *The Electrician*, August 24, 31, 1906.   "The Measurement of Phase Differences."

W. E. SUMPNER.   *Proc.* Phys. Soc. Lond.   Vol. XX., p. 1, 1906. " On the Theory of Phasemeters."

This Paper contains a very full treatment of the theory of phasemeters, especially as applied to polyphase circuits. The author's conclusions as regards these instruments are as follows :—

1. Phasemeters for multiphase circuits are all equally accurate on balanced loads provided they have been correctly calibrated and possess no faults due to purely mechanical causes. Their accuracy is not affected by variations in wave-form or in current-frequency. The calibration of the scale is affected by the number of coils used in the instrument, by the ratios of the ampere turns used with these coils, by the distribution of the windings, and by the magnetic nature and properties of the magnetic circuits, especially if these contain iron ; but the accuracy of the indications is not dependent upon any of these considerations if the working of the instrument is satisfactory from a mechanical point of view.

2. Phasemeters can be simply and accurately calibrated for balanced loads by means of a direct current method of test.

3. The error of phasemeters on unbalanced circuits is generally serious for loads which are badly out of balance. The error, like that of a wattmeter, increases rapidly as the power factor of the load diminishes. It can only be reduced at the expense of complication in the instrument, by increasing the number of coils used in the fixed and moving systems, and by arranging the coils and magnetic circuits to be symmetrical in regard to one another. If the true power factor of the load is cos ϕ, the reading of the instrument is $\cos \phi + \theta \sin \phi$,

where θ is the phase error due to the unbalanced load, and is product of two factors, one of which is the maximum value of θ determined by the amount the load is out of balance, and the other may have any value between $+1$ and -1 and determines whether the instrument reads high or low.

TABLE I.

Fuse-Wire Currents. (SIR W. H. PREECE.)

Melting currents in amperes.	Approximate gauge of wire in B.W.G. fused by the stated currents in the case of the metals			
	Tin.	Lead.	Copper.	Iron.
1	36	35	47	40
2	34	32	43	36
3	30	28	41	33
4	26	26	39	31
5	24	24	38	29
10	20	20	33	24
15	18	18	30	22
20	17	17	27	20
25	16	16	26	19
30	15	15	25	18·5
35	14·5	14	24	18
40	14	13	23	17·5
45	13	12·5	22·5	17
50	12	12	22	16
60	11·5	2/15	21	15
70	10·5	2/14	20	14
80	10	2/13	19	13·5
90	9·5	3/15	18·5	13
100	9	4/16	18	12
120	8	3/13	17·5	2/15
140	2/10	4/14	17	2/14
160	4/18	4/13	16·5	2/13½
180	3/11	4/12	16	2/13
200	4/12	5/13	15	2/12
250	5/12	5/12	13½	5/16

TABLE II.

Electro-chemical Equivalents.*

Elements.	Valency and symbol.	Atomic weight.	Chemical equiva-lent.	Electro-chemical equivalent in milli-grammes per coulomb	Coulombs per gramme.	Electro-chemical equivalent in grammes per ampere hour.
Electro-Positive.						
Hydrogen	H^1	1	1	0·010384	96293·00	0·03738
Potassium	K^1	39·04	39·04	0·40539	2467·50	1·45950
Sodium	Na^1	22·99	22·99	0·23873	4188·90	0·85942
Aluminium	Al^3	27·3	9·1	0·09449	1058·30	0·34016
Magnesium	Mg^2	23·94	11·97	0·12430	804·03	0·44748
Gold...............	Au^3	196·2	65·4	0·67911	1478·50	2·44480
Silver	Ag^1	107·66	107·66	1·11800	894·41	4·02500
Copper (Cupric) ...	Cu^2	63	31·5	0·32709	3058·60	1·17700
,, (Cuprous)..	Cu^1	63	63	0·65419	1525·30	2·35500
Mercury (Mercuric)	Hg^2	199·8	99·9	1·03740	963·99	3·73450
,, (Mercurous)	Hg^1	199·8	199·8	2·07470	481·99	7·46900
Tin (Stannic)	Sn^4	117·8	29·45	0·30581	3270·00	1·10090
,, (Stannous) ...	Sn^2	117·8	58·9	0·61162	1635·00	2·20180
Iron (Ferric)	Fe^3	55·9	18·64	0·19356	5166·4	0·69681
,, (Ferrous)......	Fe^2	55·9	27·95	0·29035	3445·50	1·04480
Nickel	Ni^2	58·6	29·3	0·30425	3286·80	1·09530
Zinc	Zn^2	64·9	32·45	0·33696	2967·10	1·21330
Lead................	Pb^2	206·4	103·2	1·07160	933·26	3·85780
Electro-Negative.						
Oxygen	O^2	15·96	7·98	0·08286	...	...
Chlorine	Cl^1	35·37	35·37	0·36728	...	...
Iodine	I^1	126·53	126·53	1·31390	...	...
Bromine	Br^1	79·75	79·75	0·82812	...	...
Nitrogen	N^3	14·01	4·67	0·04849	...	...

* Taken by permission of "The Electrician" Publishing Company (Ltd.) from the "Electrical Trades' Directory and Handbook."

CHAPTER IV.

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THE MEASUREMENT OF ELECTROMOTIVE FORCE.

§ 1. Electromotive Force Measurement.—Being provided with a standard of electrical resistance and means for defining or obtaining an electric current, the value of which in amperes or recognised units is known, we can obtain the consistent unit of electromotive force by measuring the fall of potential down the unit of resistance when the unit current flows through it. In practice we may proceed to recover the practical unit of electromotive force, called the *International* or *standard volt* as follows:—

Prepare a coil of manganin wire which has a resistance of one International ohm. This wire should be of a gauge or cross section sufficient to carry a current of 1 or 2 amperes without sensible heating, and should preferably consist of a bare manganin wire of about No. 16 S.W.G. wound on a wooden frame. The best form for this coil is to construct a short skeleton wooden drum about 8in. in diameter on which may be wound in grooves the necessary length of some 18ft. to 20ft. of the bare manganin wire.

The drum may be made by putting eight wooden pegs into holes bored near the circumference of two circular wooden discs 10in. or 11in. in diameter. Instead of winding on the drum a length of wire having an exact resistance of 1 ohm, it will be sufficient to measure with great care the resistance of the length of wire actually wound on it, provided its resistance is nearly 1 ohm. The ends of the wire

should be silver-soldered to stout copper rods which form the terminals, and between these rods the resistance must be accurately measured.

This resistance coil is then to be joined up in series with a Kelvin deciampere-balance, which has been previously most carefully standardised by silver deposit (*see* Chapter III., § 3, p. 347), and a current having a known value in International amperes is then sent through this resistance. The product of the values of the known current and known resistance gives us, then, the value in International volts of the potential difference between the copper terminals of the resistance coil.

In this manner we recover or create a known voltage between two points. It is, however, more convenient to preserve a standard voltage in the form of a standard Clark or Weston cell (*see* Chapter I., § 8, p. 86), and to employ the known voltage obtained as above mentioned to evaluate the electromotive force of the cell. This can be done in the following manner :—

The standard cell to be checked is joined up in series with a very sensitive galvanometer and a wire of high resistance—say of 10,000 ohms—the two being placed as a shunt across the ends of the above-mentioned 1 ohm resistance. It is well to insert a key, so as to close the circuit of the cell only when required. A current is then passed through the Kelvin deciampere-balance, a carbon rheostat, and the manganin resistance in series with it, and is adjusted to have such a value—approximately 1·5 amperes —that, when it flows through the manganin 1 ohm resistance, it creates a fall of potential down it equal to the electromotive force of the Clark cell. The Clark cell must be so joined up (*see* Fig. 1) that its positive or mercury pole is in connection with that end of the manganin resistance by which the current enters it.

When this arrangement of apparatus is complete the experimental measurement consists in adjusting the current through

the ampere balance until the galvanometer in series with the
Clark cell shows no deflection when the key K is down.
This being the case, the product of the observed ampere-
balance current and the resistance of the manganin wire R
is equal to the value of the electromotive force of the Clark
cell at the temperature of the day. The Clark cell should
be provided with a thermometer by means of which its
temperature can be taken. In this manner the recovered
standard of voltage can, so to speak, be transferred to a
standard cell and be rendered permanent.

In an electrical testing laboratory the cells used as stan-
dards of electromotive force should be checked at intervals
so as to re-determine their absolute electromotive force values.

A   Kelvin Deciampere-balance.
R   1 ohm Manganin Resistance.
G   High-resistance Galvanometer.
Ck Clark or Weston Standard Cell.

K   Key.
r   Resistance of 10,000 ohms.
C   Adjustable Carbon Rheostat.
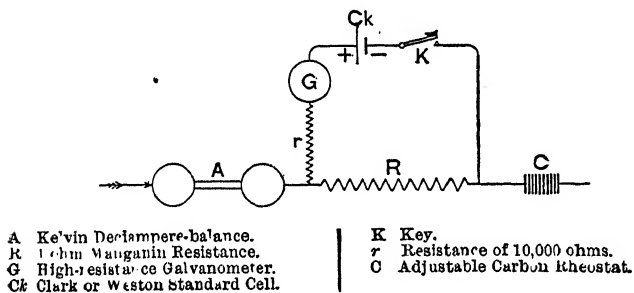
Fɪɢ. 1.

It was by a method of the above kind that Lord Rayleigh
made his original measurements of the electromotive force of
the Clark cell. An account of his work is found in a Paper
by Lord Rayleigh and Mrs. H. Sidgwick in the *Phil. Trans.*
of the Royal Society, Part II., 1884, entitled " On the
Electrochemical Equivalent of Silver, and on the Absolute
Electromotive Force of Clark Cells." In this Paper Lord
Rayleigh first describes the construction of an absolute
ampere-balance for the evaluation of the current in absolute
measure. This consisted of two fixed circular horizontal coils,
the distance between their planes being equal to the radius

of either of them. Between these coils was supported another smaller circular coil, with its plane horizontal and its centre on the line forming the centres of the fixed coils, and midway between them. This third coil was suspended from the beam of a weighing balance, as shown in Fig. 20, p. 71, Chapter I. The movable coil had the current led into and out of it by flexible leads. The current to be measured was passed through all three coils in series, so that the electrodynamic forces tended either to lift or lower the movable coil and thus decrease or increase its apparent weight. By reversing the current through the fixed coils the direction of the force could be reversed, and hence half the difference between the apparent weight of the movable coil when the current through the fixed coils is reversed gives the force in gravitation units exerted by the fixed upon the movable coil.

The general theory of the instrument is as follows :—Consider first two coils only—viz., a fixed and a movable co-axial coil. Let n, n' be the respective number of turns of wire on each, i, i' the current in each, and M the coefficient of mutual induction. Then the attraction or repulsion between the coils, in absolute units is equal to

$$ii'nn'\frac{d\mathrm{M}}{dx},$$

where x is the distance between the parallel planes of the coils.

The full expression for the attraction between two parallel co-axial circular coils of mean radii A, a, and with centres respectively at distances B, b from a point on their common axis, is given by Maxwell ("Electricity and Magnetism," Vol. II., § 304) :—

Let $\mathrm{C}^2 = \mathrm{A}^2 + \mathrm{B}^2$. Then

$$\frac{d\mathrm{M}}{db} = \pi^2\frac{\mathrm{A}^2a^2}{\mathrm{C}^4}\left\{1.2.3\frac{\mathrm{B}}{\mathrm{C}} + 2.3.4\frac{\mathrm{B}^2 - \frac{1}{4}\mathrm{A}^2}{\mathrm{C}^3}b + 3.4.5\frac{\mathrm{B}(\mathrm{B}^2 - \frac{3}{2}\mathrm{A}^2)}{\mathrm{C}^5}(b^2 - \frac{1}{4}a^2) + \ldots\right\},$$

where a and b are supposed to be small compared with A.

In the present case there are two fixed coils, and if we take the origin midway between them, then

$$\frac{d\mathrm{M}}{db} = \pi^2\frac{\mathrm{A}^2a^2}{\mathrm{C}^4}\left\{1.2.3\frac{\mathrm{B}}{\mathrm{C}} + 3.4.5\frac{\mathrm{B}(\mathrm{B}^2 - \frac{3}{4}\mathrm{A}^2)}{\mathrm{C}^5}(b^2 - \frac{1}{4}a^2).\right\}$$

Also, approximately, for the fixed coils we have $\mathrm{B}^2 = \frac{1}{4}\mathrm{A}^2$.

Hence

$$\frac{d\mathrm{M}}{db} = 6\pi^2\frac{a^2}{\mathrm{A}^2} \times 0\cdot2862.$$

In other words, the force of attraction f of the fixed coils on the movable is given by $f = hnn'i^2a^2/\mathrm{A}^2$, where $h = 6\pi^2 \times 0\cdot2862$, and i is the current through the coils.

Accordingly, if β is the ratio of the galvanometer constants of the coils—that is, if $\beta = \dfrac{an}{An'}$, we have $f = h\beta^2 i^2 n'^3/n$.

In the coils used by Lord Rayleigh

$$A = 24 \cdot 81016 \text{cms.} \quad n' = 225$$
$$a = 10 \cdot 2473 \text{cms.} \quad n = 242$$

For the elaborate details of the process of weighing the movable coil and reducing the results to calculation we must refer the reader to the original Paper by Lord Rayleigh, which is an example of accurate scientific research of the most perfect kind.

Having obtained the means of defining in absolute measure a steady electric current, this current was passed through a resistance, and the fall of potential compared, as above described, with the electromotive force of a Clark cell when not sending current.

A	Ampere-balance.	R₁, R₂	Resistance Boxes.

A Ampere-balance.

R Resistance Coil.

Ck Clark Cell.

G Galvanometer.

R₁, R₂ Resistance Boxes.

X Commutator.

L Two Leclanché Cells.
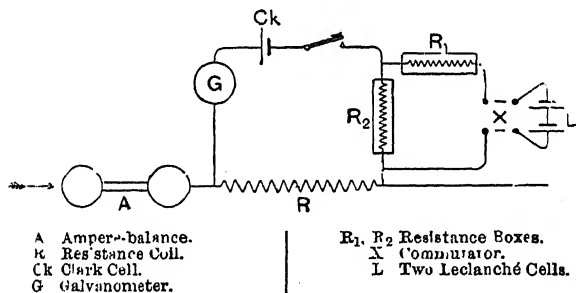
Fig. 2.

In the above arrangement, as employed by Lord Rayleigh, it was not found convenient to alter the main current flowing through the ampere-balance when once started; hence the fall of potential down the resistance was not exactly equal to the electromotive force of the Clark cell. A compensating electromotive force was, however, introduced as follows :— Let A (*see* Fig. 2) be the ampere-balance and R the resistance through which the current traversing the balance is flowing. Let Ck be the Clark cell and G a sensitive galvanometer of

high resistance. Then L is another battery, say of two Leclanché cells, the terminals of which are short-circuited by a large resistance consisting of the coils in two resistance boxes R_1, R_2. A derived circuit is taken from this Leclanché cell circuit, as shown in the figure, so as to insert an assisting or opposing electromotive force in the Clark cell circuit, which can be varied as to direction by a commutator X and as to magnitude by varying the resistances R_1, R_2, unplugged out of the circuit of the two resistance boxes—subject always to the condition that $R_1 + R_2 = 10,000$ ohms. In this way the current flowing out of the Leclanché cells is kept constant, but a variable fraction of their electromotive force can be used with or against that of the Clark cell Ck, so as to bring the current in the circuit of the galvanometer G to zero. Then, knowing the resultant electromotive force in the galvanometer circuit, which is equal to the product of the main current and the resistance R, we have only to find the ratio between the electromotive force of the Clark cell and that of the fractional part of the Leclanché cells. This can be done at once by the use of a potentiometer, as described in a subsequent section of this chapter, with any required degree of accuracy.

By the above process, Lord Rayleigh* found the electromotive force of the Clark cell, at 15°C., to be 1·435 volts, and at t°C. it is given by the equation

$$E_t = 1·435\{1 - 0·00077(t - 15)\} \text{ volts.}$$

Later experiments decided the Board of Trade Committee on Electrical Standards to recommend that the value of the Clark cell electromotive force be taken as 1·434 International volts at 15°C. If, however, the electrochemical equivalent of silver per ampere-second is taken as 0·001119 gramme, instead of 0·001118 gramme as adopted by the Board of Trade, then the corresponding value of the electromotive

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* "The Clark Cell as a Standard of Electromotive Force," by Lord Rayleigh. *Trans.* Roy. Soc., Part II., 1885.

force of the Clark cell will be a number nearer to 1·432 true volts at 15°C. The Board of Trade values of the electro-motive force of the Clark cell at various temperatures are given in Table I. at the end of this chapter.

§ 2. **Practical Recovery of a Standard or Known Poten-tial Difference.**—If the electrical laboratory possesses, as it should do, a Kelvin deciampere-balance, the most convenient practical method of recovering a known potential difference is to standardise this balance most carefully by passing a constant current through it of about 4 or 5 amperes, and then determine by the copper sulphate method of electrolysis the absolute value of this current with all the precautions described in Chapter III.

Having standardised the balance, it may be employed to measure the ampere-value of a current sent through a resist-ance of known value—say, 1 ohm—constructed of manganin wire of a sufficient cross-section not to be sensibly heated by the current. If it is desired to obtain a known electromotive force or potential difference of high value—such, for instance, as 2,000 volts—then the ampere-balance must be one which is suitable for alternating as well as for continuous currents. A wire resistance must then be made having a known value of 2,000 ohms or thereabouts, and an alternating current of 1 ampere, measured by the balance, must be passed through it. The exact difference of potential between the terminals of the high resistance then becomes known. Generally speaking, however, the expense of constructing a large current-carrying resistance of 2,000 ohms will be an obstacle to the employment of this method. There is no question, however, that it is the most satisfactory process for conducting with certainty the calibration of high-tension voltmeters.

The wire resistance must be so constructed as to be as little inductive as possible. This may be done as follows:— An ebonite or fibre or pasteboard tube has wound upon it a

silk-covered manganin wire, or, if bare wire is used, a silk
thread must be wound between the convolutions to prevent
them from touching each other.   The coil so prepared is
covered with one layer of mica or shellac paper and another
layer of wire wound upon it in an opposite direction.   The two
layers of wire are joined up at one and the same end, so that if
a current is sent through the two layers in series it flows in
opposite directions round each layer.   A double-layer coil so
constructed is practically non-inductive, or but little induc-
tive.   When traversed by a current there is little or no
magnetic field in the neighbourhood of the wire.   A series of
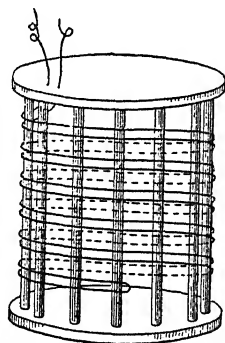
Fɪɢ. 3.—Skeleton Wooden Drum with Bare Manganin Wire wound
non-inductively on it.

such coils may be joined up in a frame and used as the
current-carrying resistance.   It is exceedingly useful to
provide for the testing laboratory a number of resistances
made in this manner of No. 16 S.W.G. manganin wire having
resistances of 1 ohm, 10 ohms, 100 ohms, 1,000 ohms and, if
possible, 2,000 ohms.   This size of wire will carry safely
1 ampere for a short time with but little change in resistance.
    Instead of winding silk-covered wire on a tube, it is better
to wind bare wire on a skeleton bobbin made by joining
together two round wooden discs by a number of wooden rods,
as shown in Fig. 3.   The turns of wire should be kept apart by

:d silk thread or cord wound between them. The
.d be wound double, two wires being wound on in
1d these joined up at one end. The whole wire is,
nearly non-inductive, and the bobbin can be
in a vessel of paraffin oil, by which means its
re can more easily be kept constant and ascertained.

⁀ Potentiometer Measurement of Electromotive

⁀arious forms of potentiometer have already been
in Chapter I. Whatever form the instrument takes
essentially of a resistance, which may be a series of
Long wire stretched over a divided scale. Down
ance a uniform fall in potential is made by means
ry working through a rheostat in series with the
.is battery is called the *Working Battery.* The
,attery should consist of one or more lithanode
cells of ample size. The cells should be highly
by being placed upon pieces of ebonite. The cells
vell charged, and then about 25 per cent. of the charge
by discharging the cell slowly. No good results
ained when using a fully and freshly charged cell,
ere are then irregular changes in its electromotive
1e cells should be joined up in series with the
eter wire through a carbon plate rheostat.
e rheostats in which wire is wound off one cylinder
lier, or in which a contact spring presses against a
1d in a helix on a cylinder, give trouble in dusty
ause the dust gets under the contact and creates a
:sistance. Whatever form of rheostat is used, the
e to cause a perfectly steady current to flow through
iometer wire or resistance, and be so adjusted as to
the wire is not sensibly heated. If a long stretched
ployed as the potentiometer wire, the greatest care
taken to secure uniformity in its resistance per
: of length, and this uniformity must be proved and
for granted.

The potentiometer is *set*, or the current in the wire brought to a standard value, by means of a standard cell—preferably a Weston cadmium cell made in the Rayleigh H-form, as then temperature correction is negligible. If a Clark cell is employed, it must have a thermometer placed in it, or in the water bath in which it stands, in order that its temperature may be known and the proper correction applied to its nominal value at 15°C. The standard cell must have its positive pole connected to that end of the potentiometer wire to which the positive pole of the working battery is attached. The other pole of the cell is connected through a high resistance galvanometer, or through a sensitive galvanometer with a high resistance in series with it, to a slider or contact maker on the potentiometer wire. The electromotive force of the standard cell being known, it is clear that, if a point is found on the potentiometer wire at which, when connected, the galvanometer shows no current, the fall of potential down the length of potentiometer wire thus intercepted must be equal to the electromotive force of the cell. The current in the potentiometer wire can be so adjusted that the fall of potential down it can be read off directly on a divided scale placed against the wire. Thus the electromotive force of the standard cell is, say, 1·434 volts; the potentiometer current can be adjusted by the rheostats to make the fall of potential down the 1,434 division of the wire equal to the above value.

When this is the case the potentiometer becomes direct reading, and any other electromotive force or potential difference falling within the range of the potentiometer wire can be measured at once by merely attaching the terminals of a circuit formed of the galvanometer in series with this new source of electromotive force to two points on the potentiometer wire so selected that the galvanometer shows no current. The positive terminal or highest potential point of this source of electromotive force must be joined to that end of the wire nearest to the positive terminal of the working

battery. The source of electromotive force may be another cell, or it may be a difference of potential between two points on another wire traversed by a continuous current.

§ 4. Measurement of Small Potential Differences.—

To measure an electromotive force or potential difference of about 1 volt or less we attach wires to the terminals of this source of electromotive force (E.M.F.) or potential difference (P.D.), and connect them as above described to the potentiometer, and when it is set we can read off at once on its scale the value of the E.M.F. or P.D. The potentiometer is thus easily employed to give us the terminal voltage of a primary or secondary cell, or the value of a current flowing through a known low resistance, or of a very small current flowing through a known higher resistance, provided always that the P.D. to be measured is less than the whole fall down the potentiometer wire. In making a number of measurements of this kind the setting of the potentiometer must be continually tested by coming back on to the Clark or other standard cell.

In all practical forms of potentiometer, such as Crompton's or Elliott's, there is a double pole switch on the instrument so arranged that, without disturbing the galvanometer connections, any cell or terminal wires from any circuit can be quickly substituted for the standard cell in the galvanometer circuit. Hence there is no loss of time in continually re-checking the setting of the potentiometer current, and, if it has varied, bringing it back to its original value by a touch of the rheostat.

The potentiometer is especially useful in measuring small E.M.F.s, such as that of a thermo-electric couple. The Author designed for certain low temperature researches a combined resistance bridge and potentiometer by means of which a thermo-electromotive force could be measured and immediately afterwards the resistance of a platinum wire giving the temperature of the junction. The

arrangements of this combined bridge-potentiometer are as
follows :—

A length of 2 metres of bare manganin wire (No. 36
S.W.G.) is stretched over a scale (*see* Fig. 4). This wire has
a resistance of 26·576 ohms and a diameter 0·0193 centimetre.
Over the slide wire moves a slider contact P. The other two
arms of the bridge consist of a resistance coil, C, of manganin
wire of 5,000 ohms, and a platinum wire used as a platinum
thermometer. The bridge is provided with current from a
2-cell lithanode battery, B, and a Pitkin-Holden galvano-
meter, G, having a resistance of 4 ohms, is used with it. In
the diagram (Fig. 4) X Z W Y is the 2-metre slide wire
having its ends attached to terminal blocks X and Y. B is
the 2-cell battery which is connected through a current
reverser, D, with the ends of the slide wire, having interposed
between one terminal and the block Y either a resistance, R_2,
of 200 ohms when used as a bridge, or a resistance R_1, R_4 and
R_3 when used as a potentiometer. R_1 is a variable resistance
of about 5,000 ohms, R_4 a resistance of 18,200 ohms, and R_3 a
resistance of $10 \times 25·576$ ohms, divided into four sections
having ratios of 1, 2, 3 and 4 in magnitude. The galvano-
meter is connected to terminals U and V when the
instrument is used as a bridge and P is the slide contact
maker on the wire; the other two arms being the 5-ohm coil
C and the platinum wire P_1 used as a thermometer.

When the instrument is used as a potentiometer the
thermo-couple or other source of E.M.F. is joined in
series with the galvanometer between the slider P and the
end of the resistance coil R_4. In this case the fall of potential
down the slide wire is measured by the Clark cell Ck. In
order to make the changes of connection rapidly a seven-cup
mercury switch is employed, so that by shifting a pair of
copper forks all the necessary changes of connection are made
instantly.

Referring to the diagram, it will be seen that if we connect
the mercury cups 4 to 5 and 1 to 7 the arrangement is a
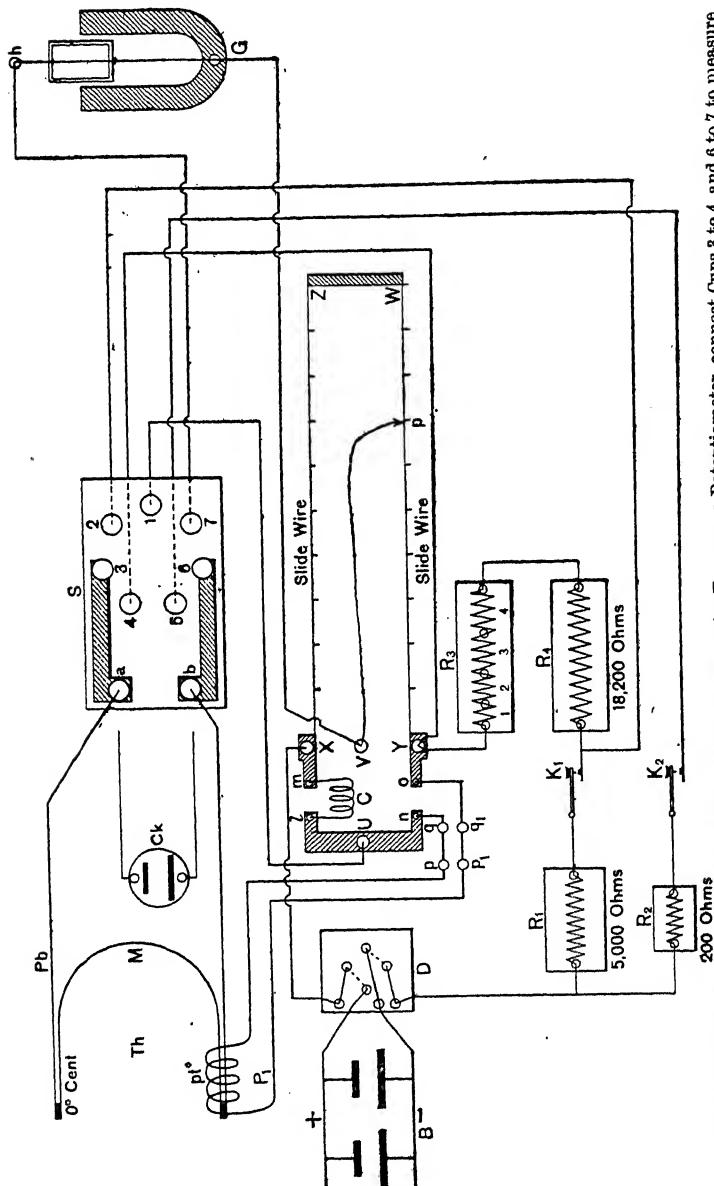
To use as a Wheatstone's Bridge, connect Cups 4 to 5 and 1 to 7. To use as a Potentiometer, connect Cups 3 to 4, and 6 to 7 to measure E.M.F. and Cups 2 to 3 and 6 to 7 to check by Clark's Cell.

Fig. 4.—Arrangement of Combined Resistance Balance and Potentiometer for Thermo-electric Measurements. (Fleming.)

Wheatstone bridge.   If we connect cups 3 to 4 and 6 and 7 it is changed to a potentiometer to measure E.M.F., and if we connect cups 2 to 3 and 6 to 7 we can *set* the potentiometer by the Clark cell.

This form of combined bridge and potentiometer, devised by the Author,[*] is of great use in measuring thermo-electromotive forces from single couples, and at the same time the temperature of the thermo junction may be obtained from the value of the resistance of a platinum wire, P_1, wound round it. The instrument may be adapted for a variety of uses in measuring E.M.F.s which are functions of temperature.

Potentiometers in which the whole or part of the resistance is an exposed slide wire have the disadvantage that dust settles on the wire, and then the sliding contact, by which the connection to the galvanometer shunt circuit is made, is uncertain in resistance.   It must be borne in mind that, as the position of balance is approached, the effective E.M.F. in the galvanometer circuit becomes vanishingly small, and hence is unable to break through a dust or oxide film and express itself as a deflection of the galvanometer coil.   Accordingly, for workshop purposes a potentiometer is preferable which has no slide wire, but in which the resistance inserted in the circuit of the working battery is wholly of insulated wire coils included in a box.

A useful form of potentiometer, in which an external or exposed slide wire only forms a 150th part of the whole potentiometer resistance, is made by Messrs. Elliott Bros., and consists essentially of 149 coils of wire joined in series with each other, and with a slide wire equal in resistance to one of them.   A perspective view of the instrument is shown in Fig. 5, and a diagram of connections in Fig. 6.

Each of the 149 sections or coils is about 3in. in length, and they are all adjusted so as to be absolutely equal in

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[*] See *Phil. Mag.*, July, 1895, p. 95.   "On the Thermo-electric Powers of Metals and Alloys between the Temperature of Boiling Water and the Boiling Point of Liquid Air."   By J. A. Fleming and J. Dewar.

resistance one to the other. The slide portion of the wire KM (Fig. 6) is extremely short, as can be seen in Fig. 5, and is equal in resistance to one coil. Inequalities in the drawing of this wire are compensated for by dividing the scale, over which arm L travels, so that the individual
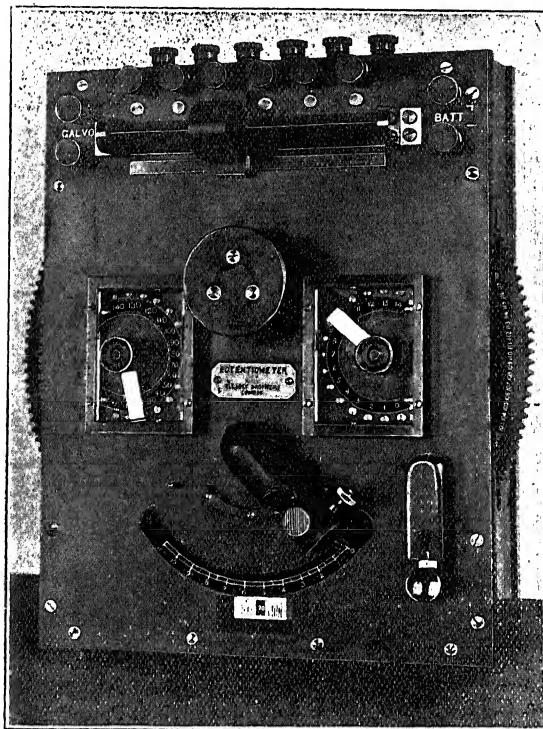
Fig. 5.—Elliott Potentiometer.

divisions of the divided wire RRR are exactly multiples of the divisions on the scale. As an example, the fall of potential over 140·7 sections of the divided wire is exactly five times that over 28 divisions of this wire, and 14 divisions on the scale.

A reference to Fig. 6 shows clearly how the arrangement is effected in practice.   AB are two terminals, to which the working battery—*i.e.*, the one accumulator cell—should be attached.   At C is a small fuse, which serves to protect the slide wire from injury should too high an E.M.F. be applied to A and B by accident.   DE are the galvanometer terminals, and across these is connected a short circuit key X, which in
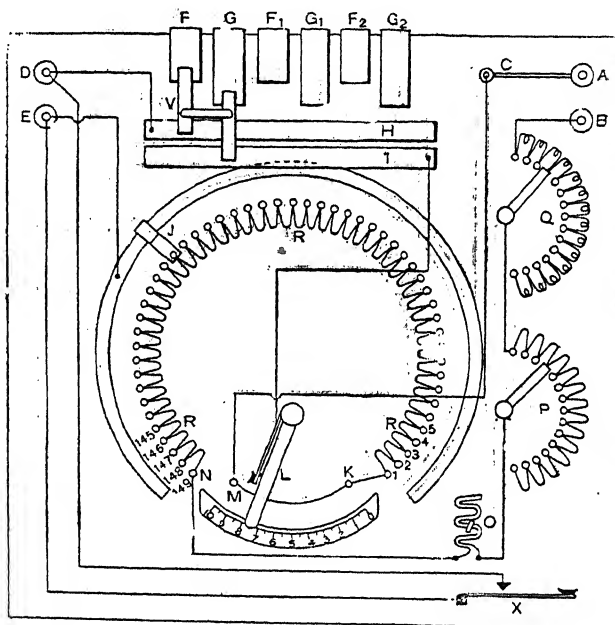
FIG. 6.—Diagram of Connections in the Elliott Potentiometer.

its normal, or free, position keeps the galvanometer short-circuited, and thus protected against violent deflections.   A d'Arsonval galvanometer should always be employed in all these tests.   F, G, F_1, G_1, F_2, G_2, &c., are the terminals to which wires leading to the various sources of potential differences to be compared should be attached.   A multiple double-pole switch V permits of the two common bars H and

I being connected to any pair of these terminals at will. RRR is the divided wire, which is laid round in a circular position, and divided in 149 parts of equal resistances, small contacts being placed at each of these 149 points. The whole of this part of the divided wire, which, of course, is the essential part of the whole apparatus, is perfectly protected from mechanical injury by being inside the case.

At one extremity of this wire N—at the 149th contact—it is connected to O, a small fine adjustment rheostat, and so on to P and Q, two other adjustable rheostats in series, and so proportioned that the total resistance value of O is rather greater than that of one section of P, and that the total value of all the sections of P are slightly greater than that of one section of Q. P, Q, and O are not adjusted in any definite values—they serve simply as adjustments. In practice the whole of Q, P, and O are approximately 200 ohms altogether. The divided wire itself, RRR, being about 30 ohms resistance, one end of rheostat Q is joined to terminal B.

The other extremity of the wire RRR is taken up through the top of the instrument at K, where it is led round a curved segment, and where the moving contact arm L can travel over it. A scale is fixed to the top of the instrument, and a pointer is attached to L, so that when the moving contact is on the stud K, the pointer attached to the arm L stands at zero on the scale. When the arm L is moved till the pointer stands at the figure 10, then the moving contact has passed over a length of the divided wire exactly equal in resistance to any of the other 149 sections between K and N. A contact J can travel round the circle of the divided wire and make contact with any of the 149 small contacts fixed to it. This contact J is attached to a large toothed wheel, the edges of which can be seen in Fig. 5 on the right and left of the instrument. This affords a ready means of shifting the position of contact J, and its position with reference to N and K can be seen through a small window in the front of the instrument, through which a number shows corresponding

to the number of the contact on which J lies ; a device is provided to cause J to make contact definitely on either one or other of any pair of adjacent contact studs on the divided wire. A wire is led from J to one on the galvanometer terminals E.

The travelling contact on arm L is connected through a small key W to the bar of the multiple switch, the other bar H being connected to the second galvanometer terminal D. The key is provided with a small clamping device, so that it can be kept down if desired, and the galvanometer deflections manipulated with key X. Care must be taken that no source of E.M.F. is attached by mistake to terminals F, G, &c., as, in the event of the key W being clamped down and switch V being in these terminals, a comparatively powerful current might flow through the galvanometer and the slide wire, probably damaging both. The key W, therefore, should only be clamped down when making a series of tests where there is no chance of a wrong connection having been made outside the instrument.

It will be seen that there exists always between A and B a closed circuit through which the current furnished by the working battery passes—from A through C to M, K, RRR, N, O, P, Q, and so to B. This circuit is of variable resistance, owing to the adjustment of rheostats O, P, and Q ; but in all cases the whole of the wire RRR is in circuit.

To make an E.M.F. measurement we proceed as follows :— Fig. 7 illustrates the connections. The working battery is connected to AB as usual, the standard cell and resistance to FG. The small separate resistance box is employed, the two terminals marked " Potentiometer " being connected to FG, wires attached to the source of E.M.F. to be measured being connected to terminals on this box, according to the various ranges used. The terminals marked " Potentiometer " have between them a small fractional part of the whole resistance in the box. Setting the figure 143 at the window, and arm L at 4 on the scale as before, when a

balance is obtained with a multiple switch on the standard cell at FG, then each movement of the toothed wheel means 0·01 volt, and 0·0001 corresponds to the small whole divisions on the scale.

Suppose an E.M.F. of about 100 volts is to be measured, the positive wire should be joined to the terminal marked + on the separate resistance box, and the other to the terminal marked 150. Then, having balanced as above, the instrument is direct reading in volts—that is to say, if a balance is obtained with 101 at the window and arm L at
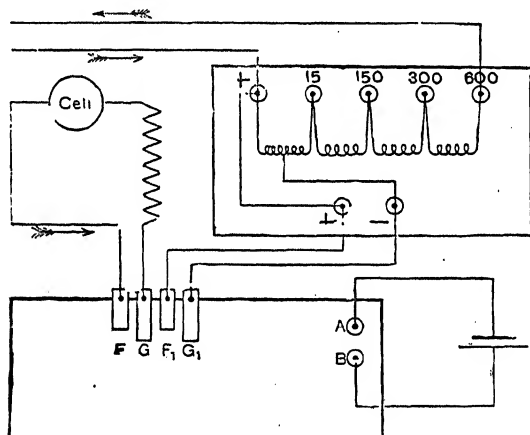
FIG. 7.—The Elliott Potentiometer and Divided Resistance arranged for Measuring Voltages.

5·3 on the scale, then the E.M.F. under test equals 101·53 volts. For readings using the + to 15 terminals on the separate resistance box, the readings must be divided by 10 ; if using terminals + to 300, readings must be multiplied by 2 ; if using + to 600, multiply by 4.

Supposing it is desired to determine exactly an E.M.F. of about 2·2 volts. Obviously, if the standard cell is balanced with 143 at the window and 4 on the scale, the range of the potentiometer is insufficient to compare this directly. If the

terminals + to 15 on the separate resistance box are used, then the E.M.F. of 2·2 volts would balance with figure showing at the window.   For great accuracy, the following method may be adopted:—Connect two accumulator cells in series to AB; then with switch V (Fig. 6) on FG, corresponding to the terminals of the standard cell, adjust P, Q and O so that a balance is obtained with 71 at the window and arm L at 7 on the scale (1·434/2 = 0·717).   Assuming that the E.M.F. of the standard is taken at 1·434 volts at the working temperature, then let wires be brought from the source of E.M.F. to be tested to terminals FG; then, if a balance is obtained on moving the multiple switch to FG with 111 at the window and arm L at 4 on the scale, the value of the E.M.F. under test = 1, 114 × 2 = 2·228 volts—that is to say, with the standard balanced at the position corresponding to half its value, all readings at the window and scale must be double to obtain correct values in volts.

This process can be carried further, but not more than 6 volts should ever be applied to AB.

In workshop potentiometers it is desirable to avoid the employment of any slide wire, however short, and to construct the whole potentiometer of coils of wire contained in a box. In the case of a potentiometer designed by the Author, the instrument takes the form of a box having on the ebonite or marble top surface a set of block contacts arranged in circles, and each set associated with a radial arm contact like one of the simple forms of Wheatstone bridge.   There are two sets of concentric block contacts, arranged as in Fig. 8, and a pair of concentric brass rings to which the radial arms connect the plugs.   Under the top of the box are a series of coils interconnecting the blocks, as in a dial pattern Wheatstone bridge, the only difference being that each dial is double.   There are three double dials, the coils in between the blocks of the first dial being each of 10 ohms, those between the blocks of the second being 1·0 ohm, and those of the third being each 0·1 of an ohm.

The several dials are so joined up that they form one resistance divided into two sections, but of such a nature that increasing the resistance of one section decreases by an equal amount the resistance of the other section. There are 10 blocks to each dial and, accordingly, the maximum resistance which can be created in either section is 99·9 ohms; and, no matter how the dial radial arms are set, the joint resistance of the two sections always has the same value, viz., 99·9 ohms.
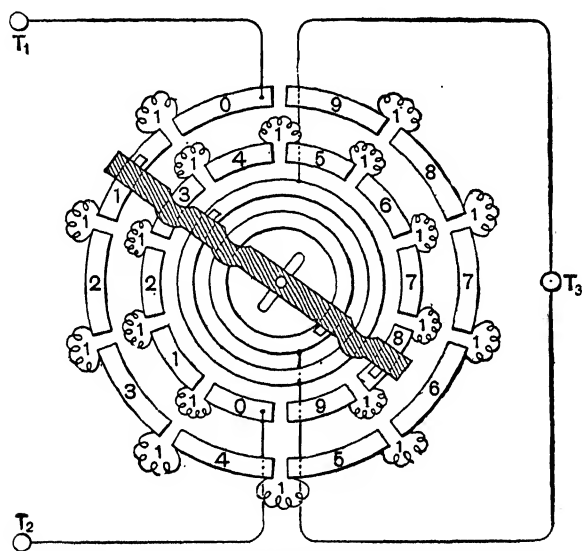
FIG. 8.—A Single Dial of the Fleming Potentiometer.

If, then, a single secondary cell is connected to the ends of this divided resistance, and a high resistance galvanometer and standard cell are connected to the terminals of one section of the resistance, it is possible to move the radial arms in such a manner that the relative magnitude of the resistances of the two sections is varied, but the total value of the two sections remains the same.

Let the standard cell be a Helmholtz calomel cell, which is constructed to have an E.M.F. of 1 volt. Then it

is possible to adjust the value of one section of the resistance by moving the radial arm so that the high resistance galvanometer indicates no current when the secondary cell is connected to the terminals of the whole resistance. For this is the case when the fall of potential down the section of the resistance to which the galvanometer is a shunt is 1 volt, and this is so when the sections of the resistance have a certain ratio near equality.

Suppose, then, that another cell is substituted for the standard cell, and again the radial arms are so moved that the galvanometer shows no current. This movement of the radial arms will not alter the value of the current flowing through the whole resistance, but will alter the ratio of the resistances of the two sections. This ratio may be made such as to bring the galvanometer to zero once more.

Let R_1 be the resistance of that section of the whole resistance to which the galvanometer is a shunt when the standard 1-volt cell is in series with the galvanometer, and let R_2 be the resistance of the same section when the second cell is substituted for it. Let us suppose the total constant resistance of the two sections of the box is 99·9 ohms. Let V be the E.M.F. of the cell being tested. Then it is clear that, since the standard cell has an E.M.F. of 1 volt, the volt-fall down the whole resistance in the box is equal to $99·9/R_1$. Hence the E.M.F. V of the cell tested against it must be equal to R_2/R_1 volts.

Accordingly, if the secondary cell attached to the ends of the whole resistance is large and very constant in E.M.F., there is no need to have rheostats and other complications to keep the potentiometer current constant The only pieces of apparatus necessary for measuring any E.M.F. are the standard 1-volt cell, a high resistance galvanometer and keys, the divided resistance above described, and a large secondary cell which has been charged and then partly discharged.

§ 5. The Calibration of a Low-Tension Voltmeter.—

Suppose it is desired to calibrate or check the scale accuracy of a voltmeter reading, say, from zero up to 150 volts, by reference to the potential difference of a standard cell, we proceed as follows:—A resistance, $r_1 r_2 r_3$ (*see* Fig. 9) must be provided, which is generally called a *volt-box*, consisting of a number of bobbins of manganin wire contained in a box. These coils of wire are arranged in series so as to form a resistance of 10,000 or 20,000 ohms, which is divided

V	Voltmeter.		*a, b*	Potentiometer Wire.
B	Testing Battery.		Ck	Clark Cell.
G	High-resistance Galvanometer.		*b*	Potentiometer Working Cell.
r_1, r_2, r_3	Divided Resistance.			
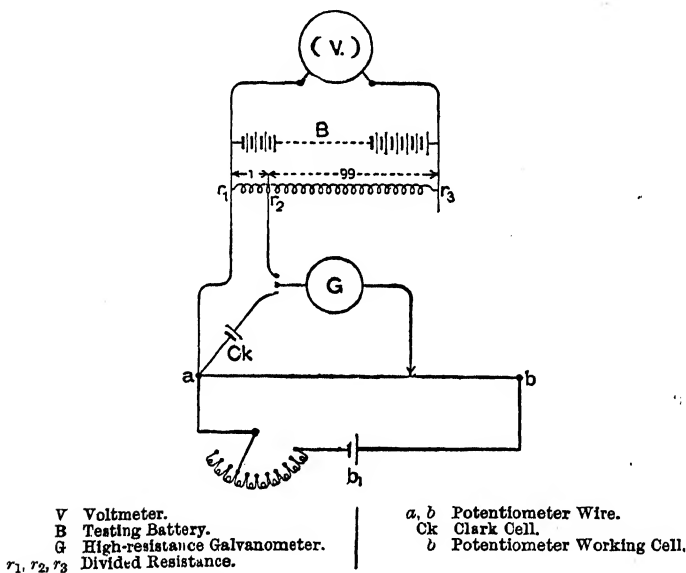
FIG. 9.—Potentiometer Arrangement for Checking Voltmeter.

in certain ratios by terminal attachments. Thus one set of terminals may be so arranged that the whole resistance is divided into two sections in the ratio of 99 : 1, another set in the ratio of 9 : 1, another 999 : 1, and so on. This resistance is joined across the terminals of the voltmeter V to be checked, and a proper potential difference applied by means of secondary cells B to give any required reading of

the voltmeter. For this purpose a set of 100 small lithanode cells are very convenient, giving any required E.M.F. from 2 to 200 volts.

Suppose, for instance, we require to check a voltmeter, reading up to 100 volts, in the neighbourhood of 100 on the scale. We should select those terminals of the volt-box which give us a ratio of 99 : 1 in the whole resistance. Wires would then be taken from the smaller section $r_1 r_2$ of the resistance to a potentiometer. We then know that, whatever may be the value of the fall of potential down the small section $r_1 r_2$ of the resistance as measured by the potentiometer, the total fall of potential down the whole wire $r_1 r_3$, and therefore the actual true voltage applied to the voltmeter terminals, is 100 times as great. In Fig. 9 is shown a diagram of the connections to be made.

By varying the number of secondary cells in this battery B attached to the terminals of the voltmeter V, we can make various known or measured potential differences on these terminals, and compare these values with the scale reading of the voltmeter. A table can be drawn up of the true potential differences which correspond to various scale readings. We then proceed, as in the case of an ammeter, to make a curve of errors.

Take a straight horizontal line on a sheet of paper and divide it into equal parts, representing the scale readings of the voltmeter (*see* Fig. 10). At each point set up a perpendicular the length of which is made proportional on same scale to the *difference* between the scale reading of the voltmeter and the true value found for the potential difference making that reading. Let this perpendicular be drawn upwards when the *error* of the voltmeter is positive and downwards when the error of the voltmeter is negative. Hence, if, corresponding to a scale reading of 100, the true potential difference is 101·2, then the error is positive— that is, we have to add 1·2 volts to the scale reading to arrive at the true voltage corresponding to the scale reading

100. In this manner the voltmeter can be calibrated throughout the scale and a curve of errors found for it.

If the voltmeter is one constructed on an electromagnetic principle, then it is necessary to take a series of ascending and descending scale readings to ascertain if there is sensible hysteresis error. Also tests should be made with the voltmeter in various positions, or with magnets near it, to discover how far it is affected by position or the presence of other magnetic fields.

In the case of an electrostatic voltmeter it is necessary to be on our guard against errors introduced by contact difference
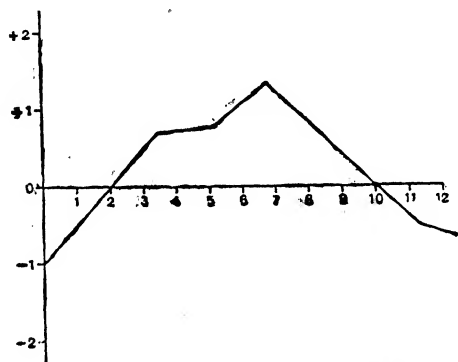
Fig. 10.—Error Curve of a Voltmeter.

of potential. Take the case of a Kelvin electrostatic voltmeter. In this instrument the "needle," or movable portion of the instrument, is of aluminium, and the fixed quadrants are made of brass; hence, when a secondary cell battery is joined in between the terminals of the instrument, the scale reading will be determined by the voltage of this battery *plus* or *minus* the Volta contact potential difference of an aluminium-brass couple. This latter amounts to about 0·3 of a volt. Hence we find that an electrostatic voltmeter gives scale readings slightly different when the "needle" is positive from that which it does when the "needle" is made negative

by reversing the connections of the working battery.   Hence, in checking the voltmeter, note must be made of the mode in which the working battery is connected.

§ 6. Calibration of a High-Tension Voltmeter.—Supposing we have calibrated in the manner described in the previous section an electrostatic voltmeter reading up to 100 volts. We may then employ this voltmeter as an intermediate standard instrument to check a high-tension voltmeter reading, say, up to 2,000 volts.   For this purpose we require a divided resistance which can be safely placed across a 2,000-volt alternating-current circuit.   This can be made as follows :—

Provide 100 coils of double silk-covered platinoid wire No. 36, each wire 100ft. in length, coiled up into a coil of
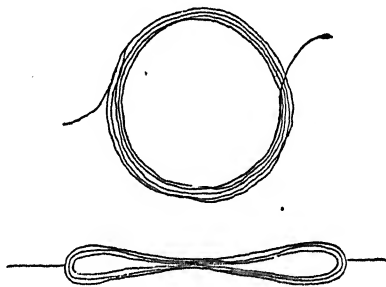
Fig. 11.

6in. in diameter, leaving both ends out.   Squeeze up each coil into a sort of hank (*see* Fig. 11) and bind it in the middle with tape.   Boil each coil or hank well in melted paraffin wax.   The resistance of each hank of wire will be about 400 ohms.   Make a wooden frame so that 25 of these hanks can be spaced 1in. apart and be held by silk string, as shown in Fig. 12.   Join up the wires of the hanks in series.   Connect four such frames of wood with cross bars and join up all coils in series.   Bring the ends of the whole series and of each hank to well-insulated terminals on the outside of the wooden

frame. The whole wire will then form a resistance of 40,000 ohms in 100 sections, and each section will be perfectly insulated from its neighbour. The wire will bear placing across a 2,000-volt circuit for any length of time without over-heating or risk of failure of the insulation.

This divided resistance can then be placed across the terminals of the high-tension electrostatic voltmeter, and a calibrated low-tension electrostatic voltmeter joined across the terminals of one section, say $\frac{1}{20}$th of the whole wire. Hence, when the high-tension voltmeter is connected to a
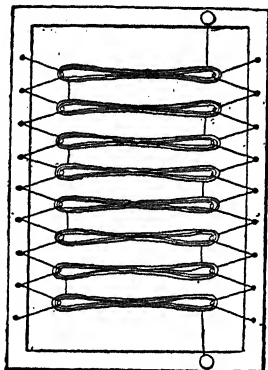
Fig. 12.—Inductionless Safety Resistance.

2,000-volt alternator or transformer, the reading of the low tension voltmeter will give the true volt fall down $\frac{1}{20}$th of the resistance, and therefore the true voltage at the terminals of the high-tension voltmeter.

It should be noticed, however, that there is a possible source of error in the above process which arises from the capacity of the low-tension voltmeter. This may be explained as follows:—

Let us suppose, in the first place, for the sake of avoiding mere mathematical complexity, that the divided resistance is entirely non-inductive. Let R and R_1 be the resistances of the two sections of this resistance, and let the low-tension electrostatic voltmeter be joined over the section of which the resistance

is R (*see* Fig. 13). Let the capacity of the voltmeter be represented in the figure (Fig. 13) by a condenser C. Then let the instantaneous values of the currents through R, R_1 and C be i, i_1 and i_2, and let the instantaneous values of the potential fall down R, R_1 and $R+R_1$ be v, v_1, and v'.

We have then the following equations:—

$$\left.\begin{array}{l} v+v_1=v' \\ i_1-i=i_2 \end{array}\right\} \text{by the principle of continuity,}$$

$$\left.\begin{array}{l} Ri=v \\ R_1 i_1=v_1 \end{array}\right\} \text{by Ohm's law,}$$

and

$$i_2=C\frac{dv}{dt} \text{ from definition.}$$

Also, let $v'=V' \sin pt$, the frequency being as usual $p/2\pi$.
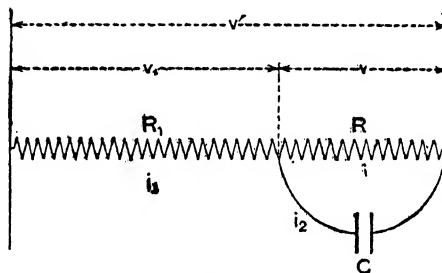
FIG. 13

Obviously, then, we have the following equation:—

$$C\frac{dv}{dt}=\frac{v'}{R_1}-\left(\frac{1}{R_1}+\frac{1}{R}\right)v. \quad \cdots \cdots \cdots \text{(i)}$$

Differentiating with regard to t, we obtain, on re-arranging terms,

$$C\frac{d^2v}{dt^2}+\left(\frac{R_1+R}{RR_1}\right)\frac{dv}{dt}=\frac{dv'}{dt}\ \frac{1}{R_1}; \quad \cdots \cdots \text{(ii)}$$

and eliminating $\dfrac{dv}{dt}$ between (i) and (ii), and remembering that $C\dfrac{d^2v}{dt^2}=-Cp^2v$, if the variation of v is harmonic, we obtain finally the equation

$$\left\{C^2p^2+\left(\frac{R+R_1}{RR_1}\right)^2\right\}v=\frac{R+R_1}{R_1^2R}V' \sin\ pt-\frac{Cp}{R_1}\ V' \cos pt.$$

In virtue of a well-known transformation (*see* "The Alternate Current Transformer," Fleming, Vol. I., p. 161) we can write the above equation in the form

$$\left\{C^2p^2+\left(\frac{R+R_1}{RR_1}\right)^2\right\}\ v=\ \sqrt{\frac{C^2p^2}{R_1^2}+\left(\frac{R+R_1}{R_1^2R}\right)^2}\ V' \sin\ (pt+\theta).$$

Hence, if V is the maximum value of v, it follows that

$$\frac{V}{V'}=\frac{1}{R_1}\ \frac{1}{\sqrt{C^2p^2+\left(\dfrac{R+R_1}{RR_1}\right)}}:$$

or, if we write K_1 for $1/R_1$ and K for $1/R$, we have

$$\frac{V}{V'} = \frac{K.}{\sqrt{C^2 p^2 + (K_1 + K)^2}}.$$

If, then, $C = 0$, or if the low-tension voltmeter has no sensible capacity, we

have

$$\frac{V}{V'} = \frac{R}{R_1 + R};$$

in other words, the maximum or root mean-square value of the volt fall down the section of the resistance R is to the fall down the whole resistance in the ratio of those resistances. If, however, C is not zero, or if p is large and if R and R_1 are large, then the root-mean-square values of the volt falls are no longer in the exact ratio of the resistances.

There is no need to complicate the proof by considering the case when these sections of the divided resistance have slight inductance. The same principle holds good.

Hence, although the use of the divided resistance is a convenient method of stepping from one voltmeter to another, it is necessary to take the precaution to check the high-tension voltmeter by the only unexceptionable method—viz., by connecting it to the terminals of a high known resistance through which a known measured current is passed.

§ 7. **Self recording Voltmeters.**—In many cases it is necessary to secure a continuous record of the difference in potential between two points, and this can be accomplished by the use of a self-recording voltmeter. These instruments consist of two parts—a voltmeter part and a paper-carrying drum which is made to revolve by clockwork uniformly. The voltmeter part carries a pen, which is displaced by the voltage applied to the instrument. Hence, if the pen is at rest and the drum revolves, the pen will draw a line on the paper which will be a straight line when the paper is removed from the drum and spread out. If the pen is displaced by a voltage applied to the instrument, then the line is a sinuous, irregular line. If the displacements of the pen are exactly proportional to the voltage, then the ordinaters of the curve will measure this voltage. If not, then paper ruled up in lines placed at intervals equal to 1 volt or 10 volts is

employed, so that an inspection of the line drawn by the pen suffices to determine the voltage. The voltmeter part may be a hot-wire voltmeter, as in the Pitkin-Holden instrument (*see* Fig. 14). In this case the expansion of the wire caused by the current flowing through it is made to move the pen arm by an amount increased by the use of a multiplying gear.

In other cases the voltmeter mechanism is electro-magnetic; as in the Kelvin recording voltmeter (*see* Fig. 15) or Elliott instrument (*see* Figs. 16 and 17). In the Kelvin instrument the control is by gravity, and the electro-magnetic action of a solenoid on an iron core lifts or lowers the pen.

In testing a self-recording voltmeter the clock mechanism must first be examined, to see whether it keeps correct time. Then the voltmeter part must be separately tested by the potentiometer, to ascertain if this portion of the mechanism works properly.

Self-recording voltmeters are employed to detect the irregularities of pressure in connection with electric supply stations, and to decide whether complaints as to abbreviated lives of glow lamps are due to the qualities of the lamps or to excessive variation of service pressure. A self-recording voltmeter is useful in connection with tests of secondary batteries to determine the exact time at which the voltage per cell has fallen to 1·8 volts, below which voltage no current readings are of any value. They are also necessary in con-nection with life-tests of incandescent lamps.

§ 8. **Extra High-Pressure Voltmeters.**—In connection with cable and transformer testing it becomes necessary to employ voltmeters for measuring extra high pressures and voltages such as 20,000 to 60,000 volts. Instruments for this purpose have been designed by Lord Kelvin depending on the attraction exerted between a fixed disc and one suspended over and parallel to it—from the arm of a steelyard. The

FIG. 15.—Kelvin Recording Voltmeter.
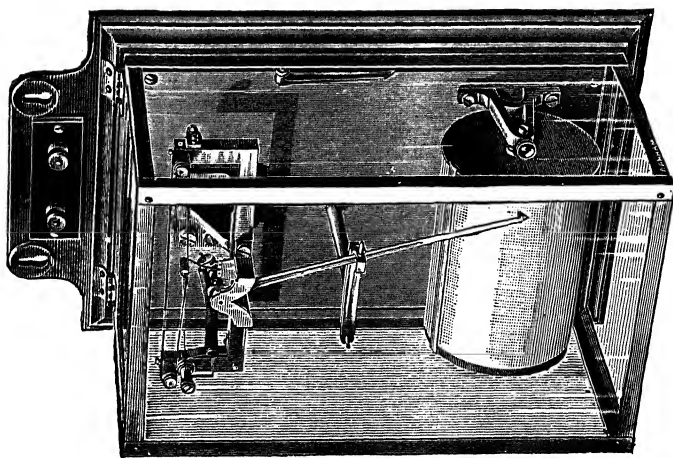
FIG. 14.—Pitkin-Holden Hot-Wire Recording Voltmeter.
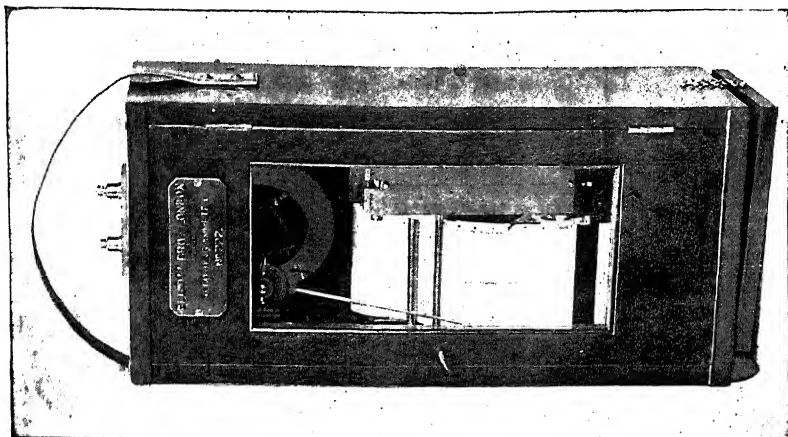
Fig. 17.—Elliott Recording Voltmeter (case closed):
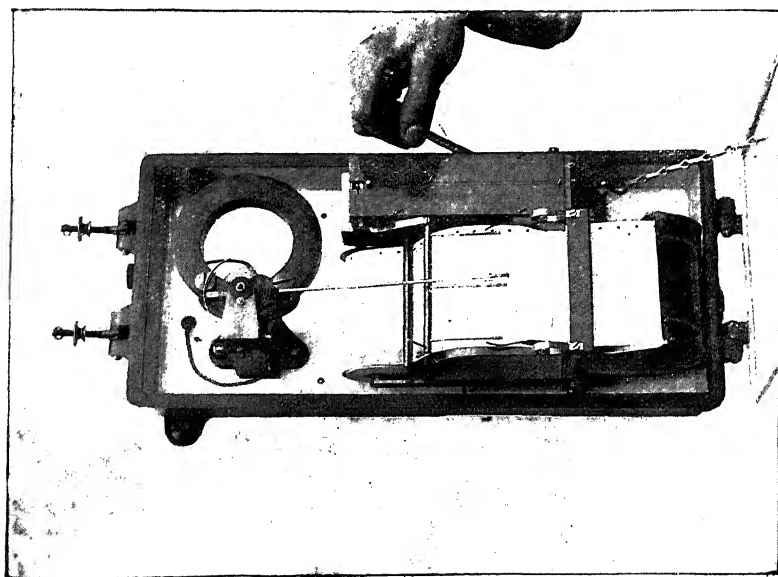
.16.—Elliott Recording Voltmeter (case open).

discs are included in a case which is well earthed (*see* Fig. 18).

The movement of the suspended disc is limited by stops. The control is a gravity control. The instrument is provided with certain weights which, when hung on the beam, cause the deflection of the scale beam under the electrostatic forces to have a definite and indicated value in volts.
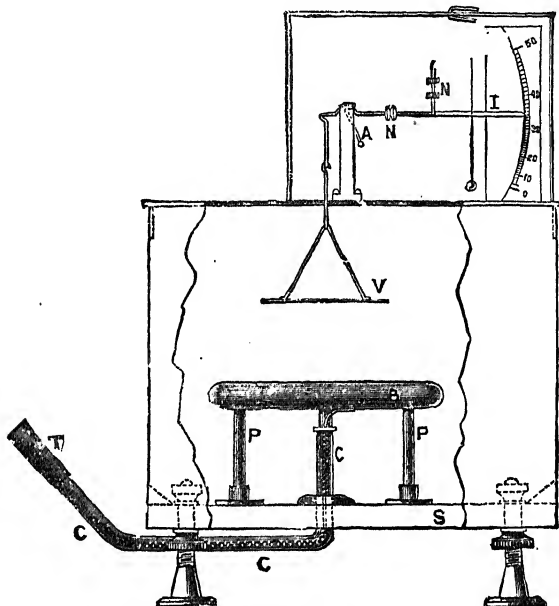
FIG. 18.—Kelvin High-Tension Voltmeter.

Since the attractive force between two such parallel discs varies as the square of the difference of potential, the instrument is equally well adapted for use with continuous or alternating-current voltages. A modification of this instrument has been made by Messrs. Pirelli, of Milan (*see* Fig. 19), for measuring voltages of 25,000 or so. A fixed plate, A, exerts an attraction on a movable one, B. The two plates are enclosed in a cylindrical copper case, M, and the whole is placed in a glass cup filled with vaseline oil. C is a glass
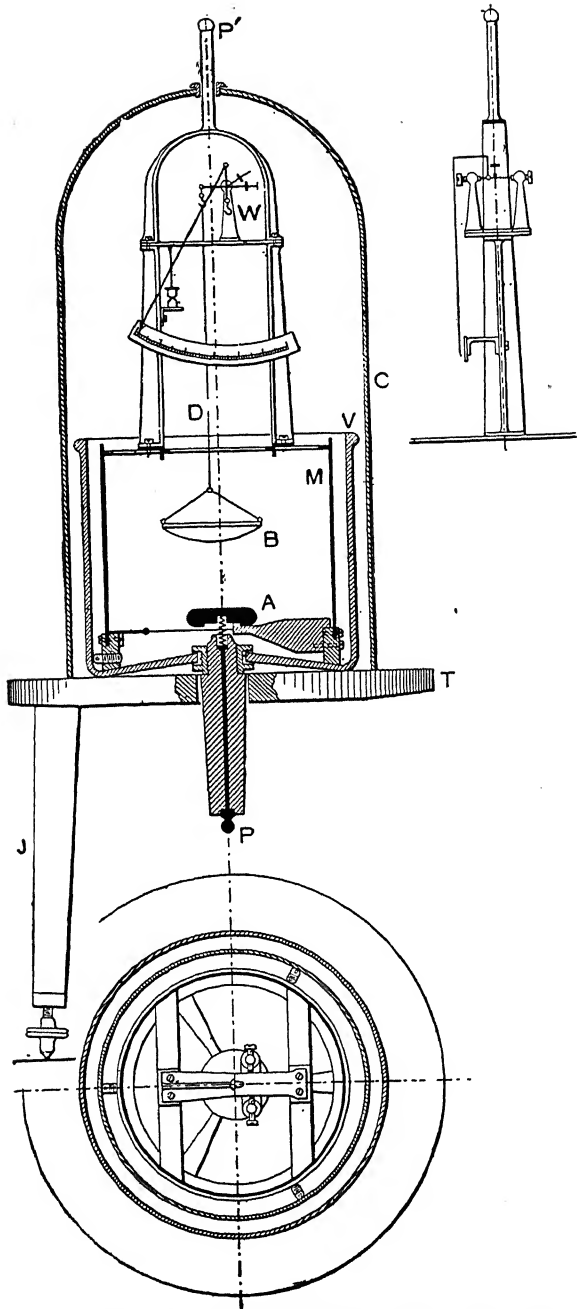
FIG. 19.—Pirelli & Co.'s Electrostatic Voltmeter.   Scale : 2in.=1ft.

shade from the top of which the pole P′ projects, this being connected with the movable plate by a thin wire, D. The apparatus rests on an ebonite stool, T, having ebonite legs. Two weights are provided: with one weight one scale division corresponds to 500 volts, and with the other a scale division equals 1,000 volts. The instrument can thus read up to 60,000 volts.

Some instrument of the above type is indispensable in testing cables or condensers with high voltage. The instrument is best calibrated by the employment of a chain of overlapping instruments, so that we can step up from an electrostatic voltmeter capable of being calibrated directly by
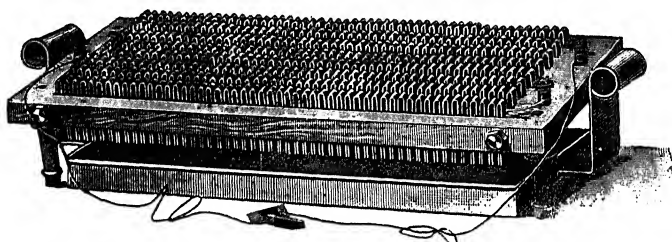
FIG. 20.—Kelvin Water Battery.

a Clark cell to one of the above type. Thus, if we have three electrostatic voltmeters—one reading from 500 to 2,000 volts, one from 1,000 to 5,000, and one from 4,000 to 20,000, we can reduce the readings to their equivalent in terms of the international volt as defined by a Clark or Weston cell.

Without a chain of overlapping voltmeters there is some difficulty in verifying the indications of an extra high-tension voltmeter, because the construction of a resistance capable of having these very high potentials put on its ends is an expensive matter. One way in which it can be done is by the use of a series of condensers charged by a battery of small cells. Lord Kelvin has designed a simple form of water battery consisting of small slips of copper and zinc placed so close together (*see* Fig. 20) that, when

dipped in water, a small drop is held up by capillary action between the alternate plates. This arrangement may be made to furnish E.M.F. up to 1,000 volts by having a sufficient number of plates. The E.M.F. of the battery can be measured by a standardised voltmeter of the electrostatic type. The cells have, however, such high internal resistance that any attempt to take from them a current, even though exceedingly small, results in a great fall in terminal potential difference.

In addition to these cells we must be provided with a number of condensers made with glass, paraffined paper or mica as dielectric. These condensers must first be tested to ascertain the rate at which the terminal potential difference of the condenser falls off with lapse of time, after being charged, due to internal or external leakage. If the condenser is charged with the water battery and connected to an electrostatic voltmeter we can observe the rate at which the voltmeter deflection decays. This is partly due to leakage in the condenser, and partly to leakage in the voltmeter. The voltmeter must be separately tested, and should have such good insulation that, if charged and left to itself, the potential, as indicated by the needle, does not fall to half its value in a quarter of an hour.

If the condensers are all found to be equally free from leakage, then they may be arranged in series and all be highly insulated. Each one is then separately charged in the same direction, say to 1,000 volts, by the battery, and the result is that the potential differences are added together, and between the extreme terminals of the series of condensers we have a potential difference represented by nV, where n is the number of condensers and V is the E.M.F. of the batteries. In this experiment both battery and operator must be very highly insulated by being placed on blocks of paraffin wax or sheets of ebonite.

It is easy to arrange a highly insulated commutator, consisting of a block of paraffin wax having small holes bored in it, to act as mercury cups, and, by means of wire

forks attached to ebonite handles, to so connect up the condensers that they can be charged in parallel and then arranged in series.

Various forms of commutator have been devised to effect such a change quickly. An arrangement was employed by G. Planté in his researches, and more recently by Prof. Trowbridge. A more effective though expensive arrangement is to employ small secondary cells. Series of 50 small lithanode secondary cells may be arranged in trays so as to give 100 volts. These may be supported on shelves on blocks of paraffin, and sets of these 50 cells may be joined up in series. In this manner, high voltages may be created of known value. These may be multiplied by the use of condensers charged in parallel and then joined in series. In this way, by employing series of cells and condensers charged in parallel and then arranged in series, Prof. J. Trowbridge has built up potential differences amounting to two or three million volts, capable of giving electric sparks 7ft. in length. Under these very high E.M.F.s he finds the air at ordinary pressures becomes conductive and behaves like the rarefied air in a vacuum tube under lower electromotive force.

It is not beyond the bounds of possibility to provide in an ordinary electrical testing laboratory a continuous voltage of approximately 10,000 volts. This is best done by setting up small secondary cells in sets of 40 cells. These can be put up in stout glass test tubes, which are carried in holes bored out in blocks of paraffin wax. These cells are joined up in series, and each set can be charged off an ordinary 100-volt circuit through a carbon filament lamp or high resistance. The sets of 40 cells are joined in series, being arranged on ebonite shelves in a sort of cabinet. One hundred and twenty-five of these sets can be arranged in the cabinet, and give, when charged, an E.M.F. of 10,000 volts. The test-tubes in which the cells are set up may be 1in. in diameter and 6in. high, and contain two small plates of lithanode in each cell. Each shelf in the cabinet can be

made to hold one dozen sets of 40 cells and on 10 or 11 shelves the sets requisite to give 10,000 volts can be arranged. The cells can be used to charge mica or glass condensers, and by means of 10 condensers, each having a capacity, say, of 0·1 of a microfarad, charged in parallel and then arranged in series, we have the means of building up a known potential of 100,000 volts, and for calibrating an electrostatic voltmeter for measuring extra high pressures.

§ **9. Laboratory and Switchboard Voltmeters.**—A classification of voltmeters intended for laboratory and dynamo room use, sufficient for present purposes, is as follows:—

A. Classification by Type.

(I.) Continuous-current voltmeters.

(II.) Alternating-current voltmeters.

(III.) Universal voltmeters.

B. Classification by Principle.

(i.) Electrostatic voltmeters.

(ii.) Electromagnetic voltmeters.

(iii.) Electro-thermal voltmeters.

C. Classification by Range and Use.

(*a*) Laboratory or table voltmeters.

(*b*) Switchboard voltmeters.

(*c*) Extra high-tension or testing voltmeters.

It is impossible to describe here all the numerous forms of voltmeter which have been devised. There are certain types which have survived in the struggle for existence because they have proved most convenient.

In laboratory work, for use with continuous currents only, one of the best forms of table voltmeter is the Weston voltmeter. This instrument is of the electromagnetic type, and consists of a well-aged magnet, which provides a permanent field. In this field is supported on jewelled centres an axis which carries a circular coil of wire. The

coil carries an index needle. In series with the coil is a high non-inductive resistance. The passage of a current through the coil causes it to turn so as to place its magnetic axis more or less in line with the field. This motion is resisted by the torque of a steel spring like the hair spring of a watch. The instruments are so made that the scale division reading in volts or fractions of a volt are equal and there is no dead or undivided portion of the scale. The instruments are very dead-beat. One of the most useful forms is the voltmeter reading from 0 to 150 volts from one pair of terminals, and by the use of another pair reading from 0 to 15 volts over the same range of scale. These voltmeters have a very high resistance—from 10,000 to 15,000 ohms. Hence the actual current taken is very small. In the next place, for alternating-current laboratory measurement the multicellular electrostatic voltmeter of Lord Kelvin is very valuable. This instrument has already been described in detail (*see* Chapter I., p. 132).

Other types of voltmeter available like the electrostatic instruments both for alternating and continuous voltage are the electro-thermal voltmeters represented by the Cardew and Hartmann and Braun hot-wire voltmeters. The Cardew voltmeter consists of a platinum-silver wire well aged by being repeatedly heated and cooled. This is carried on a support which in one form consists of a pair of compound metal rods made of one-third of iron and two-thirds of brass. The platinum-silver wire is fixed to one end of the support and the other end of the wire is attached to a motion-multiplying gear. The object of making the rods partly of iron and partly of brass is to give them the same resultant coefficient of expansion with heat as the platinum-silver wire, so that no external changes of temperature cause any difference of expansion in the wire and rods ; but the indicating mechanism is only caused to operate when the wire expands more than the rods. The rods, wire and mechanism are enclosed in a brass case.

In another and better form of instrument the support which carries the wire is not a pair of rods but a tube, partly of brass and partly of iron, which is split longitudinally so as to enable the wire to be easily inserted. This form is called the tube instrument. The split tube is enclosed in another outer casing tube. The resistance of the wire is generally about 300 ohms. Hence, if its ends are attached to a circuit of 100 volts, the wire passes about one-third of an ampere. This causes it to be heated and to expand. The heat radiated by the wire heats the rods or supporting tube and causes it also to expand. After a short time a stationary condition is reached, in which, whilst the wire and rods or tube are both hot, the wire is hotter than the supports, and hence the magnifying mechanism indicates on the dial by the position of the needle a certain relative elongation. Corresponding to each particular voltage on the wire there is a position of the indicating needle on the dial. Hence the instrument can be calibrated as a voltmeter.

In the rod instrument, if the current is switched off the indicating needle goes back to zero and then passes back beyond it. This is due to the fact that the thin wire cools more quickly than the rods, and hence, for a short time, the rods are expanded relatively to the wire. In the tube form of instrument this effect is not so apparent. For the same reason the reading of the instrument must not be taken until the voltage has been kept on it for a few minutes. One objection to the Cardew form of voltmeter is the large power taken up by it. If the wire has a resistance of 300 ohms when hot and with 100 volts on the terminals, then the instrument is taking up about 33 watts, and if kept on the circuit for three hours it uses nearly one-tenth of a Board of Trade unit of electric energy. Hence, relatively to many other electromagnetic instruments, and to electrostatic ones, it is uneconomical for continued use. The Cardew voltmeter should always be fixed with its tube horizontal, as in this position the air convection currents in the tube are less
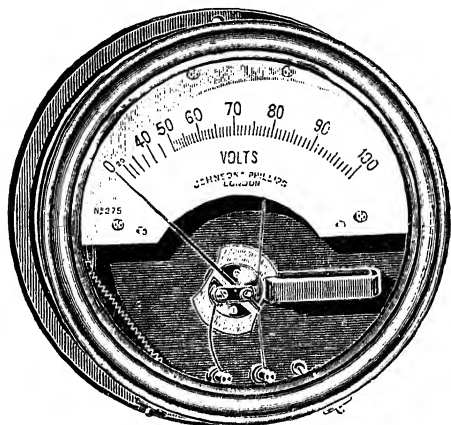
Fig. 21.—Hartmann and Braun Hot-Wire Voltmeter.
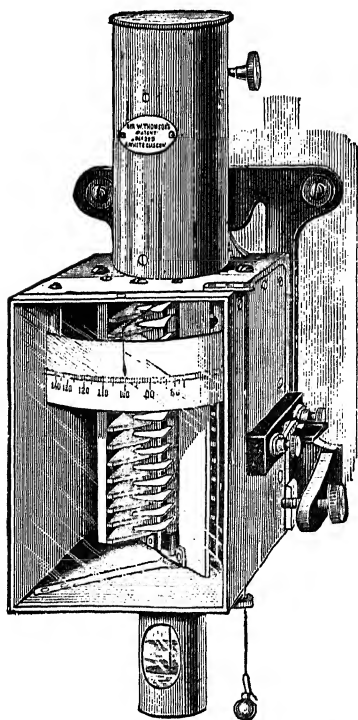
Fig. 22.—Kelvin Multicellular Vertical pattern Voltmeter.
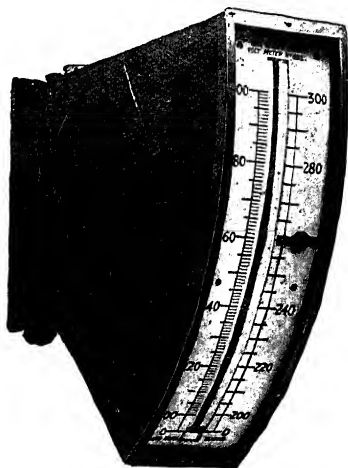
Fig. 23.—Kelvin Edgewise Voltmeter.

violent than when in a vertical position. If placed with the tube upright, then the air convection currents cause small and frequent changes in temperature in the wire which make the indicating needle "breathe," or move slightly to and fro. The instruments are usually calibrated for use between 40 and 150 volts, and hence are not suitable for measuring very low voltages.

Another form of thermal voltmeter is that of Hartmann and Braun (*see* Fig. 21). In this instrument the platinum-silver wire is fixed at the ends, but when heated by a current it "sags," and the sag is detected and measured by a delicate multiplying mechanism. The needle is kept from vibration by a damping copper disc attached to it which moves between the poles of a permanent magnet.

These hot-wire voltmeters, like the electrostatic, are suitable for use with alternating currents of any frequency, not very high, as well as with continuous currents, since their indication depend upon the heating power of the current, which is proportional to the square of the current, and therefore to the square of the potential difference of the terminals.

For switchboard purposes a voltmeter is preferred which has a scale in a vertical plane. If the voltmeter is to be kept continuously in connection with a circuit, electrostatic instruments have a great advantage over electromagnetic or electrothermal, in that they take up no sensible amount of power. Moreover, in contrast with electromagnetic instruments, they can be employed for direct as well as alternating currents, and in the latter case their indications are independent of the frequency.

In the class of voltmeters suitable for switchboard work we may especially include the vertical multicellular pattern of Kelvin electrostatic voltmeter (*see* Fig. 22), which is made for various ranges of voltage.

A type of voltmeter for switchboard work, called the Edgewise pattern (*see* Fig. 23), is often used on switchboards because of the small space it takes up. This instrument,

together with the Kelvin engine-room voltmeter, is of the electromagnetic type, and depends for its action upon the attraction of a small, carefully annealed rod of iron by a solenoid.

In testing a voltmeter for station or workshop use it is necessary to pay attention to the following points :—

First, the accuracy of the scale readings must be checked with the potentiometer if the voltmeter is for continuous currents, or by reference to a carefully-calibrated electrostatic voltmeter if the instrument is for alternating currents. In so doing the voltmeter should be first tested with gradually increasing voltage and then with gradually diminishing voltage, to ascertain if there is any hysteresis error. This is especially necessary in the case of electromagnetic instruments containing iron in their construction.

In the next place, the voltmeter should be tested with the voltage applied in both directions on the terminals, if the instrument permits this being done. This is essential in the case of electrostatic instruments to detect any contact, potential or volta-effects due to contact of different metals.

In the case of alternating-current voltmeters the effect of varying the frequency should be examined. In all cases the disturbing effects of varying position, or of the proximity of magnets or wires conveying continuous or alternating currents, should be carefully employed. In the case of switchboard instruments this is very necessary, as some types of instrument indicate correctly when isolated but very incorrectly when in the neighbourhood of conductors conveying strong currents.

In a vertical pattern electrostatic voltmeter, in which the needle moves on pivots, it is essential that the needle axis should be carried in jewels or on friction wheels. The electrostatic forces brought into play are not large, and if pivot friction exists the instrument will be sluggish and will require a great deal of tapping to make the needle take up its right position when the voltmeter is in the circuit.

Another source of error in electrostatic voltmeters is that due to the electrification of the glass front by friction.

Prof. Ayrton has studied this question, and has devised various transparent varnishes which are sufficiently conducting when dry to prevent an electric charge being held on them. The glass front of the voltmeter is covered with a layer of this varnish, and it is impossible to disturb the instrumental readings by electrifying the surface by touching or rubbing. Two of Prof. Ayrton's formulæ for a varnish of the above kind are given below :—

(i.) Dissolve ¼oz. of transparent gelatine in 1oz. of glacial acetic acid by heating them at 100°C. To this add half the volume of dilute sulphuric acid (one part of strong acid to eight of water by volume) and apply this whilst warm to the glass shade. When this film has become hard, apply over it a coating of Griffiths' anti-sulphuric enamel.

(ii.) Thin the gelatine solution prepared as above by the addition of acetic acid (say, two volumes of acid to one of solution), and, after drying the glass, float this solution over it. Drive off the excess of acetic acid by warming. Allow the glass to cool, and repeat the process. Thin anti-sulphuric enamel by the addition of ether and float it over the gelatine layer. Expel the ether by heating and apply a second layer of anti-sulphuric enamel.

A glass disc so coated is quite as transparent as one not coated, but it cannot be electrified by touching or rubbing it or holding near it an electrified body. The varnish film acts as a metallic screen would do in preventing electrification of the interior portions of the voltmeter.[*]

A good switchboard voltmeter should comply with the following conditions :—

(i.) It should not be affected by external magnetic fields, such as exist in the neighbourhood of switchboards. It is quite common to find that the stray fields on the front of a station switchboard

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[*] *See* Ayrton and Mather "Transparent and Conducting Screens for Electric and other Apparatus," *Proc.* Inst. Elec. Eng., April 12, 1894 ; also *The Electrician*, Vol. XXXII., p. 693.

have a strength of 5 to 10 C.G.S. units—that is, 25 to 50 times the earth's horizontal field.

(ii.) It should not be affected by outside electrostatic fields, or by rubbing the dial glass.

(iii.) It should have no temperature error.

(iv.) It should have a high resistance, so as to measure equally accurately the pressure at the station 'bus bars or at the ends of long thin pilot wires.

(v.) It should have as large and open a scale as possible.

(vi.) It should be dead-beat if possible without the use of dashpots.

(vii.) If employed with alternating voltages, its readings should be independent of frequency.

(viii.) Its readings should not be affected by variations in external temperature or by vibration, and it should be easy to pack, transport and fix up on a switchboard, and take up as little room as possible.

Hardly any voltmeter complies perfectly with the whole of these conditions. The electrostatic instruments comply well with (i.), (iii.), (iv.) and (vii.), and the electrothermal with (iii.) and (vi.). Many electromagnetic instruments of the movable coil permanent magnet type are much affected by external magnetic fields. Hence such switchboard voltmeters should be calibrated in position and checked constantly against a tested electrostatic instrument.

If a switchboard is equipped with electromagnetic voltmeters it is well to have at least one electrostatic instrument in connection with them.

In selecting these instruments, if not electrostatic, regard should be taken of their resistance and of the amount of power they take up and the energy absorption in watt-hours per year of ordinary use. A voltmeter which has a resistance of only 1,000 ohms takes up 10 watts when used on a 100-volt circuit, and absorbs therefore 1 B.T.U. in 100 hours of use. Hence the switchboard absorption of power may

amount to a not inconsiderable amount if voltmeters of low resistance are largely employed.

Prof. Ayrton and Mr. Mather have devised an electro-magnetic astatic voltmeter for switchboard purposes which is, as far as possible, free from liability to disturbance by external magnetic fields. For details of this instrument the reader is referred to the Paper describing it in the *Proceedings* of the Institution of Electrical Engineers, April 12, 1894, or *The Electrician,* Vol. XXXII., p. 688, 1894.

TABLE I.

Electromotive Force of the Clark Cell at Various Temperatures, based on the Board of Trade Value at 15°C.

Temperature.	E.M.F.	Temperature.	E.M.F.
6°C.	1·442	16°C.	1·4317
7°C.	1·441	17°C.	1·4307
8°C.	1·440	18°C.	1·4297
9°C.	1·439	19°C.	1·4287
10°C.	1·438	20°C.	1·4278
11°C.	1·4365	21°C.	1·4267
12°C.	1·4355	22°C.	1·425
13°C.	1·4345	23°C	1·4235
14°C.	1·4335	24°C.	1·4226
15°C.	1·4326	25°C.	1·4216

The values from 10°C. to 21°C. are given to three and also to four places
of decimals.

CHAPTER V.

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THE MEASUREMENT OF ELECTRIC POWER.

§ 1. Electric Power : Mean Power and Power Factor.—
When an electric current exists in a circuit there is an
absorption of power by the circuit due to its electrical resis-
tance, and this power is ultimately dissipated as heat. If the
circuit contains one or more sources of counter-electromotive
force against which a current is urged by the external impressed
electromotive force, additional work is done and power is
expended on the circuit. Considering, first, the case of
unvarying continuous or unidirectional electric currents, we
can estimate the power expended on the power-absorbing
circuit by measuring separately the current through the
circuit and the difference of potential of its ends.

If V be this difference of potential, then the work done
in raising a quantity of electricity, Q, through a difference
of potential V is QV, and if this work is uniformly per-
formed in a time T, then QV/T is the rate of doing work.
But $Q/T = C$, or is the measure of the current in the power-
absorbing circuit. Hence, the measure of the power being
taken up is the numerical value of the product CV.

Accordingly, in the case of unvarying unidirectional
currents the practical measurement of the power absorbed
electrically by any circuit consists in measuring the current
in the circuit and the fall in potential down it, and taking
the numerical product of these values. These two readings
may be taken by separate instruments, an ammeter and a

voltmeter, or the product may be directly obtained at one reading by the use of a *wattmeter,* or electric power-measuring instrument.

If the power taken up from instant to instant varies, the *mean value* taken at equidistant intervals may be calculated or obtained from a *power curve.* Thus, if the horizontal distances in the diagram in Fig. 1 represent *time* and the vertical distances *power*, then, if observations are taken at certain times of the power being absorbed by a circuit, and ordinates are drawn to scale to represent these values, the curve obtained by joining their upper ends is a *power curve.* The mean ordinate of this curve is the *mean power*, and the product of the mean power and the whole time of observation gives us the total energy absorbed by the circuit in that time.
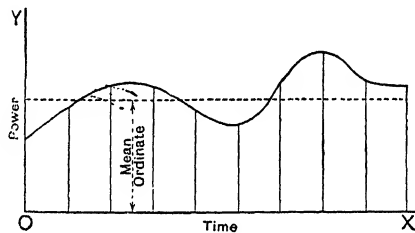
FIG. 1.

If the current flowing in the power-absorbing circuit is an alternating current, then we have two cases to consider: first, when the circuit is non-inductive, and second, when it is inductive. In the first case the mean power taken up by the circuit is measured by the product of the root-mean-square (R.M.S.) values of the current and the terminal potential difference or voltage; in other words, by the product of the ammeter and voltmeter readings, employing proper instruments for recording these quantities. When the circuit is inductive, the true mean power is measured by the product of the root-mean-square values of the current and terminal voltage and a factor called the *power-factor.* In

the case when the current and voltage vary periodically in a simple harmonic manner the power-factor is the cosine of the angle of phase-difference of the current and terminal voltage.*

Hence, generally, we may say that in all cases the mean power taken up in a circuit is measured by the product of the three quantities—the R.M.S. value of the terminal voltage, the R.M.S. value of the circuit current, and the power-factor. If the circuit is non-inductive the power-factor is unity, and if the current is continuous the R.M.S. value is the same as the unvarying value of this quantity. If the current is alternating the product so obtained is the mean power during this phase.

§ 2. Measurement of Power in the case of Unvarying Continuous or Direct Currents.—If a circuit is traversed by a continuous or unvarying current, in order to measure the power taken up we have to measure the current in the circuit and the terminal voltage. The current can be
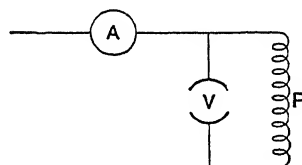
Fig. 2.

measured by a correct ammeter and the voltage by a voltmeter. The voltmeter should preferably be of the electrostatic type. If, however, the voltmeter is electromagnetic, or takes up a current, then a correction will be needed in the ammeter reading.

The voltmeter and ammeter should be arranged as shown in Fig. 2, where A is the ammeter, V the voltmeter and P

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* The term *power-factor*, defined as above, was first suggested and so used by the Author in a Paper entitled "Experimental Researches on Alternate Current Transformers." See *Proc.* Inst. E.E., 1892, Vol. XXII., p. 606.

the power-absorbing circuit.   Then it is necessary to deter-
mine the current taken up by the voltmeter, and to deduct
this value from the ammeter reading to obtain the true
current taken by the power-absorbing circuit alone.   This
correction is the more necessary in cases in which the current
taken by the power-absorbing circuit is small.   It may, how-
ever, be neglected in many other cases in which the voltmeter
current is very small in comparison with the total current.

If the voltage or the current are either of them very small
it may be quite impossible to obtain really good power
measurements when using commercial ammeters and volt-
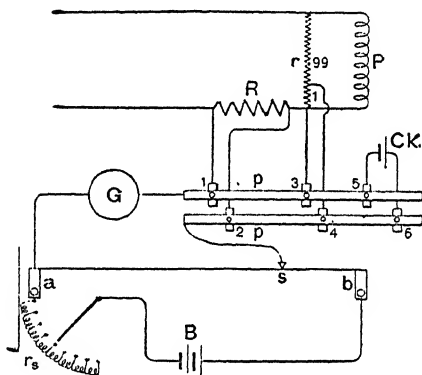
FIG. 3.

meters.   This is the case in power measurements made on
small candle-power low-voltage incandescent lamps.   Under
these conditions it is better to employ a potentiometer for
taking the current and voltage readings.   The following
arrangement is a very effective one for obtaining quickly
good power readings in the case of incandescent lamps.

The lamp, or power-absorbing circuit P (see Fig. 3), has
placed in series with it a manganin resistance R of about
1 ohm, and across its terminals a manganin resistance r of, say,
10,000 ohms, divided into two sections, having resistances in
the ratio of 1 to 99.   From the ends of the series resistance

and of the small section of the divided resistance, wires are brought to a potentiometer, and the volt-fall down these known resistances measured. If the series resistance is exactly 1 ohm, these volt-falls give us at once the current through the lamp in amperes and $\frac{1}{100}$th part of the terminal voltage on the lamp. These voltage measurements are of course referred to that of the Clark or Weston cell, Ck, used to *set* the potentiometer.

It is necessary always to apply the above-mentioned correction, and to deduct from the current, through the series resistance, the current passing through the divided resistance attached to the terminals of the lamp. The measurements can be depended upon to a fraction of 1 per cent., provided that the standard cell and the resistances have been previously carefully checked or standardised against correct standards. The potentiometer measurements, and the setting of the potentiometer, can be very quickly accomplished when using a Crompton form of potentiometer.

The scheme of connections is shown in Fig. 3. The resistance R in series with the power-absorbing circuit P should, if possible, be a 1-ohm resistance, because then calculations are simplified, and the volt-fall down this resistance is the numerical value of the current through the circuit. The divided shunt resistances may be either divided in a 99:1 or 9:1 ratio. By the use of a double-pole switch on the potentiometer, or parallel bars and plugs *pp*, the readings of the volt-fall down the series and down the decimal fraction of the divided resistance can be taken successively with great expedition.

The same arrangement may be applied also in measuring the power absorbed by a continuous-current motor, provided that the load on it is steady.

§ 3. **Measurement of Continuous Current Power by the Wattmeter.**—If it is desired to obtain the measurement of the power absorbed in a circuit traversed by a direct or

unidirectional current by one observation, then a wattmeter of the Siemens type may be used. This instrument has already been described (*see* p. 181, Chap. I.). In using it to measure continuous current power the fixed circuit of the instrument is joined in series with the power-absorbing circuit, and the movable or shunt coil of the wattmeter is joined across the terminals of that circuit. The fixed coil is then traversed by the current through the power circuit, and the movable coil by a current which is proportional to the P.D. of the ends of this circuit. The electromagnetic torque between the fixed and movable circuits of the wattmeter has then to be balanced by a mechanical torque produced by giving the movable head of the wattmeter a twist. Let θ be this
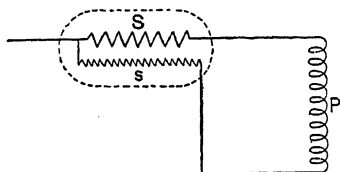
Fig. 4.

twist in angular degrees or divisions of the wattmeter circular scale. The electromagnetic torque between the wattmeter coils is proportional to the numerical product of the currents in these coils. Let C be the current in the series coil, let c be the currents in the shunt coil, and let R and r be the resistances of these circuits respectively. Then let G be such a constant that $G\theta = Cc$. G is called the wattmeter constant. It can be determined by passing known constant currents, or one and the same current, through the wattmeter coils and observing the twist which must be given to the wattmeter head to bring back the movable coil to its normal position, with its axis at right angles to that of the fixed coil.

There are two ways in which the movable or shunt coil can be connected. First, it may be joined up as shown in

Fig. 4, where P is the power-absorbing circuit, S the series coil of the wattmeter, and s the shunt coil. In this case the power absorbed in P is equal to the product of the current C and the potential difference of the ends of P, which last is equal to $cr - CR$. Hence, if we call W the power absorbed in P in watts, we have

$$W = C(cr - CR) = Ccr - C^2R ;$$

but if G is the wattmeter constant, then $G\theta = Cc$, and hence

$$W + C^2R = G\theta r.$$

In other words, the twist which will have to be given to the head of the wattmeter under the above conditions to bring back the movable coil to its zero position is proportional to the sum of the power absorbed in P and that in S the series coil, and inversely as the resistance of the shunt coil.
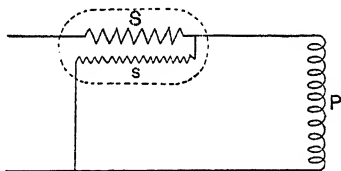
FIG. 5.

If, on the other hand, the shunt coil of the wattmeter is connected up as shown in Fig. 5, then we have, using the same notation,

$$W = cr(C - c) = Ccr - c^2r,$$
$$\therefore \quad W + c^2r = G\theta r,$$

and hence the twist θ is proportional to the sum of the power absorbed in the circuits P and s, and inversely as the resistance of s.

Accordingly, it will be seen that the scale reading of the wattmeter is never proportional simply to the power absorbed in the circuit under measurement, but it always reckons in as well the power absorbed in one of its own circuits.

This correction may be negligible, but it becomes of importance in the case of circuits taking either very small currents or very small voltages.

A precaution which must not be neglected in using the dynamometer wattmeter in measuring continuous current power is to set the instrument in such a position that the horizontal magnetic field of the earth does not exercise any action upon the movable coil when this last is traversed by a current. For when a continuous current is passing through the movable coil it becomes a magnet and is directed by the magnetic field of the earth. This may be discovered and neutralised in the following manner:—Before beginning an experiment pass a continuous current through the shunt coil of the wattmeter only, and observe whether there is any tendency in this coil to move one way or the other when this current flows through it. If so, turn the wattmeter bodily round into various positions. This may be easily accomplished by placing it on a turntable. A position can be found in which the movable coil of the wattmeter is not changed in position by the passage through it of a current. This will be the case when the magnetic axis of the movable coil coincides with the direction of the earth's magnetic field at that place. The wattmeter must then be used in this position, and we shall know that the current through the movable coil has no effect by itself and apart from the action of that in the fixed coil in causing a displacement of the suspended coil of the wattmeter.

The wattmeter can of course be calibrated or its constant discovered by sending through its two circuits a current the value of which in amperes is known. We may define the wattmeter constant G, used in the equations above, as the reciprocal of the numerical value of the twist which must be given to the head of the wattmeter to bring the movable coil back to its normal position when a current of 1 ampere flows through both coils joined in series. For the wattmeter constant G is defined by the equation

$$G\theta = Cc$$

where C is the current through one coil and c that through the other. Hence, if $C = c = 1$, we have

$$G = 1/\theta,$$

θ being the restoring twist which must be given to the wattmeter head. Otherwise we may standardise the wattmeter by observing the torsion necessary to be given to the head when it is connected to a power-absorbing circuit through which a known current is flowing and down which there is a known volt-fall.

§ 4. Measurement of Alternating-Current Power.—In the case of single-phase alternating-current power measurements, when the current flows in a circuit having a power-factor of unity, or one which is practically non-inductive, we can measure the power taken up by the numerical product of the root-mean-square (R.M.S.) value of the current and the R.M.S. value of the voltage or fall of potential down it. This can be done with any ammeter and voltmeter suitable for measuring the alternating current and voltage in question.

If, for instance, incandescent lamps are being operated by means of a single-phase alternating current, the power taken up in them, reckoned in watts, is obtained by taking the product of the values of the terminal potential difference in volts, as read by an electrostatic voltmeter, and the current in amperes as given by a hot-wire or other suitable alternating-current ammeter ; or else a wattmeter can be used, with certain precautions, named below, as to its construction, to measure directly the same quantity. If, however, the circuit is not non-inductive, but has a power factor sensibly less than unity, then the product of the R.M.S. values of the current through it and the fall of potential down it does not give the true power taken up in that circuit, but gives what is called the *apparent power* or, as it is also called, the *volt-amperes*.

The true power taken up can be obtained by the employment of a properly constructed wattmeter.* In the construction of a wattmeter for use with alternating currents it is most important that there should be no metal near to the fixed and movable coils. Instrument makers generally pay no attention to this detail. They delight to devise wattmeters in which the working parts are enclosed in brass cases or are wholly made up of metal. When such an instrument is used with alternating currents, eddy electric currents are set up in the metal portions of the instrument near the coils, and these react upon the movable coil when it is traversed by an alternating current and create additional and disturbing mechanical forces which displace it. Hence the readings of such a wattmeter may be, and generally are, quite erroneous when it is used with alternating currents, and no reliance can be placed upon them.

An alternating-current wattmeter must be constructed entirely of non-conducting material, and no metal work should exist in proximity to the coils of the instrument.†

We can show experimentally that this induction of eddy currents in neighbouring conductors must be a source of error in wattmeter readings when employing alternating currents. Suspend a coil of insulated wire wound on a rectangular or circular frame, and let an alternating current flow through it. The coil may be suspended by a wire or by a bifilar suspension. When traversed by the alternating current, hold a sheet of copper near the coil. It will be found to be repelled. This repulsion arises from the reaction of the eddy currents set up in the copper, and is an effect of

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* For a full discussion of the formulæ for obtaining the true power taken up in an alternating-current inductive circuit the reader may consult the Author's treatise on "The Alternate Current Transformer." *See* Vol. I., 3rd Edition, pp. 147-157.

† For an illustration of the errors which may arise by neglecting the above precautions, the reader is referred to a Paper by the Author entitled "Experimental Researches on Alternating Current Transformers." See *Proc. Inst. E. E.*, London, Vol. XXI., 1892, p. 666.

the same character as that which gives rise to the phenomena of *electromagnetic repulsion.**

It is evident, therefore, that, if mechanical forces are brought to bear on the movable coil of an alternating-current watt-meter due to any other cause than the mutual action between it and the fixed coil, errors in its indications must ensue.

§ 5. Measurement of the Power taken up in the case of High-Tension Alternating Current Circuits.

—When employing a wattmeter to measure the power taken up by an inductive or non-inductive circuit, the voltage of supply being high, as in the case of alternating-current transformers, certain precautions are necessary for safety and economy. It is, of course, possible to place the series coil of the wattmeter in series with the power-absorbing circuit and to join the shunt coil of the wattmeter in series with a suitable induc-tionless high resistance, and to place this last circuit as a shunt in the ends of the power-absorbing circuit. If, however, the volt-fall down the power-absorbing circuit is large, this will in general necessitate a great expenditure of power in the wattmeter shunt circuit and associated resistance. The Author therefore devised, in 1892, the following method of working, which has many advantages.* It depends upon the well-known fact that, in the case of a good closed iron circuit alternating-current transformer, not much loaded up on its secondary side, the primary voltage is always exactly opposite in phase to the secondary voltage and proportional to it.

Hence we may make use of such a transformer (called an auxiliary transformer) to reduce voltage in a known ratio, but still preserve its phase. The wattmeter is accordingly arranged as follows :—

The series coil of the wattmeter is joined in series with the power-absorbing circuit (*see* Fig. 6). Across the circuit terminals supplying the voltage is connected the primary coil

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* For a full discussion and description of these effects, the reader is referred to the Author's treatise on "The Alternate Current Transformer," Vol. I., 3rd Edition, p. 307.

of the auxiliary transformer **T**, and its secondary circuit is joined in series with the shunt coil *s* of the wattmeter and with one or more incandescent lamps, **L**, of suitable voltage. Under these conditions the series coil S of the wattmeter W is traversed by the same current as that through the power-absorbing circuit P, and the shunt coil of the wattmeter is traversed by a current which is proportional to the difference of potential of the ends of the power-absorbing circuit and in step with it. Hence the torque, acting on the wattmeter movable coil, will be proportional to the mean power taken up in the power-absorbing circuit. It is desirable to annul certain electrostatic effects by connecting one end of the
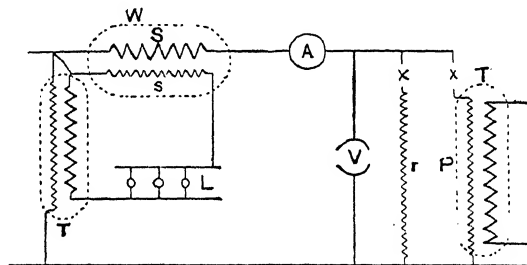
FIG. 6.

series coil of the wattmeter with one end of the secondary circuit of the auxiliary transformer nearest to it.

The wattmeter is then calibrated as follows :—A circuit must be provided, consisting of a practically inductionless resistance, *r*, capable of being placed safely across the high-tension circuit. An electrostatic voltmeter, V, and an ammeter, A, must be provided suitable for measuring the volt-fall down the resistance, called the standardising resistance, and the current through it. We begin by connecting this inductionless, power-absorbing circuit to the wattmeter, and observe the current through it and volt-fall down it, whilst at the same time we take a wattmeter reading. We then know that the true power taken up in

this inductionless resistance is given by the product of the readings of the ammeter and voltmeter, and we can obtain at once the wattmeter constant by comparing this product with the wattmeter scale reading. We then substitute for the inductionless circuit the inductive circuit P, and take a second wattmeter reading.

A simple calculation then enables us to deduce the value of the true power taken up in the inductive resistance, for the wattmeter readings in the two cases are proportional to the true power taken up respectively in these circuits; and in one of these—viz., the non-inductive case—the actual power is independently obtained in watts as the product of the numerical values of the current through it and its terminal potential difference.

It is desirable that the wattmeter readings in the two cases should not be very different, or, at any rate, that the power taken up in the inductionless circuit used to standardise the wattmeter should not be less than that taken up in the inductive power-absorbing circuit under investigation.

§ 6. **Power Measurements in the case of Circuits of Small Power Factor.**—There are greater difficulties in measuring accurately by a wattmeter the true mean power taken up in a circuit of small power factor when supplied with alternating current than in making the same measurement when the power factor is large. This arises from the fact that in all cases the small residual inductance of the shunt circuit of the wattmeter causes the current in that circuit to be not quite in step with its terminal potential difference. If the power-absorbing circuit, being examined, has a large power factor, or one approaching unity, then a small shift or lag in the phase of the shunt coil current behind the phase of the potential difference between the ends of the power-absorbing circuit does not affect the reading of the wattmeter to the same percentage extent as it does if the power factor is small.

Consider the case of simple harmonic variation. Then, if I is the R.M.S. value of the current through the power-absorbing circuit, and V that of the potential difference of its ends, and if ϕ is the phase difference of these quantities, then the true power, W, taken up on the circuit is $IV \cos \phi$, where $\cos \phi$ is the power-factor. Hence;

$$W = IV \cos \phi,$$

and
$$dW/d\phi = -IV \sin \phi.$$

Therefore
$$\frac{dW}{W} = -\tan \phi \, d\phi.$$

Accordingly, for a given variation, $d\phi$ of the phase angle ϕ, the error in the power measurement, which is measured by dW/W, is greater in proportion as ϕ is greater.

If, therefore, we are employing a wattmeter of the dynamometer type to measure the true power taken up in an open iron-circuit transformer or in a condenser, great precautions must be exercised or else the wattmeter readings will be valueless. For, in the case of an open iron-circuit transformer or choking coil, the power factor is small, and the current flowing into the circuit lags behind the impressed electromotive force, whilst in the case of a condenser the power factor is also small, but the current is in advance in phase of the impressed electromotive force. If any eddy currents are set up in metallic parts of the wattmeter by the current in its movable or shunt coil, this will cause an increase in the phase difference of the shunt coil current and the current in the series coil, if the series coil current normally lags behind the shunt coil current; and, accordingly, the reading of the wattmeter will be less than it ought to be. This will be the case when the wattmeter is employed to measure the power being taken up in a choking coil or transformer of small power factor. The reverse is the case with a condenser.

Then, since the series coil current is normally in advance of the shunt coil current, any eddy currents established in

metal parts of the wattmeter by the shunt coil current will tend to decrease the phase difference of the shunt and series coil currents, and make the wattmeter readings larger than they should be.

There is, therefore, always a tendency for the power measurement made with the wattmeter on a choking coil or transformer or inductive circuit of small power factor to be too small, and for that made on a condenser or circuit of small power factor having capacity to be too large.

In these cases wattmeter readings should always be carefully criticised and not too readily accepted as correct evaluations of the true power absorption of the circuit.

One way in which this difficulty of making power measurements on circuits of small power factor may be minimised is by joining in parallel with the circuit under another inductive circuit, or circuit having capacity, as the case may be. The current-phase displacements in the case of the inductive circuit and the permittive circuit (one having capacity) are in opposite directions. Hence, if an inductive circuit in which the current lags behind the impressed electromotive force is joined in parallel with a permittive circuit or condenser in which the current is in advance in phase of the impressed electromotive force, the whole combination has a larger power factor than either of them separately, and the small power factor of one element annuls more or less that of the other. Hence the difficulties which arise from the small power factor in wattmeter measurement may be reduced by taking two wattmeter measurements—one on a combined circuit and one on a single circuit.

Thus, if it is required to measure the power taken up in a concentric cable or other condenser, due to dielectric hysteresis or other causes, when it is subjected to an alternating electromotive force, we may proceed as follows :—An inductive circuit must be provided, which should be one without any iron core—in fact, be simply a large coil of insulated copper wire of many turns. The power factor of this inductive

circuit will be small but positive—that is, the current will lag behind the impressed electromotive force. The true power taken up in this inductive circuit must first of all be carefully measured by the wattmeter, or its power factor determined, as described in the following section. It is then joined in parallel with the cable or condenser, and the true power taken up in both together measured. The difference of the two measurements then gives the true power taken up in the condenser.

If the inductive circuit used is one with no iron core, then the true mean power taken up in it is given at once by taking the product of the mean-square value of the current in the circuit and the ohmic resistance of the circuit.

The advantage of combining the inductive circuit with the cable or condenser is that in the latter case the capacity current is in advance in phase of the impressed electromotive force, whilst in the case of the inductive circuit it is in arrear. Hence the opposite phase differences more or less annul each other, and the combined circuit has a larger power factor than either of them separately.

Instead of joining the ironless inductive circuit in parallel with the cable or condenser, it may, as suggested by Mr. Mather, be connected in series with it.* If the wire of which the inductive circuit is formed is sufficiently stranded to prevent eddy currents being set up in its mass, then the actual power absorption in the circuit is sufficiently nearly found by taking the product of its resistance and the mean-square value of the currents through it.

It is obviously desirable that this ironless choker should be wound in the form of a coil of maximum self-induction, as given by Maxwell.

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* The method of employing a choking coil to increase the power factor of a cable or condenser was suggested by Prof. Ayrton in a discussion on a Paper by Mr. Mordey on " Capacity in Alternate Current Working." The plan of using the choker in series is due to Mr. Mather. See *The Electrician*, Vol. XLVI. 1901, pp. 512, 518 and 667.

For a further discussion of the theory of the dynamometer wattmeter the reader is referred to the Author's treatise on "The Alternate Current Transformer," Vol. I., 3rd. Ed., Chap. III., p. 168. It is there shown that if the time constant of the wattmeter shunt circuit is denoted by T_s and that of the circuit under test is denoted by T_r, and $p = 2\pi$ times the frequency, then, when simple harmonic currents are being considered, a correcting factor F must be applied to the wattmeter readings such that

$$F = \frac{1 + p^2 T_s^2}{1 + p^2 T_s T_r}.$$

Hence, if T_r is greater than T_s, F is a proper fraction, and the wattmeter reading is too high and is corrected by multiplying by F.

The practical utility of this formula is not, however, very great, as the time-constants of most inductive circuits cannot easily be measured, and if the circuit contains an iron core, or is wound on an iron core, the time-constant or ratio of inductance to resistance is not constant.

For a discussion of the practical precautions to be employed in using the wattmeter the reader is referred to a Paper by the Author read before the Institution of Electrical Engineers, London, in 1892 (see *Proc.* Inst. E. E., Lond., Vol. XXI., pp. 623-675).

Also a Paper may be consulted by Mr. C V. Drysdale, "On the Theory and Use of the Alternate Current Wattmeter" (see *The Electrician*, 1901, Vol. XLVI, p. 774).

A useful discussion on power measurements in the case of cable dielectrics took place after the reading of a Paper by Mr. Mordey on "Capacity in Alternate Current Working" (see *Proc.* Inst. E. E., Lond., 1901, and *The Electrician*, Vol. XLVI., p. 467, *et seq.*).

In this discussion the difficulties of such measurements and the precautions which must be taken in dealing with wattmeter readings are well brought out.

§ 7. Power Measurements by Direct Measurement of the Power Factor.

—It has been shown by Lord Rayleigh[*] that we can use a shunt and series coil combined with a soft iron indicating needle as a means of measuring either the phase-difference of two currents or of the power factor of an inductive circuit.

Let two circular coils of wire be placed with their axes in one straight line and their planes parallel, and between them let a soft iron needle be suspended by a glass fibre or torsion wire, so that its centre is on the common axis of the coils and its length at an angle of 45deg. to this line. If, then, a current is passed through one of these coils it will create a magnetic force proportional to the currents and a magnetisation in the iron nearly proportional to the magnetic force. Hence the couple or torque tending to place the axis of the soft iron needle in the direction of the coil axis is proportional to the square of the current in the coil.

Suppose, in the first place, that the current i varies harmonically and is expressed by the function $i = I \sin pt$, then the mean torque on the soft iron needle, and therefore its angular displacement, if small, varies as the average value throughout a complete period of $I^2 \sin^2 pt$, or as $\frac{1}{2}I^2$. Suppose, then, that two separate simple periodic currents, differing in phase, are passed through the two coils. Let these currents be represented by the functions $I_1 \sin pt$ and $I_2 \sin (pt - \theta)$. The torques produced by these currents separately on the soft iron needle will vary as $\frac{1}{2}I_1^2$ and as $\frac{1}{2}I_2^2$. Also the joint effect, when both currents act together, is to produce a torque on the needle proportional to the average value throughout a complete period of the function

$$\{I_1 \sin pt + I_2 \sin (pt - \theta)\}^2,$$

which varies as $\frac{1}{2}I_1^2 + \frac{1}{2}I_2^2 + I_1 I_2 \cos \theta$.

If, then, the first current is allowed to act alone on the iron needle, it will produce a small displacement which may

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[*] See *The Electrician*, Vol. XXXIX, p. 180, or *Phil. Mag.*, May, 1897.

be represented by D_1; and, in the same way, the second current acting alone will produce a displacement D_2; whilst both acting together will produce an effect D_3. We have then the following equations

$$D_1 = C_1^2 I_1^2, \quad D_2 = C_2^2 I_2^2,$$
$$D_3 = (C_1^2 I_1^2 + C_2^2 I_2^2 + 2C_1 C_2 I_1 I_2 \cos \theta),$$

where C_1 and C_2 are constants depending on the form of the coils.

Therefore $$\cos \theta = \frac{D_3 - D_1 - D_2}{2\sqrt{D_1}\sqrt{D_2}}.$$

Accordingly, by observing the angular displacements of the iron needle due to each coil acting separately and then that due to the two acting together we can find the phase-difference between the currents in the two coils.

The same formula holds good even when the currents are not simply periodic, that is, have not a simple sine curve form, but we then derive from the deflections the *power factor* and not merely the cosine of an angle of phase-difference. For in this last case we have

$$D_1 = h^2 \int i_1^2 dt, \text{ and } D_2 = k^2 \int i_2^2 dt,$$

where i_1 and i_2 are the instantaneous values of the currents in the coils and p and k are constants depending on the form of the coils, and the integrals are taken throughout one complete period. Also we have

$$D_3 = \int (h i_1 \pm k i)^2 dt,$$

the $+$ or $-$ sign being used according as the coils act with or against each other.

In the first case, then, we have

$$\frac{D_3 - D_1 - D_2}{2\sqrt{D_1 D_2}} = \frac{\int i_1 i_2 dt}{\{\int i_1^2 dt \times \int i_2^2 dt\}^{\frac{1}{2}}}.$$

The quantity on the right-hand side of the above equation is the expression for the power factor of a circuit in which i_1 is the current through that circuit and i_2 is the fall in voltage down a shunt across that circuit, or the potential difference of the two ends of the circuit.

The above method gives us, therefore, a means of directly determining the power factor of an inductive circuit for any particular form of single-phase periodic electromotive force. We have to introduce in series with that circuit a coil which will carry the current flowing through the inductive circuit, and we have to place as a shunt across the circuit another

nearly inductionless high-resistance coil, with its plane parallel to that of the series coil and its axis coincident. A short, soft iron needle is then to be suspended by a fibre of glass or quartz, or by a metallic torsion wire of phosphor bronze, in the axial line of the coils. The needle may have a mirror attached to it, so that its small deflections may be read on a scale in the usual way. The coils should be so arranged that the axis of the needle is at 45deg. to the line joining the centres of the coils drawn perpendicular to their planes. The series coil should be capable of being short-circuited at pleasure and the shunt coil disconnected.

We can then observe the small deflections produced on the soft iron needle by the separate actions of the series and shunt-coil currents, and also their joint effect. These deflections give us the quantities called D_1, D_2 and D_3 in the formula above. Then $\dfrac{D_3 - D_2 - D_1}{2\sqrt{D_1 D_2}}$ is the power factor of the power-absorbing circuit connected to the coils.

The coils should be capable of being moved parallel to themselves independently, so as to make the deflections of the soft iron needle small and the deflections due to the series and shunt currents separately approximately equal to each other.

§8. Three-Voltmeter Method of Measuring Alternating Current Power.—The following method of measuring the alternating current power absorption of a circuit was first given by Prof. Ayrton and Dr. Sumpner in 1891 :—*

Let AB (*see* Fig. 7) be an inductive circuit traversed by an alternating current. The first step is to join in series with it another nearly inductionless resistance BC, and to pass a current through both. In many cases, such as in the

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* *See* " The Measurement of the Power given by any Electric Current to any Circuit." By Prof. Ayrton and Dr. Sumpner. *Proc.* Roy. Soc., Vol. XLIX., 1891, p. 424.

measurement of the power absorption of transformers, this necessitates the possession of means for increasing the circuit pressure, so as to enable this additional resistance to be added to the circuit under test. Three measurements of potential difference are then taken, either simultaneously with three voltmeters or, better still, successively with one and the same voltmeter suitable for alternating current measurement. The voltage is measured across AB (call it V_1), down BC (call it V_2), and over AC (call it V_3). Then let v_1, v_2, v_3 be the instantaneous values of these voltages at any moment, and let i be the instantaneous value of the current and R the resistance of the inductionless
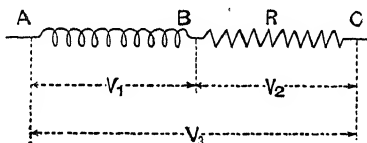
FIG. 7.

part of the circuit. Then, at any moment we have the equality

$$v_3 = v_1 + v_2;$$

therefore

$$v_3{}^2 = v_1{}^2 + v_2{}^2 + 2v_1 v_2.$$

But

$$v_2 = Ri;$$

hence

$$\frac{1}{2R}(v_3{}^2 - v_1{}^2 - v_2{}^2) = v_1 i.$$

Multiplying all through by dt, and integrating throughout a complete period T, or from 0 to T, and then multiplying by $1/T$ to obtain the mean-square values, we have

$$\frac{1}{T}\int_0^T v_1 i\,dt = \frac{1}{2R}\left(\frac{1}{T}\int_0^T v_3{}^2 dt - \frac{1}{T}\int_0^T v_1{}^2 dt - \frac{1}{T}\int_0^T v_2{}^2\,dt\right),$$

or

$$W = \frac{1}{2R}\{V_3{}^2 - V_1{}^2 - V_2{}^2\},$$

where W is the mean power taken up in the inductive part, AB, of the circuit.

The objection to this method is that great accuracy in the voltmeter readings must be attained if the resulting power value is to be correct to a small percentage. Since the formula involves the difference of squares of voltages, a small error in the measurement of the voltages themselves will make a much larger percentage error in the calculated value of the power.

If we take the equation given above for the power, viz.,

$$W = \frac{1}{2R}(V_3{}^2 - V_2{}^2 - V_1{}^2),$$

and differentiate it, we have

$$dW = \frac{1}{R}(V_3 dV_3 - V_2 dV_2 - V_1 dV_1),$$

where dV_1, dV_2, dV_3 are the errors made in the estimation of the three potential differences, or rather in their R.M.S. values.

Let $\qquad dV_3 = \pm eV_3, \quad dV_2 = \pm eV_2, \quad dV_1 = \pm eV_1,$

where e is a small fraction. The most probable value of $(dW)^2$ is then

$$\frac{e^2}{R^2}\{V_3{}^4 + V_2{}^4 + V_1{}^4\};$$

so that $\qquad \left(\dfrac{dW}{W}\right)^2 = 4e^2 \dfrac{V_3{}^4 + V_2{}^4 + V_1{}^4}{V_3{}^2 - V_2{}^2 - V_1{}^2}.$ . . . . (1)

Let the resistance R have such a value that

$$V_2 = xV_1. \qquad \text{. . . . .} \quad (2)$$

We proceed to find what value x should have that dW/W may be a minimum. Since $v_3 = v_2 + v_1$, we have

$$V_3{}^2 = V_2{}^2 + V_1{}^2 + 2V_1V_2 \cos \phi. \qquad \text{. . . . .} \quad (3)$$

Hence, eliminating V_1, V_2 and V_3 from the equations (1), (2) and (3), we have

$$\left(\frac{dW}{W}\right)^2 = 4e^2 \frac{(1 + x^2 - 2x \cos \phi)^2 + 1 + x^4}{4x^2 \cos^2 \phi}. \qquad \text{. . . . .} \quad (4)$$

Now $\cos \phi$ is independent of x. Hence, if we differentiate the numerator of (4) and equate to zero, we have the value of x which will make dW/W a minimum. We find $x = 1$ satisfies this condition ; hence the arrangement which will give the maximum sensibility is when R has such a value that $V_2 = V_1$.

In (4) put $x = 1$. Then

$$\frac{1}{e}\frac{dW}{W} = \frac{\sqrt{2 + 4(1 + \cos \phi)^2}}{\cos \phi}.$$

But $\dfrac{1}{e}\dfrac{dW}{W}$ is the ratio between the percentage error made in determining W and that made in determining the potential differences.

Suppose, then, that the inductive circuit is an ordinary closed iron circuit transformer on open secondary circuit. In this case $\cos \phi = 0\cdot75$ nearly.

Accordingly, $\frac{1}{e}\frac{d\mathrm{W}}{\mathrm{W}}=5$. Therefore, an error of 1 per cent. made in deter-mining the potential differences by the readings of one common voltmeter would involve an error of 5 per cent. in the estimation of the power taken up by the circuit. It is difficult to obtain commercial voltmeters reading to less than quarter per cent., and therefore the limitation of accuracy in the estimation of power by the three-voltmeter method is rather over 1 per cent.

In employing the three-voltmeter method to measure the power taken up in an alternating current transformer we proceed as follows :—Suppose the transformer under test to take a current at 2,000 volts on its primary circuit, and that we have available the current from an alternator or circuit having a pressure, say, of 100 volts. It is then necessary to
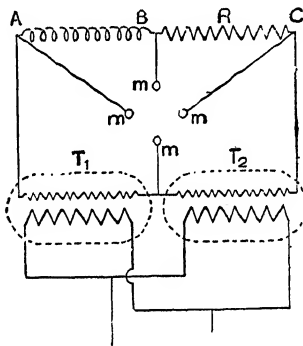
FIG. 8.

connect two step-up transformers, T_1, T_2 (*see* Fig. 8) with low-tension sides in parallel and high-tension sides in series, so as to create a voltage of 4,000 volts. Across the terminals of this 4,000 volt circuit we join in series an inductionless resistance, R, and the primary circuit of the transformer to be tested. It is convenient to bring potential wires or leads from the ends of the circuits AB, BC and AC to mercury cups, *mm*, well insulated. A carefully standardised electrostatic voltmeter then has potential wires attached to its terminals, and these wires may be connected to two brass pins carried on an insulating handle, by means of which the

voltmeter can be connected in between any two mercury cups. The voltmeter reading of the required voltages can then be quickly taken.

If the voltmeter range is only up to about 2,000 volts, then the voltage AC must be measured in two parts by measuring the PD of the terminals of each of the supply transformers separately and adding these voltages together. Hence, if V and V' are these last readings and V_1 and V_2, as before, the volt-fall down the inductive circuit under test and the inductionless resistance R, then the power W taken up in AB is given by

$$W = \frac{1}{2R}\{(V+V')^2 - V_1{}^2 - V_2{}^2\}.$$

The previous discussions of the theory of this method shows, however, that it is not well adapted for accurate power measurements in those cases in which the power factor of the tested circuit is small. A modification of the three-voltmeter method which does not necessitate the use of such a large auxiliary resistance has been described by Mr. A. Campbell.[*]

§ 9. **The Three-Ammeter Method.**—In cases in which it is not possible to obtain the augmented voltage required by the three-voltmeter method, a variation of it, called the three-ammeter method, proposed by the author in 1891, may be used.[†]

In this case the circuit under test, the inductionless resistance R, and three ammeters are arranged as in Fig. 9. The first ammeter, A_1, measures the current before division, the second, A_2, the current flowing through the inductionless resistance, and the third, A_3, the currents through the circuit under test.

By a similar process of reasoning to that employed in the case of the three-voltmeter method it can be shown that, if

[*] See *Proc.* Phys. Soc., London, 1901, or *The Electrician*, Vol. XLVI., p. 13, "On a Method of Measuring Power in Alternating Current Circuits."
[†] S e *The Electrician*, May 8, 1891.

W is the power taken up in the tested inductive circuit and R is the inductionless resistance, then

$$W = \frac{R}{2}\{I_1{}^2 - I_2{}^2 - I_3{}^2\},$$

where I_1, I_2, and I_3 are the readings of the three ammeters A_1, A_2 and A_3 respectively.

For if i_1, i_2, i_3 are the instantaneous values of these currents, we have always $\qquad i_1 = i_2 + i_3$;

therefore $\qquad i_1{}^2 = i_2{}^2 + i_3{}^2 + 2i_2 i_3.$

But $i_2 = v/R$, where v is the potential difference of the ends of the inductive circuit. Hence

$$\frac{R}{2}(i_1{}^2 - i_2{}^2 - i_3{}^2) = i_3 v.$$

Multiplying all through by dt, and integrating over a period, and then dividing by T, so as to obtain the mean values of each term, we arrive at the formula given above.
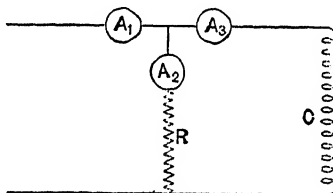
Fig. 9.

The three-ammeter method labours under the same diffi-culty as the three-voltmeter method. Very small errors in the absolute determination of the three currents make a very much larger percentage error in the result. It is not adapted for power tests on circuits of small power factor.

§ 10. Dynamometer Methods of Measuring Power.—

Mr. Blakesley gave, in 1891,[*] a method of measuring alter-nating current power which is independent of the frequency or wave-form and can be employed on any inductive circuit. It involves the use of a split dynamometer or wattmeter, in which the two circuits are traversed by two different currents.

* See *Phil. Mag.*, April, 1891 p. 346.

Let P, Fig. 10, be the inductive circuit under test, let R be an inductionless resistance joined in parallel with it, and let an ammeter, A, be employed to measure the current flowing through the inductive circuit. Then a dynamometer, D, of the Siemens type has its two circuits joined as shown in the diagram.

Let i be the current at any instant in the inductive circuit, i_1 the current in the inductionless resistance, and i_2 the current flowing to both. Then at any instant

$$i_2 = i_1 + i,$$

or
$$ii_1 = i(i_2 - i) = ii_2 - i^2.$$

Therefore
$$Ri_1 i = R(ii_2 - i^2).$$
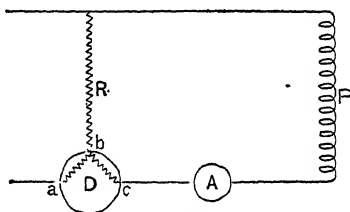
FIG. 10.

Let v be the potential fall down the inductionless resistance, R; this is also the potential difference of the ends of the inductive circuit, hence $v = Ri_1$. Accordingly, from the two last equations we have

$$iv = Rii_2 - Ri^2.$$

Multiplying all through by dt and integrating throughout the period, or from $t=0$ to $t=T$, and dividing by T, we obtain the mean value throughout a period of each of the quantities in the above equation. Hence

$$\frac{1}{T}\int_0^T iv\,dt = \frac{R}{T}\int_0^T ii_2\,dt - \frac{R}{T}\int_0^T i^2\,dt,$$

or
$$W = RD - RI^2,$$

where W is the mean power taken up in the inductive circuit in watts, I the R.M.S. value in amperes of the current

through it, D the dynamometer value or mean value of the product of the currents in the two coils throughout a period, and R the ohmic resistance of the non-inductive shunt employed.

If G is the dynamometer constant and θ is the twist which must be given to the torsion head to bring back the movable coil to its zero position when steady currents C and c flow through the coils, then

$$G\theta = Cc = D.$$

Accordingly, in the equation for W we can write $RG\theta$ for RD, and we have

$$W = RG\theta - RI^2.$$

If we employ a hot-wire ammeter or another dynamometer to measure the current, I, the method is perfectly general and independent either of the frequency or wave-form of the alternating current employed.

§ 11. **Power Measurement in the case of Polyphase Circuits.**—In the majority of cases the practical measurement of the mean electric power taken up in polyphase circuits offers no greater difficulties than in the case of single-phase alternating-current circuits, although it may involve a multiplication of instrumental readings. The theoretical treatment of the problem is rather more complicated, however, in the case of polyphase circuits, by reason of the phase relations of the various currents involved. In the very simple case of two-phase alternating currents the power taken up in the inductive or inductionless circuit supplied can be estimated by the employment of two watt-meters, one placed in each circuit. Thus, for instance, whether the outgoing line consists of three or four conductors, we can measure the power supplied to a two-phase transformer in motor by the employment of two wattmeters connected into the circuits, as shown in Fig. 11.

Let M_1, M and M_2 be the two-phase leads and ABC the power-absorbing circuit, whether motor or transformer. Then,

if two wattmeters, constructed with all the precautions already described for use with alternating currents, are inserted in the two sides of the two-phase circuit, the sum of their readings will be the total power given to the circuit ABC.
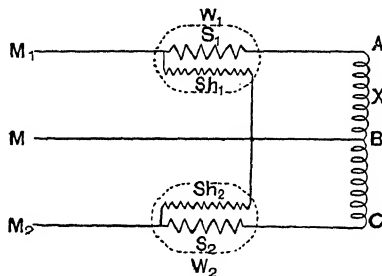
Fig. 11.

A similar arrangement can be applied in the case of three-phase circuits when connected on the *star pattern* with a common return.

For if OA, OB, OC, Fig. 12, are the three-phase power-absorbing circuits and M_1, M_2, M_3 the leads, and M the common return, then three wattmeters W_1, W_2, W_3 may be employed to measure simultaneously the power given to the
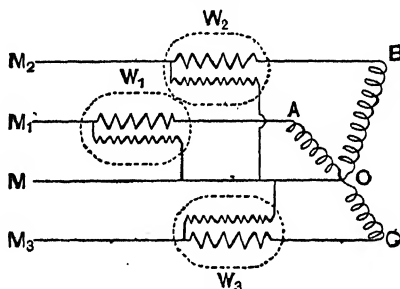
Fig. 12.

circuits OA, OB, OC. The sum of these wattmeter readings is the power taken up by the three-phase circuit.

The case of a three-phase circuit arranged on the delta pattern is, however, of particular interest, because then two

wattmeters arranged as in Fig. 11 will give the power absorption, whether the circuit be inductive or induction-less. The problem may, however, be considered on first principles.

Let ABC (Fig. 13) be a three-phase circuit arranged delta fashion, and let M_1, M_2, M_3 be the leads through which current
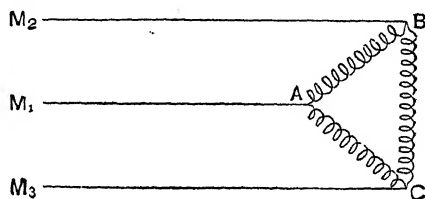
FIG. 13.

is supplied to it. We may consider, then, in the first place, the following problem: Given the ammeter values of the currents in M_1, M_2 and M_3 and the potential differences of the ends of AB, BC, CA find the currents in the delta branches and the mean electrical power taken up in the circuit ABC.
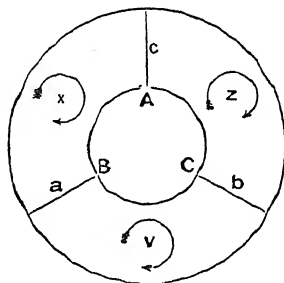
FIG. 14

Let us consider the general relation between the currents in M_1, M_2, M_3, AB, BC, and CA. Represent the three-phase system by a network of conductors arranged as in Fig. 14. Let the instantaneous current values in the lines M_1, M_2, M_3 be represented by a, b, c, and those in AB, BC, CA by x, y

and z; and let the outer circle represent the armature circuit of the three-phase dynamo or other source of electromotive force. Then we may represent the relations between the instantaneous values of the currents by the equations

$$x - y = a, \quad \ldots \quad \ldots \quad \text{(i.)}$$

$$y - z = b, \quad \ldots \quad \ldots \quad \text{(ii.)}$$

$$z - x = c. \quad \ldots \quad \ldots \quad \text{(iii.)}$$

Hence, if we take large letters to represent the maximum values of these currents, and assume a simple periodic mode of variation, it is clear that, on a vector diagram of currents, the line currents A, B, C will be represented by the sides of a triangle, and the delta currents X, Y and Z by lines drawn to the angular points of this triangle from some point in the interior.

To find the currents X, Y and Z in terms of the line currents A, B and C we may proceed algebraically as follows :—Square each of the equations (i.), (ii.) and (iii.) above, multiply each by dt, integrate throughout a complete period, and divide by T the periodic time. Thus

$$\frac{1}{T}\int_0^T x^2 dt + \frac{1}{T}\int_0^T y^2 dt - \frac{2}{T}\int_0^T xy\, dt = \frac{1}{T}\int_0^T a^2 dt. \quad \text{(iv.)}$$

In other words, equate the mean-square values.

In the next place, make certain assumptions for the sake of dealing with the simpler problem first, and therefore elucidating more difficult ones later on. Let us assume the circuits AB, BC, CA are inductionless, and that the currents and electromotive forces vary in a simple harmonic manner. Then the currents x, y and z may be expressed thus

$$x = X \sin pt, \quad \ldots \quad \ldots \quad \text{(v.)}$$

$$y = Y \sin (pt - 120°), \quad \ldots \quad \text{(vi.)}$$

$$z = Z \sin (pt - 240°), \quad \ldots \quad \text{(vii.)}$$

when $p = 2\pi n$ as usual, n being the frequency.

If we substitute the values for x, y and z given in the last equation in (iv.) we arrive at equations of the form

$$X^2 + XY + Y^2 = A^2, \quad \ldots \quad \text{(viii.)}$$
$$Y^2 + YZ + Z^2 = B^2, \quad \ldots \quad \text{(ix.)}$$
$$Z^2 + ZX + X^2 = C^2. \quad \ldots \quad \text{(x.)}$$

If these equations are solved for X, Y and Z in terms of A, B and C, they will give us the expression for the R.M.S. values of the currents in the delta circuits, which for shortness we will call the delta-currents, in terms of the line currents. Now, unfortunately, the above equations are very intractable. They *can* be solved algebraically, and those who are fond of algebra can amuse themselves by finding the solutions, which are not very simple. But where algebra fails common sense steps in, and a graphical solution sufficient for practical purposes can easily be obtained.

It is clear, from an inspection of the equations (viii.), (ix.) and (x.), that they are equivalent to the following geometrical problem:—Given a triangle whose sides are A, B, C in length, find a point, P, within that triangle, such that lines drawn from P to the angular points of the triangle are all at 120deg. angular distance from each other, and determine the lengths X, Y and Z of these lines in terms of the sides of the triangle.

It may be possible to solve this geometrical problem by purely Euclidean methods, but it is a waste of labour to attempt it.

From an electrical engineering point of view the following graphical method gives a solution quite accurate enough for all practical purposes :—Procure a celluloid circular protractor and cut out a sector subtending an angle of 120deg. Make a scratch on the protractor forming an angle of 120deg. with both the edges of the sector. The protractor will then look as in Fig. 15. Observe with an ammeter, or with three ammeters, the currents in the main leads M_1, M_2, M_3. These are the currents A, B, C. On paper set off to scale a triangle whose

sides represent these currents. Put two pins at two of the corners of this triangle, l and m (Fig. 16). Then apply the protractor with the sector edges to these pins and move it about until the scratch lies over the other corner of the triangle n. Then measure off the lengths, Pl, Pm, Pn, where P is the centre of the protractor. These lengths will be the delta currents, X, Y and Z, on the same scale on which the sides of the triangle are the line currents A, B, C.
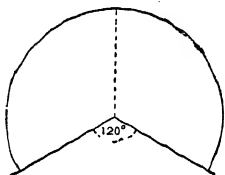
FIG. 15.

Otherwise, in default of a protractor, we may draw on paper three lines making angles of 120deg., and draw on tracing paper the triangle of line currents A, B, C. Place this tracing paper over the other, and trace off the lengths of the
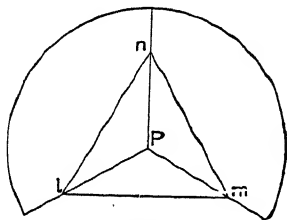
FIG. 16.

lines from the centre, P, where the three radial lines intersect, to the angular points of the triangle—viz., measure the lengths Pl, Pm, Pn (*see* Fig. 17).

By this simple, graphical construction we can find the ammeter or R.M.S. values of the currents in the delta branches when we are given those in the lines. Hence, multiplying the ampere value of these delta currents by the observed potential differences of the delta corners, and adding the

three products, we have the total power in watts taken up in the inductionless delta circuit.

It is obvious, if the resistances of the delta branches are all non-inductive and equal, that the three line currents (R.M.S. value) are equal, and also the three delta currents. Also, each delta current is equal to the quotient of each line current by $\sqrt{3}$. Accordingly, in the very simple case of a symmetrical inductionless delta circuit, the power taken up in the whole delta is equal to the product of $\sqrt{3}$ times either line current and the potential difference of the ends of either delta branch.
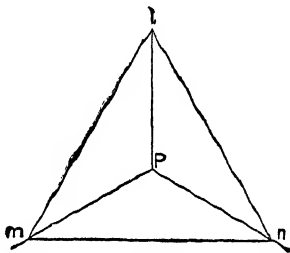
FIG. 17.

We have next to consider the case when the delta branches are not inductionless, which is an important case, as it involves the testing of three-phase motors and transformers worked or constructed on the delta pattern. Let us consider, first, the limited case when the three branches of the delta circuit have equal inductance and resistance. This is always the case with three-phase motors or transformers. A little consideration will, then, show that the same rule given above for finding the currents in the inductionless delta circuits will give also, in the case of equi-inductive delta circuits, the value of the delta currents. On the other hand, when the delta branches are inductive the current in each branch lags in phase behind the potential difference of the ends or delta corners. Hence we cannot find the

power taken up in the whole delta circuit until we have discovered the value of this phase difference.

This may be achieved in the following manner:—
Draw, as before, the triangle representing the line currents and the radial lines Pl, Pm, Pn at angles of 120deg., representing the delta currents. Let the dotted lines on the diagram (*see* Fig. 18) Pe, Pf, Pg represent the phase positions of the potential differences of the ends of the delta circuits—that is, let the line Pe represent the potential difference of the ends of the delta branch in which the current Pl exists. Then the angle θ is the angle of lag of the current Pl behind the impressed electromotive force Pe acting on that circuit.
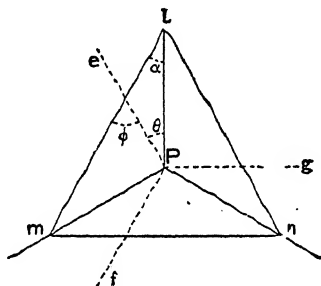
Fig. 18.

The angle θ is equal to the angle ϕ, minus the angle α. (*See* Fig. 18.)

Now the angle ϕ is the phase difference between the line current represented by the side ml and the potential difference between the adjacent corners of the delta circuit. The angle a can be measured by a protractor on the triangle of currents.

Accordingly, we may find all that we require in the following manner:—Introduce into one of the lines a series coil, and put across the terminals of one of the delta-branches another high resistance shunt coil. Employ these two coils in conjunction with a soft iron needle to determine, as already described, the phase difference between any one of the line

currents and the potential difference between any two line circuits. This phase difference is the angle ϕ. Find from the triangle of currents the angle a, and then the required phase difference $\theta = \phi - a$.

Hence we can find, assuming sinoidal currents and electromotive forces, the phase difference between any delta current and the potential difference creating it. The power taken up in the whole delta is then easily found, for it is three times that in any one branch of the delta, and this last is $XV \cos \theta$, where X is any delta current (R.M.S. value) and V the potential difference of the ends of that delta branch.

In place of a soft iron needle and series and shunt coil, a phasemeter of the type devised by Dolivo von Dobrowolsky may be employed. If the delta branches have unequal inductance and resistance, then the above measurements must be repeated on each line, and for the complete calculation of the power taken up in the delta we require, then, to know the value of each line current, the potential difference between each corner of the delta, and the phase difference of the above currents and potential differences.

The cases of most practical interest are, however, those in which each delta branch has the same inductance and resistance, as this involves the testing of three-phase transformers and motors; and the case when each branch is inductionless but not of equal resistance, as with this we are concerned in a three-phase distribution of current for lighting purposes when unequal numbers of lamps are on each phase.

But now it is interesting to notice that, although it is difficult to obtain an algebraical expression for the mean power taken up in an inductive delta three-phase circuit, given the line currents and line potential differences only and such phase angles as can easily be measured, yet it is quite easy to determine the power absorption practically by means of two wattmeters or even one instrument properly arranged.

Consider again the state of the three-phase delta circuit at any instant. Let c_1, c_2, c_3 be the line currents at any instant,

let i_1, i_2, i_3 be the delta branch currents, and let v_1, v_2, v_3 be the line potential differences or delta corner potential
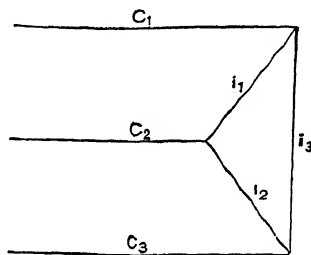
F_IG. 19.

differences respectively (*see* Fig. 19). Then it is clear that

$$v_1 + v_2 = v_3.$$
$$c_1 = i_1 + i_3,$$
$$c_3 = i_2 + i_3.$$

Let w be the power absorption in the whole delta at any instant, then

$$w = i_1 v_1 + i_2 v_2 + i_3 v_3.$$

Hence $$w = i_1 v_1 + i_2 v_2 + i_3 v_1 + i_3 v_2,$$

or $$w = (i_1 + i_3) v_1 + (i_2 + i_3) v_2.$$

Therefore $$w = c_1 v_1 + c_3 v_2.$$
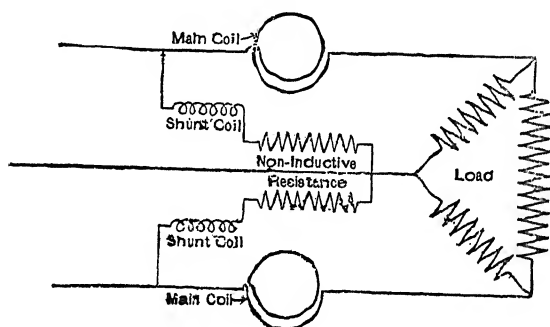
F_IG. 20.—Connections for Wattmeter Measurement of Power in Three-Phase Circuit.

If therefore we arrange two wattmeters as in Fig. 20, one of them will give a reading proportional to the average value of

$c_1 v_1$, and the other will give a reading proportional to the average value of $c_3 v_2$, and accordingly the sum of the watt-meter readings will be proportional to the total mean power taken up in the whole delta circuit whether the branches be inductive or not.

Hence we may arrange two suitable alternating current wattmeters to give the required measurement, or we may arrange one wattmeter and a series of switches which throws over the wattmeter quickly from one side to the other of the delta mains.*

Lord Kelvin has devised a form of duplex wattmeter suitable for this purpose which is described in the next section.

§ 12. **Practical Forms of Wattmeter.**—The existing prac-tical forms of wattmeter may be classified under two head-ings:—(i.) electrodynamic, (ii.) electrostatic instruments.

The electrodynamic instruments consist of two circuits or coils of wire, one called the series coil and the other called the shunt coil.   One of these coils or circuits is fixed, and the other is suspended and free to move over a small range under the influence of the electrodynamic stress existing between the coils when both are traversed by currents. Generally speaking, the series coil is fixed, because it has to carry the larger current, and the difficulty of getting this current in and out of a movable coil is greater than in the case of the smaller current used in the shunt circuit.   The movable coil must be restrained and brought back to a fixed zero position by a couple or force due to gravity or a spring control.   In any case the conditions which must be complied with are that a small displacement of the movable coil must bring into existence an opposing mechanical force which increases with the displacement.   The electrodynamic stress between the coils should be a maximum when the coils are

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* See *Science Abstracts*, Vol. I., p. 554, M. Aliamet " On Three-phase Power Measurement."

in their sighted or zero positions. For if the electrodynamic force increases with the displacement, it may increase faster than the opposing mechanical force, and then the equilibrium will be unstable. It is also desirable, though not necessary, that the mutual induction between the coils should be zero when they are in the sighted or normal position.

In the case of wattmeters intended for use with alternating currents, there must not be any metal enclosing case or metal
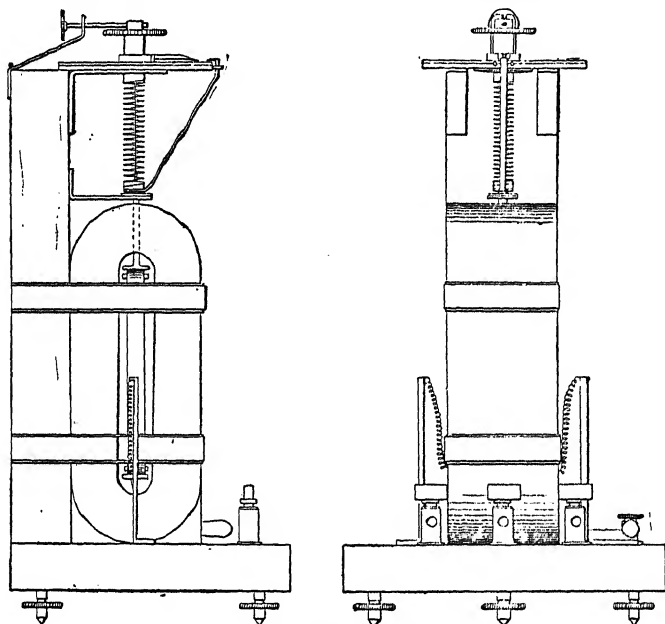
FIG. 21.—Siemens Workshop Wattmeter.

work of any kind near the coils, and the coils themselves must be wound on non-conducting formers or cores. If large-sized wire has to be used for the series coil, it must be stranded or formed of a cable of silk-covered wire twisted together.

One of the simplest wattmeters for workshop use is the Siemens wattmeter. It consists of a wooden base and support which carries the fixed coil (*see* Fig. 21) and a

movable coil hung from a torsion head by means of a few fibres of floss silk. In some instruments the ends of the movable coil dip into mercury cups. In the one shown in Fig. 21 the current is led into and out of the movable coil by light flexible connections. The restoring couple is applied to the movable coil by means of a spiral metal spring, one end of which is attached to the movable coil, and the other to the torsion head. In using the instrument the series coil is joined in the circuit of the power-absorbing circuit, and the movable coil is connected as a shunt across the ends of that circuit and the series coil taken together. As already explained, when so connected the reading of the wattmeter is proportional to the power absorbed in the circuit under test and that in the series coil of the wattmeter. If the wattmeter is employed with continuous currents, care must be taken to see that it is placed in such a position that the magnetic field of the earth has no influence on the movable coil when traversed by a current. When both coils have currents through them, the movable coil is twisted round through a small angle as far as a pair of stops will allow it to move. The torsion head is then twisted round in the opposite direction until the movable coil comes back to its original position, and the angular displacement of the torsion head noted. A table is furnished with each instrument, which will then give the power being taken up in watts in the circuit under test.

In the wattmeter designed by the Author for use with alternating currents, special precautions are taken to obviate sources of error. The wattmeter consists (*see* Fig. 22) of a teak case about 15in. high and 10in. wide with glass doors on each side. The case stands on levelling screws. On the top surface is a celluloid divided scale. A hollow axis through the centre of this scale carries on its outside an adjustable index arm, and to a support on the inside is suspended the movable coil. This is in a rectangular form, and consists only of a few turns of

insulated wire kept in shape by shellac. This coil is hung up to the torsion head by a few fibres of floss silk, and to it is attached an index arm of aluminium wire. The torsion spring is a spiral gilt steel chronometer spring, and this is affixed by one end to the torsion head axes and by the other
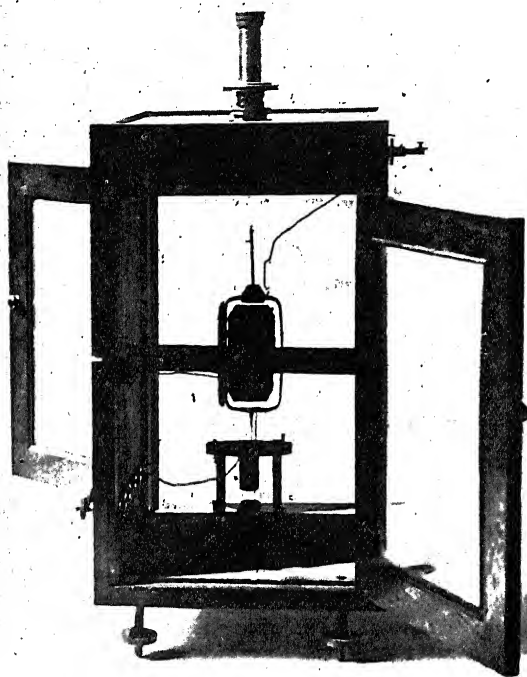
Fig. 22. – Fleming Alternating Current Wattmeter.

to the suspended coil. The ends of the coil dip into mercury cups made of vulcanised fibre. The fixed coil is carried on a wooden bar, and has its axis at right angles to the movable coil. The wires bringing the currents to the coils are twisted together. There are no metal parts or screws of any kind

near the coils. This wattmeter is used, as originally suggested by the Author, in conjunction with an auxiliary transformer, the secondary circuit of which is connected through two or three incandescent lamps with the movable coil of the wattmeter. The primary circuit of the transformer is connected as a shunt across the ends of the power-absorbing circuit and the series coil in series with that circuit. The wattmeter is standardised, as already described, by the use of an inductionless standardising resistance and an alternating current ammeter and voltmeter.

Lord Kelvin's ampere balances are also constructed as wattmeters by forming the fixed coils of thick copper strip, and using them as the series coils, whilst the balanced or movable coils are formed of thinner wire and constitute the shunt coil (*see* Fig. 23). The shunt coil is made to have a small resistance, and is joined up in series with a large non-inductive resistance outside.

Lord Kelvin's form of alternating current wattmeter for large powers is shown in Fig. 24. In this case the series coil is a stranded cable bent in a U shape. The shunt coil consists of a pair of balanced coils over it attached to the scale beam arm. The electrodynamic action is the same as in the other balances—that is to say, the forces due to the currents tend to raise one balance coil and depress the other. The series coil in the instrument shown in the figure is designed for carrying large currents. This conductor is made up of ropes of insulated copper wire, twisted together so as to form a cable with a hollow core. In order to correct any effect due to the induction of one arm of the coil upon the other the twisting is done in a very careful manner, so that the strands of the cable which are inside on passing the left-hand movable coil on one side are outside on passing the right-hand movable coil on the same side, and are in the reverse direction on the other arm of the U. The core of the cable is hollow, and brass tubes are passed along each arm of the U as far as the bend. The main object of these tubes
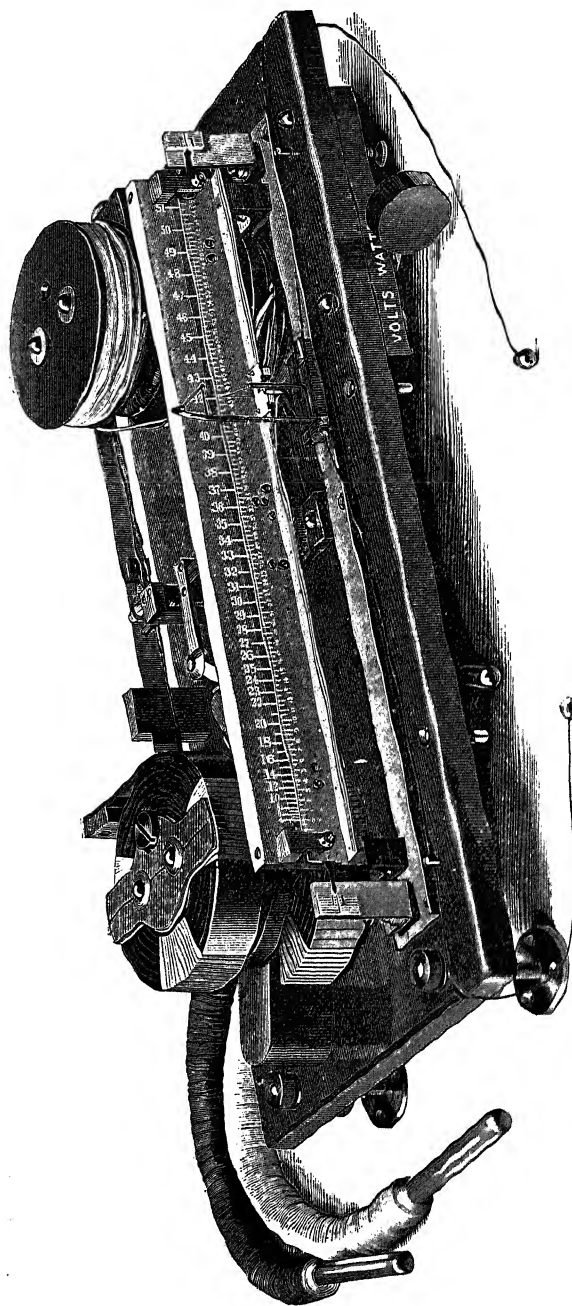
Fig. 23.—The Kelvin Wattmeter.
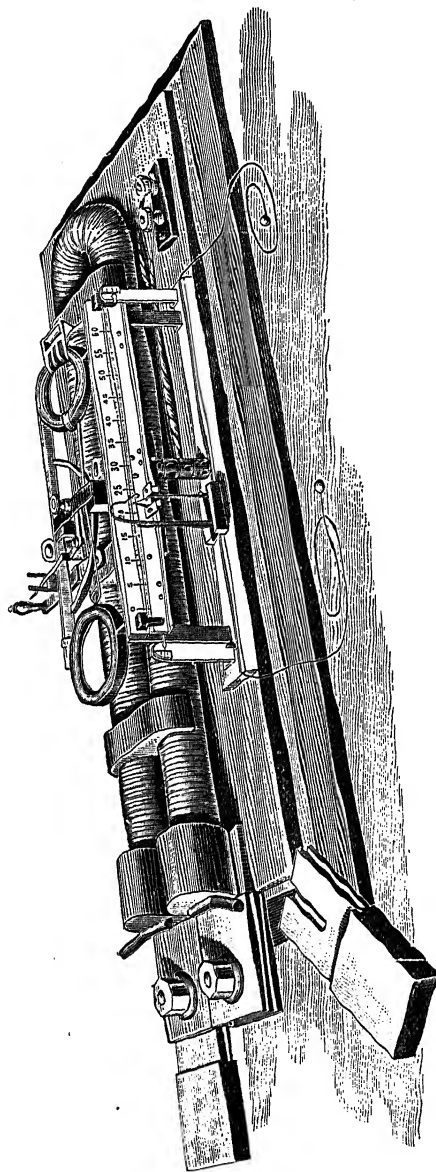
Fig. 24.—The Kelvin Alternating Current Wattmeter.

is to prevent any deformation in the cable, but they also serve as a means of blowing air through to keep the conductor cool, if it should ever be necessary to use it for much heavier currents than those for which the instrument is primarily intended.

Another form of dynamometer wattmeter is that devised by the Author and Mr. Gimingham. In it there are two helices of wire wound on non-conducting cores. These are so wound as to have similar magnetic poles in the centre,
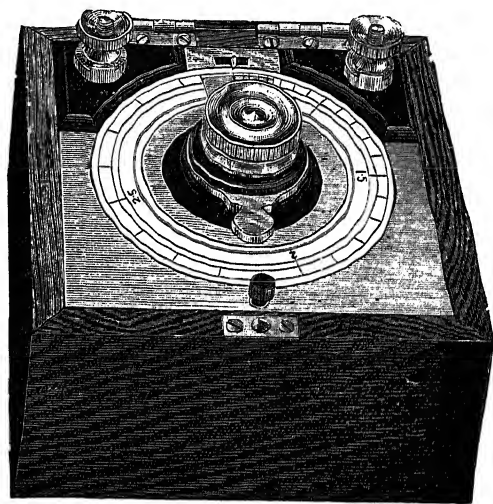
Fig. 25.—Fleming and Gimingham's Voltmeter (lid removed).

and are placed parallel to each other. These coils form the series coil of the wattmeter. They are embraced by two circular coils attached to the ends of a bar suspended on a needle point by means of a jewelled centre. These coils are made with aluminium formers, and are wound with fine wire. They constitute the shunt coils. The current is got into and out of this movable coil by means of very fine flexible leads, which do not prevent the movable coil from swinging freely within narrow limits. To the movable coil

is attached a spiral steel torsion spring, as in the Siemens wattmeter, and its upper end is fixed to a torsion arm moving over a divided scale. The scale can be divided to read directly in watts. The operation of reading consists in turning the torsion head until the movable coil is brought back to its normal or zero position, as shown by a small index needle attached to it. The external appearance of the instrument is shown in Fig. 25. The figure, however, represents the voltmeter designed by the same inventors, but the wattmeter only differs from it in having four terminals instead of two, one pair for the series circuit and one pair for the shunt circuit.

The above-described instruments are not direct reading. The observer has to move or slide some part of the instrument in the process of taking a reading. They are, therefore, not adapted for switchboard purposes. In this latter case an instrument must be employed which shows directly by a needle upon a scale or dial the power passing through it. A form of direct-reading wattmeter has been devised by Lord Kelvin. It consists of a coil of one or two thick turns of copper wire, and a spectacle-shaped fine wire coil in series, with an external resistance. The instrument is adapted as a central station wattmeter, giving indications of power passing through it by means of the movement of a needle attached to the fine wire coil over a scale. The interior is shown in Fig. 26. It has a main circuit formed of a double rectangle of copper rod having sufficient area to carry 200 amperes, and a shunt circuit with two fine wire coils astatically arranged. The main coil is mounted on a slate back so that the rectangles are horizontal. The shunt coils are mounted on a light but strong aluminium frame in the manner shown in Fig. 27. One end of this frame has a circular knife-edged hole fixed to it, and the other end has a straight knife-edge. These two knife-edges rest on two phosphor-bronze hooks attached by insulating supports to the outside ends of the double rectangle. By this method of suspension complete freedom from friction is obtained,
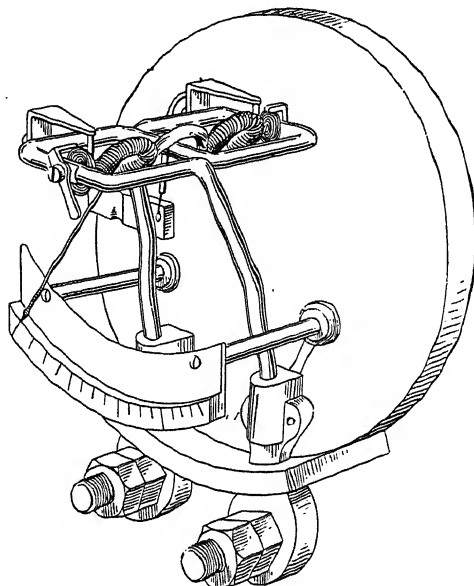
FIG. 26.—Kelvin Engine Room Wattmeter.   General View with Case
Removed.
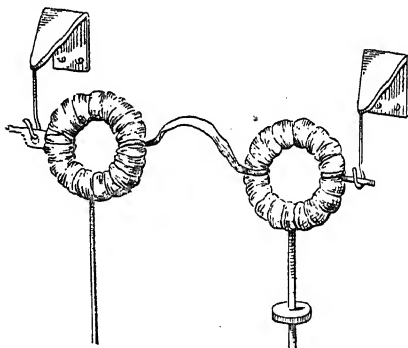
FIG 27.—View of Fine Wire Shunt Coils showing details of Suspension
Springs removed.

while the movable system is kept in a definite position with-
out end guides.

Each fine wire coil has about 1,000 turns of insulated
wire, and its resistance is about 100 ohms. The current is
conducted in and out from the movable system by two
flat palladium spiral springs, which also supply the restoring
force for governing the sensibility of the instrument. Not
more than $\frac{1}{20}$th of an ampere is allowed to pass through the
fine wire circuit, and in order to regulate this a large non-
inductive resistance is rolled on the case of the instrument,
which offers a large cooling surface. The scale has nearly
uniform divisions, and is graduated to read directly in watts
or kilowatts as required.
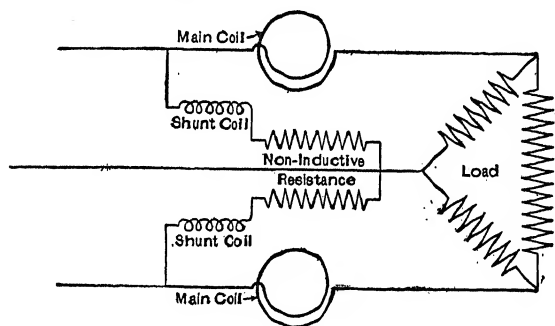
FIG. 28.—Connections of Kelvin's Three-Phase Wattmeter.

Lord Kelvin has modified his single-phase balance watt-
meter to make it suitable for three-phase measurement.
It has two sets of fixed coils mounted on the opposite sides
of an ebonite or marble slab. Suspended inside these coils
are two sets of movable coils carried on the same spindle,
the pointer being also carried on the spindle. The only
opening in the ebonite slab is for the spindle. The fixed
coils are in the two arms of the main circuit, and the shunt
coils are connected across through non-induction resistances,
as shown in Fig. 28. With instruments for low pressures
the inductionless coils are in the case, and for high pressures
they are outside and separate.

The use of the Kelvin quadrant electrometer, or some modification of it, as a wattmeter was suggested almost simultaneously by Profs. Ayrton, FitzGerald and Potier. Maxwell showed that, in the case of the Kelvin electrometer, if A and B are the potentials of the quadrants and C that of the needle, then the deflection of the needle should vary as

$$(A-B)\left(C - \frac{A+B}{2}\right).$$

It was shown, however, by Dr. J. Hopkinson, and also by Profs. Ayrton and Perry, that the above law is not fulfilled by every instrument of the quadrant type, but that electrometers can be constructed which do obey it. A form of Kelvin quadrant electrometer has been devised by Messrs. Ayrton, Perry and Sumpner (*see* Fig. 29), which strictly obeys the above law of deflection. Assuming that, for any particular instrument, the law has been verified, we may use it as a wattmeter as follows :—Let AB be an inductive circuit through which an alternating current can be set flowing. It is desired to measure the power taken up in AB. Join in series with AB an inductionless resistance BC, and connect the quadrants of the electrometer to the terminals A and B. Then take two readings, one with the needle joined to B and one with it connected to C. Observe the deflections in each case: call them θ and θ'. Then we have

$$\theta = K\left\{ (V_A - V_B)\left(V_B - \frac{V_A + V_B}{2}\right) \right\},$$

$$\theta' = K\left\{ (V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right) \right\},$$

where K is an instrumental constant and V_A, V_B, V_C signify the potentials at the points A, B, C respectively. Subtracting the equations, we have

$$\frac{\theta - \theta'}{K} = (V_A - V_B)(V_B - V_C).$$

Now $(V_A - V_B)$ is the fall of potential down the inductive

circuit, and $V_B - V_C$ is the fall of potential down the non-inductive circuit, and is proportional to the current through
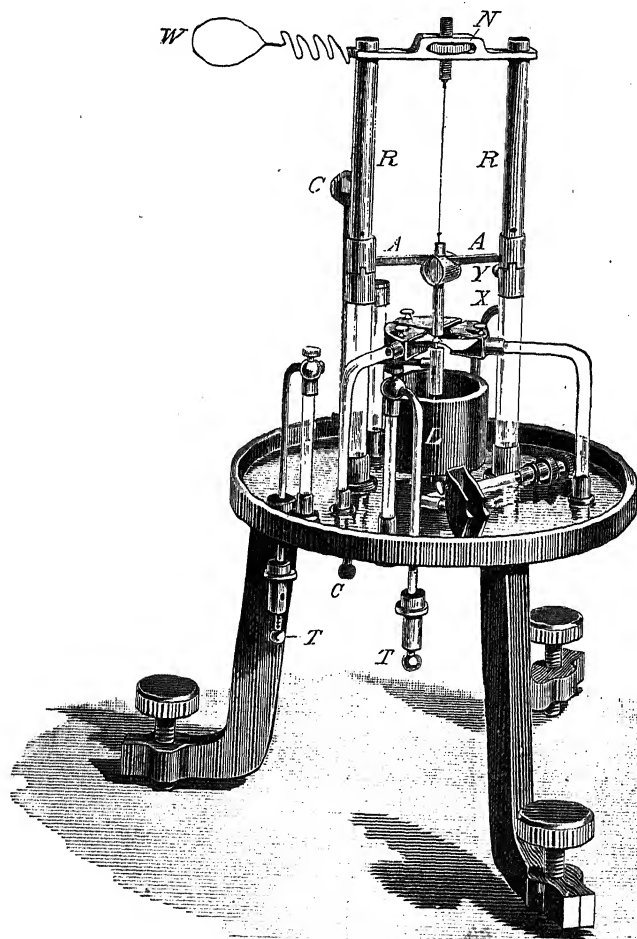
Fig. 29.

the inductive circuit. The difference of the readings, viz., $\theta - \theta'$, is proportional, therefore, to the mean value of this

product, assuming that the free periodic time of the needle is large compared with that of the alternating current.

It has been suggested by MM. Blondlot and Curié that the power may be obtained by a single reading with a double electrometer containing two needles and two pairs of quadrants, the quadrants being connected to the terminals A and B and one "needle" to B and the other needle to C. A careful verification of the right to use the Kelvin quadrant electrometer in the above manner as an alternating-current wattmeter has been given by Prof. E. Wilson (see *Proc.* Roy. Soc., London, Vol. LXII., 1898, p. 356), but the experimentalist employing any particular instrument should independently verify for himself its obedience to the theoretical law. Mr. G. L. Addenbrooke has devoted particular attention to the improvement of the quadrant electrometer for alternating current measurement, and has arranged a convenient form of electrostatic wattmeter for this purposes. (*See* abstract of a Paper read at the International Congress of Electricity in Paris, 1900, *The Electrician*, Vol. XLV., p. 901.)

The various forms of continuously-recording wattmeters or watt-hour-meters employed as house-meters will be considered in the chapter on "Electric Quantity and Energy Measurement."

§ 13. **Wattmeter Testing.**—Owing to the various disturbing actions which tend to render wattmeter readings incorrect, no prudent experimentalist will engage in a course of experiments with any one particular instrument without previously making a careful examination of its behaviour under various conditions. Let us assume, in the first place, that the wattmeter is of the dynamometer type and that it is to be employed in measuring continuous-current power with the series coil joined in series with the power-absorbing circuit and the shunt coil joined across the ends of a circuit consisting of the power-absorbing circuit and the series coil. The wattmeter should be placed upon a turntable so as to move

it round in azimuth into different positions. The first experiment which should be made is to pass the normal current through the shunt coil, no current going through the series coil; and notice should be taken whether the interruption or reversal of this current through the shunt affects its position when freely suspended and movable. If this is the case, then the wattmeter must be turned round its vertical axis and tests made in different positions to discover if the displacement is due to the earth's magnetic field. If so, an orientation can be found in which the position of the shunt coil is not disturbed by reversing the current through it. If no such position of the wattmeter can be found, then the disturbance may be due to currents flowing in neighbouring wires, and these should be looked for and removed. The next step is to measure the resistance of the series coil and calculate the C^2R loss in it when the current to be passed through it is used.

As already explained, the torsion which has to be applied to the wattmeter head to bring the movable shunt coil back to its zero position is proportional to the power taken up in the power-absorbing resistance, plus the power absorbed in the series coil when the connections are made as above described. It is necessary, therefore, to ascertain what proportion the power absorbed in the series coil bears to that taken up in the circuit under test, in order that the value of the correction may be estimated. Again, it must not be taken for granted without investigation that the twists given to the wattmeter head as measured in angular displacements of the head are proportional to the power taken up in the power-absorbing circuit; but special experiments must be made with different power absorptions in a circuit under test, and the quotient of true power taken up by the wattmeter head displacement taken. This quotient should be constant throughout the range of currents within which the wattmeter will be used. This test is best made by passing currents through the wattmeter coils, the values of which are

independently observed, and also the twist θ given to the wattmeter head to restore the movable coil to its zero position. Then, if C and c are these currents, we have to prove that, for the particular instrument in question, $Cc/\theta =$ a constant, in order that the wattmeter may be relied upon to give consistent results when used to measure power.

In the case of a wattmeter to be used with alternating currents, an additional examination has to be made into its construction and behaviour before accepting its readings as valid. Assuming it to be constructed in accordance with the rules already laid down, we have to ascertain whether its power readings when taken on a circuit of small power factor are in agreement with those taken on circuits of large power factor. For instance, let it be supposed that a wattmeter is to be employed for measuring the power taken up in concentric cables when employed with alternating currents. In this case the power-absorbing circuit, which is the dielectric of the cable, has a small power factor. We have to assure ourselves first that any wattmeter reading taken on this circuit means the same in true power absorption as when the same indication is found on a power-absorbing circuit of high power factor. A cautious electrician will not take this for granted, knowing, as he should do, that it is quite possible for errors of 300 or 400 per cent. to be made in evaluating by means of a dynamometer wattmeter the power absorption in the case of a small power factor circuit. In order to test the behaviour and trustworthiness of a wattmeter when employed with alternating currents it is essential, therefore, to possess a power-absorbing circuit of known small power factor and known power absorption with various voltages on its ends, and to compare the wattmeter readings taken on this circuit with those taken on a practically inductionless power-absorbing circuit made as already described.

The question arises, how is such an inductive circuit to be made? The following principles will guide the construction

of a useful form of inductive resistance for wattmeter testing. It consists in making an ironless choking coil in the form of a Gauss coil of maximum inductance. If a coil of insulated wire is made on a former of circular shape, and if the shape of the cross section of the coil is a square having a length of side equal to a, then it was shown by Maxwell (*see* "Electricity and Magnetism," Vol. II., p. 316) that the coil will have a maximum inductance if the mean diameter of the coil $= D = 3\cdot7a$.

Let a core, therefore, be prepared formed by placing cheeks on the sides of a circular disc of wood so as to form a square channel in which insulated wire may be wound, and let the square channel be fitted up with double silk or cotton-covered copper wire well shellaced or paraffined. Let the diameter of the circular former and the depth and breadth of the square channel be such that the outside diameter of the circular coil of wire formed in it is D_0 and the inner diameter D_1, and these are so chosen that $3\cdot7\left(\dfrac{D_0 - D_1}{2}\right) = \left(\dfrac{D_0 + D_1}{2}\right)$, or $\dfrac{D_0}{D_1} = \dfrac{47}{27}$; also let the width of the coil sideways or parallel to its axis be equal to $\dfrac{D_0 - D_1}{2}$. The coil will then have the proportions which will give it the maximum inductance for the quantity of wire used according to the Gauss and Maxwell rules.

The inductance of this coil may be calculated approximately as follows :—It is shown in the "Treatise on Electricity and Magnetism" by Mascart and Joubert (*see* Atkinson's English translation, Vol. II., pp. 152 and 153) that for a circular coil of wire of N turns and mean radius a, having a rectangular section of width $2b$ and depth (radial) $2c$, the inductance L is given by the following expression :—

$$L = 4\pi a N^2 \left[\left(1 + \frac{3b^2 + c}{24c^2} \right) \log_e \frac{4a}{\sqrt{b^2 + c^2}} - \lambda_1 + \frac{b^2}{4\mu_1 a^2} \right],$$

where λ_1 and μ_1 are certain functions of the ratio $\dfrac{c}{b}$ which are tabulated by Mascart and Joubert.

The deduction of the above formula from first principles would occupy too much space to give it here in full, and the reader must therefore be referred to the treatise of Mascart and Joubert for details.

Suppose, now, that the axial width of the coil section is equal to the radial depth, and that the proportions for maximum inductance are fulfilled. That is, let $b=c=\dfrac{a}{3\cdot7}$. Corresponding to $c/b=1$, Mascart and Joubert give the values $\lambda_1=0\cdot84834$ and $\mu_1=0\cdot8162$.

Making, therefore, the above substitutions in the general expression for L, we have

$$L=4\pi a N^2\left[\left(1+\frac{4}{(3\cdot7)^2\cdot24}\log\frac{3\cdot7\times4}{\sqrt2}-0\cdot8483+0\cdot0224\right]\right.$$

$$=4\pi a N^2\left[2\cdot3784-0\cdot8259\right]=4\pi a N^2\times1\cdot5525,$$

or
$$L=3\cdot1\pi D N^2,$$

where D = mean diameter of the coil and N = the total number of turns of wire.

Now let the total length of wire on the coil $=l$, then

$$l=\pi N D,$$

hence
$$L=3\cdot1lN.$$

Let d' be the diameter (over all) of the covered wire used to make the coil; then the length of the side of the square section is equal to $d'\sqrt{N}$. Accordingly, by the Gauss relation,

$$D=3\cdot7d'\sqrt{N},$$

and
$$\pi D N=l=\text{total length of wire.}$$

Substituting these in the above expression for L, viz., $L=3\cdot1lN$, we have, finally,

$$L=36d'N^2\sqrt{N}.$$

This last expression gives us, therefore, a very simple formula for calculating the self-induction of a square-sectioned circular coil made with the Gauss proportions

for maximum inductance, **viz.,** 3·7 times side of section
=mean diameter of coil.

For example, let a coil of maximum inductance be made
with cotton-covered copper wire having an over-all diameter
of 2·5mm. and, say, 1,600 turns. Calculate the inductance
and specify the form of the coil. Here

$$d'=0.25 \text{ cm.,}$$
$$N=1,600,$$
$$\sqrt{N}=40.$$
$$\therefore \ D=3.7 \times 0.25 \times 40 = 37\text{cm.,}$$

and side of the section of the coil = 10cm.

Hence the coil must have an outer diameter of 47cm.,
an inner diameter of 27cm., a thickness (axially and
radially) of 10cm., and its inductance L is such that

$$L'=36 \times \tfrac{1}{4} \times (1,600)^2 \times 40$$
$$=36 \times 16 \times 16 \times 10^5 = 921,600,000\text{cm.}$$
$$=0.9216 \text{ henry.}$$

The resistance of the coil can easily be approximately
calculated as follows:—Let ρ denote the specific resistance of
copper in C.G.S. units. At 0°C for hard-drawn high conduc-
tivity copper ρ has a value near 1,600. The resistance R
of the coil of wire is equal to $\frac{4\rho L}{\pi d^2}$, where d is the diameter
of the copper wire and l is its length. But $l = \pi DN$, and
$D = 3.7d' \sqrt{N}$ for the maximum inductance coil; hence,

$$R = \frac{6,400 \ DN}{d^2} = \frac{23,680 \ N\sqrt{N}d'}{d^2}.$$

This value must, however, be increased by at least 15 per
cent. to allow for the temperature rise in the resistance.

The difference between d' and d is about 0·01 of an inch,
or 0·0254 of a centimetre, for double cotton-covered wire,
and 0·005 of an inch, or 0·0127 of a centimetre, for double
silk-covered wire. Accordingly, when we are given the
diameter, d', of the covered wire, its total resistance can
easily be calculated.

Having, then, the value of L and R for the coil, we can calculate the impedance ($\sqrt{R^2+p^2L^2}$) of the coil for simple periodic alternating currents of a frequency $n=2\pi/p$, and find at once the current which will flow through the coil under a given alternating voltage V (R.M.S. value). We then obtain at once the true power W expended in the coil, since it is equal to V^2/R, and also the apparent power or volt amperes, since this is equal to $V^2/\sqrt{R^2+p^2L^2}$ and the power factor $R/\sqrt{R^2+p^2L^2}$.

We are thus able to design an inductance coil with any desired power factor and power absorption and to use it to check a wattmeter. The above formulæ are, however, only to be used to give an approximate notion of the power factor and power absorption; the true values for the coil in question can best be obtained experimentally as described below.

One point of considerable importance to which attention must be directed is the energy waste which arises from eddy electric currents set up in solid copper wire when above a certain diameter. If a coil of insulated wire is made, say, of size as large as No. 14 S.W.G., and if this wire is traversed by an alternating current, the field of each turn embraces and cuts that of other turns and sets up in the mass of the copper eddy currents which dissipate energy. This is in addition to the proper C^2R loss due to the ohmic resistance of the circuit. As this energy loss is not easily predicted and taken into account, it is necessary to make the conductor of such an ironless inductance coil of stranded wire. Generally speaking, it will not be necessary to insulate each strand. The film of dirt or grease on each constituent wire is usually sufficient to stop the circulation of these eddy currents. If, however, the wire is stranded, then its inductance per unit of length is not quite the same as that of a round-sectioned solid wire of the same cross-sectional area. Stranding the wire reduces the inductance because it increases the average distance of all the filamentary elements into which we may conceive the currents divided.

Hence the calculations made by the above formulæ for the inductance and resistance of the copper circuit do not quite exactly give the required quantities. If the wire is solid, then the C^2R waste in the coil with alternating currents will be under-estimated; and if the wire is stranded, then the inductance will be rather over-estimated when calculated by the above-given rules.

As an illustration, however, of the use of the above formulæ let us for the moment neglect the copper eddy current loss, and proceed to design an ironless choking coil having a power absorption of $\frac{1}{2}$ H.P., or 375 watts, when submitted to an alternating electromotive force of 2,000 volts at a frequency of 100, the power factor of the coil to be 2 per cent.

After a few trials we find the specification to be as follows:—Take No. 14 double cotton-covered copper wire, say 0·2cm. diameter, use 1,225 turns, and make it in the form of a square-sectioned circular Gauss coil. Then

$$d = 0\text{·}2\text{cm.,} \quad N = 1,225, \quad \sqrt{N} = 35.$$

The inductance $L = 36 \times 0\text{·}2 \times 35 \times (1,225)^2$
$$= 378,157,500\text{cm.}$$
$$= 0\text{·}378 \text{ henry.}$$

The resistance R in ohms at 0°C
$$= 23,680 \times 1,225 \times 35 \times 5 \times 10^{-9} = 5\text{·}076 \text{ ohms,}$$
or, say, 6 ohms when hot.

Then $\qquad p = 2\pi n = 200 \times 3\text{·}1415 = 628\text{·}3,$

the reactance $= Lp = 235\text{·}6$ ohms at 0°C,

the impedance $= \sqrt{R^2 + p^2 L^2} = \sqrt{55,508\text{·}7 + 25\text{·}7} = 235\text{·}7$ ohms.

Hence, under an alternating electromotive force of 2,000 volts the current in the coil is $2,000/235\text{·}7 = 8\text{·}48$ amperes.

The true power absorbed by the coil at 2,000 volts is $(8\text{·}48)^2 \times 5\text{·}076 = 365\text{·}3$ watts, if we neglect the increase in resistance due to rise in temperature.

If the current is not kept on more than a few moments, the true power absorption will not exceed 400 watts.

The volt-amperes, or apparent power, taken is equal to $2,000 \times 8\cdot48 = 16,960$, and the power factor$= 400/16,960 = 0\cdot023$.

The covered wire will have a diameter, d', equal to $0\cdot225$cm.

Then as to the size of the coil. The mean diameter$= D = 3\cdot7 \times 0\cdot225 \times 35 = 29\cdot1$cm., and the side of the square section $= a = 0\cdot225 \times 35 = 7\cdot9$cm. Hence the coil must have an inside diameter of $21\cdot2$cm., an outside diameter of 37cm., and a thickness of $7\cdot9$cm. It will contain 1,225 turns of No. 14 double cotton-covered copper wire. The length of the wire will be $\pi DN = \dfrac{22}{7} \times 29\cdot1 \times 1,225$cm.$= 112,035$cm.$= 1,250$yd. nearly. The weight will be about 80lb. Hence 80lb. of double cotton-covered No. 14 wire will be required to make the coil.

As a matter of practical construction, it would not be advisable to wind up this weight of cotton-covered wire in one coil, and use it on a high-voltage circuit, because the current would probably jump from layer to layer and destroy the insulation. The coil should be wound like an ordinary induction or spark-coil secondary circuit in a set of side-by-side coils, or sections, insulated from each other by thin ebonite or micanite discs. Before or during winding, the covered wire should be well paraffined, and it is an advantage to keep a coil of this description immersed in insulating oil during experiments with high-tension currents.

Moreover, as above explained, in practice the wire of which the coil is made should be stranded wire, using a strand the constituent wires of which are not larger than No. 30 S.W.G. Hence, instead of using solid No. 14 S.W.G. copper wire, a stranded 36/30 should be employed.

Assuming the copper wire to be sufficiently stranded, the actual power absorbed and the power factor at any voltage can, however, best be determined experimentally as follows:— Provide a means for regulating the alternating voltage by very small steps—say, for instance, by introducing a variable choking coil into the supply circuit, or by varying the

exciting current of the service alternator very gradually by a carbon plate rheostat. Then provide also a supply of continuous current from secondary batteries. Place in series with the inductance coil under test a hot-wire ammeter suitable for use with both continuous and alternating currents, and have at hand calibrated voltmeters for reading accurately the alternating and continuous voltages. Then begin by applying a measured continuous voltage to the inductance coil sufficient to create in it a current equal to that which it will take with the alternating voltage. In the case of the above-described coil, since the resistance is about 5 ohms, a continuous voltage of 40 volts will create a current of 8 amperes. Measure very carefully this voltage and current. As soon as the currents become constant, or very nearly so, switch off the continuous voltage and apply an alternating voltage sufficient to maintain the same heating current in the coil. In this case, since the impedance is nearly 250 ohms, about 2,000 alternating volts (R.M.S. value) will be required. Measure this alternating voltage carefully. Then, if the ventilation or cooling of the coil is rapid enough to enable these voltages to be measured when the current is practically the same in the two cases, we have at once the power factor and the true power absorbed in the coil. For, if V_c is the continuous voltage required to maintain a current, A', through the coil, and V_A is the equivalent alternating voltage, then V_c/V_A is the power factor of the coil, and AV_c is the true power absorbed by the coil under an alternating voltage V_A and an apparent power absorption AV_A.

The inductance coil so made, then, becomes a means of checking a wattmeter. For this purpose we must provide in addition a nearly inductionless resistance, taking up at the 2,000 volts a true power equal, or nearly equal, to that taken up by the inductance coil. For the above case, if we make a resistance coil by winding on a wooden frame non-inductively a length of 1,000 yards of No. 36 S.W.G. cotton-covered platinoid wire, we shall have a resistance

of about 12,000 ohms, which, if properly ventilated, will carry without sensible heating one-sixth of an ampere, or the current it will take under an alternating voltage of 2,000 volts. The power absorbed will then be about 333 watts.

We have, then, two resistances, one, the above platinoid resistance, taking up 333 watts or so at 2,000 volts, and having a power factor nearly unity, and the previously-described inductance coil, taking up 360 watts or so, and having a power factor about 0·02. Proceed then to calibrate the wattmeter under test by means of the inductionless platinoid resistance, measuring the pressure with an alternating electrostatic high tension voltmeter, and the current through it with a hot-wire ammeter or by a resistance and associated electrostatic low-reading voltmeter; in this way obtain the constant of the wattmeter on the high power-factor resistance. Then, employing the same instruments on the inductance coil, measure the power taken up calculated out by means of the wattmeter constant observed as above. If this power reading does not agree with the true power absorbed by the inductance coil at that voltage, as already determined by the use of the continuous and alternating currents, then something is wrong with the wattmeter, and it cannot be trusted when used with small power factor circuits.

An inductance coil made as described, and built up in sections so that one or more sections can be used in series as required, is a very useful implement in a laboratory in which cable testing is being conducted. For, if it is desired to measure the true power absorption in the dielectric of a cable under alternating voltage, then, since the power factor of the dielectric circuit of the cable is small, its measurement directly by a wattmeter becomes a matter of difficulty. But the difficulty is reduced if an inductance coil of suitable power factor is joined, either in parallel or in series, with the dielectric of the cable or condenser, as suggested by Prof.

Ayrton and Mr. Mather,[*] and the power factor of the two together, cable and coil, becomes greater than either of them separately by reason of the fact that the current leads on the electromotive force in the case of capacity and lags on it in the case of inductance. In order to know approximately how much we may expect to improve the power factor, we must, however, have a rough knowledge of the power factor and current taken in each case separately. Thus, suppose the above-described inductance coil, having a power factor of 0·02, and taking a current of 8·5 amperes at 2,000 volts, is joined in parallel with a length of 5 miles of a cable having a dielectric power factor of 0·02 and a capacity of 0·3 microfarads per mile. Then, under simple periodic currents, the capacity current of the cable (I) would be equal to $CpV/10^6$, where C is the total capacity in microfarads, $p = 2\pi$ times the frequency, and V is the voltage. The reader should note that this formula cannot be used to calculate the condenser current when the capacity, inductance and frequency have such values as to create electric resonance in the circuit.[†] If, however, resonance is absent, then, since $n = 100$ and $V = 2,000$, we have

$$I = \frac{5 \times 3 \times 2,000 \times 628\cdot3}{10^7}$$

$$= 1\cdot885 \text{ amperes.}$$

Hence, to obtain the best result in augmenting the power factor it would be desirable to couple the dielectric of this cable in parallel with an inductance coil having a larger number of turns (about twice as many) than the one above specified, in order that the coil current under 2,000 volts should be about equal in magnitude and opposite in phase, as regards the electromotive force, to the capacity current of the cable. With the assistance of the previously-

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[*] *See* Prof. Ayrton's remarks in *The Electrician*, January 18, 1901, Vol. XLVI., p. 476 ; also January 25, Vol. XLVI., p. 512 ; and Mr. Mather, *The Electrician*, February 22, Vol. XLVI., p. 667.

[†] *See* Mr. T. Mather, *Electrical Review*, May 31, 1901 ; also Vol. II. of this Handbook, in the chapter on "Measurement of Capacity."

explained principles, the reader will have no difficulty in doing this for himself.

If an ironless inductance coil is joined in parallel with a condenser, and the two are supplied with alternating current from an alternator at constant potential, there is a certain value of the inductance which will, when associated with a given condenser, make the current coming out of the alternator a minimum. This value may be ascertained as follows:—Let i be the instantaneous value of the current coming out of the alternator, i_2 that of the current into the condenser, and i_1 that through the inductance. Let C be the capacity of the condenser and L and R the inductance and resistance of the coil. Then, if v is the instantaneous value of the alternator voltage, we have as fundamental equations

$$i_2 = i - i_1,$$

$$C\frac{dv}{dt} = i_2,$$

$$L\frac{di_1}{dt} + Ri_1 = v.$$

Hence, assuming a simple periodic variation of i, we have

$$CL\frac{d^2i_1}{dt^2} + CR\frac{di_1}{dt} + i_1 = i = I \sin pt.$$

Then, if $i = I \sin pt$, we shall have $i_1 = I_1 \sin (pt - \theta)$, because there will be a difference of phase between i_1 and i. Accordingly, we obtain, by substitution,

$$(1 - CLp^2)I_1 \sin (pt - \theta) + CRpI_1 \cos (pt - \theta) = I \sin pt,$$

and hence, by a well-known transformation,

$$I_1\sqrt{(1 - CLp^2)^2 + C^2R^2p^2} = I.$$

But $$I_1 = V/\sqrt{R^2 + p^2L^2};$$

therefore $$I = \frac{V\sqrt{(1 - CLp^2)^2 + C^2R^2p^2}}{\sqrt{R^2 + p^2L^2}}.$$

We have, then, to find what value of L will make I a minimum. Differentiate therefore the last expression with

respect to L and equate to zero, and after some simple reductions we have as a result the equation

$$C(CLp^2 - 1)(R^2 + p^2L^2) = L\{(1 - CLp^2)^2 + C^2R^2p^2\},$$

which reduces to

$$L^2 - \frac{1}{Cp^2}L = \frac{R^2}{p^2}.$$

Hence the solution of the above quadratic equation gives us the value of L which makes I a minimum. It is

$$L = \frac{1}{2Cp^2} \pm \sqrt{\frac{1}{4C^2p^4} - \frac{R^2}{p^2}},$$

or

$$L = \frac{1 \pm \sqrt{1 - 4C^2R^2p^2}}{2Cp^2}.$$

In this last form the solution of the problem was given by Prof. Ayrton in a discussion at the Institution of Electrical Engineers in 1901. Given a condenser or cable of capacity C farads subjected to an alternating current of frequency $n = p/2\pi$, the condenser being shunted by an inductance L of resistance R; the value of L, calculated from the above equation, is that which will make the total current taken by coil and condenser a minimum.

The reader will find some valuable information on watt-meters with iron cores, as well as upon wattmeter testing, in a Paper by Dr. Sumpner on "New Iron-Cored Instruments for Alternate Current Working" (*Journal* Inst. Elec. Eng., Vol. XXXVI., p. 421, 1905). Also a description of a wattmeter with an iron core by Dr. Drysdale will be found in *The Electrician*, Vol. LV., p. 472, and one by C. Olivetti in *The Electrician*, Vol LIV., p. 1050.

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